行政院國家科學委員會專題研究計畫報告

****

匯率風險與跨時避險的研究(1/2)

****

計畫類別:整合型計畫

計畫編號:NSC 90-2416-H-002-003

執行期間:90 年 8 月 1 日至 91 年 7 月 31 日

計畫主持人:洪茂蔚

處理方式:可立即對外提供參考

執行單位:國立台灣大學國際企業學系

中華民國九十一年五月二十一日

ABSTRACT

We develop an intertemporal IAPM that prices world market hedging risk and exchange hedging risk in addition to world market risk and exchange risk. We extend Campbell's (1993) model to an international framework in which inflation rate is time-varying. We employ a recursive preference, which separates investors' attitudes towards risk from their willingness to substitute future consumption for present consumption in pricing international assets. Our model prices exchange risk and exchange hedging risk in addition to world market risk and world market hedging risk. While the world market hedging risk is the covariance of asset returns with news about the discounted value of all future world market returns, the exchange hedging risk is the covariance of asset returns with news about the discounted value of all future currency returns.

I. Introduction

Several theoretical international asset pricing models (IAPMs) focus on market risk and currency risk. Well known examples include, Solnik (1974) and Adler and Dumas (1983). It is widely accepted that we should also consider intertemporal nature of the problem as suggested by Merton (1973) and Stulz (1981). Indeed, in the international setting, if investors hedge their exposure to anticipated variation in the investment opportunity set, both the market hedging risk and the currency hedging risk should be priced in addition to the market and currency risks. Hence, we develop a theoretical model that prices world market hedging risk and exchange hedging risk in addition to world market risk and exchange risk. This allows us to explicitly separate exchange risk from intertemporal hedging risk.

Campbell (1993) develops a domestic asset pricing model in which investors are assumed to be endowed with Kreps-Porteus utility and consumption is substituted out from the model. He demonstrates that the conditional covariance of any asset return with consumption growth can be written in terms of conditional covariances with the return on the market and revisions in expectations of future returns on the market. We extend Cambpell's (1993) model to an international framework in which inflation rate is time-varying. We employ a recursive preference, which separates investors' attitudes towards risk from their willingness to substitute future consumption for present consumption in pricing international assets. Our model prices exchange risk and exchange hedging risk in addition to world market risk and world market hedging risk. While the world market hedging risk is the covariance of asset returns with news about the discounted value of all future world market returns, the exchange hedging risk is the covariance of asset returns with news about the discounted value of all future currency returns.

II. Portfolio Choice in an International Setting

In this section, we consider the problem of optimal consumption and portfolio allocation in a unified world capital market without taxes or transactions costs. Investors' preferences are assumed to be nationally heterogeneous and there is the same menu of assets for every investor in all countries. That is to say, there does not exist a representative agent in the world, but there is a representative agent in each domestic country. Therefore, we can first deal with a representative agent in each country, and then aggregate to obtain an international asset pricing model. We consider a world of L+1 countries and a set of S equity securities. All returns are measured in the L+1[st] country's currency (numeraire) in excess of the risk free rate. To facilitate discussion, we define the following terms:

π_t : the price level at time t.

$R_{i,t}$: the real return for security i at time t.

$R_{i,t}^N$: the nominal return for security i at time t.

C_t : the real consumption at time t.

C_t^N : the nominal consumption at time t.

$R_{m,t}$: the real return for market portfolio at time t.

$R_{m,t}^N$: the nominal return for market portfolio at time t.

In this setup, the optimal solution can be obtained by specifying a real pricing kernal, M_{t+1} :

$$1 = E_t(R_{i,t+1}M_{t+1}), \qquad (1)$$

where E_t is the expected value function conditional on the information available to the investor at time t.

Note that under perfect foresight the real return links the nominal return to inflation through the Fisher parity equation,

$$R^N_{i,t+1}\frac{\pi_t}{\pi_{t+1}} = R_{i,t+1}, \tag{2}$$

we obtain the pricing formula under time-varying inflation rate,

$$1 = E_t(R^N_{i,t+1}\frac{\pi_t}{\pi_{t+1}}M_{t+1}). \tag{3}$$

Furthermore, investors are assumed to have a Kreps-Porteus utility. Following Epstein and Zin (1989), it can be shown that the pricing kernel has the following form:

$$M_{t+1} = \beta\{(\frac{C_{t+1}}{C_t})^{-1/\sigma}\}^{\theta}\{R_{m,t+1}\}^{\theta-1}, \tag{4}$$

where the parameter β is the agent's subjective time discount factor , θ is defined as $\theta = (1-\gamma)/[1-(1/\sigma)]$, and γ can be interpreted as the Arrow-Pratt coefficient of relative risk aversion. It can also be shown that σ measures the elasticity of intertemporal substitution. For instance, if the agent's coefficient of relative risk aversion, γ, is greater (smaller) than the reciprocal of his elasticity of intertemporal substitution, $1/\sigma$, then he prefers early (late) resolution of uncertainty. If γ is equal to $1/\sigma$, the agent's utility becomes an isoelastic von Neumann-Morgenstern utility and he is indifferent to the timing of the resolution of uncertainty.

Plugging equation (4) into equation (3), we obtain

$$1 = E_t\left[R^N_{i,t+1} \frac{\pi_t}{\pi_{t+1}} \beta\{[\frac{C^N_{t+1}/\pi_{t+1}}{C^N_t/\pi_t}]^{-1/\sigma}\}^{\theta}\{R^N_{m,t+1}\frac{\pi_t}{\pi_{t+1}}\}^{\theta-1}\right]. \qquad (5)$$

Equation (5) is the basic pricing formula. Based on equation (5), we will follow Campbell (1993) to substitute consumption out of the pricing formula.

When $\gamma = 1/\sigma$, the Euler equations of the time additive expected utility model are also obtained under random inflation assumption:

$$1 = E_t\{R^N_{i,t+1} \frac{\pi_t}{\pi_{t+1}} \beta[\frac{C^N_{t+1}/\pi_{t+1}}{C^N_t/\pi_t}]^{-1/\sigma}\} \qquad (6)$$

Another special case is the logarithmic risk preference where $\gamma = 1/\sigma = 1$. Then, the Euler equations under fixed inflation are same as the Euler equations under random inflation and can be written in two algebraically identical functional forms:

$$1 = E_t\{R^N_{i,t+1}\beta[\frac{C^N_{t+1}}{C^N_t}]^{-1}\} \qquad (7)$$

or

$$1 = E_t\{R^N_{i,t+1}/R^N_{m,t+1}\} \qquad (8)$$

In this case, the parameter σ governing intertemporal substitutability can not be identified from these equations. Hence, under logarithmic risk preferences there is no difference between Euler equations of the non-expected utility model and the expected utility model.

If we assume that asset prices and consumption are jointly lognormal or alternatively if we assume that asset prices and consumption are conditional homoskedastic and use a second order Taylor expansion, the log-version of the real Euler equation (5) can be represented as:

$$0 = \theta \log \beta - (\theta / \sigma) \underset{t}{E} \Delta c_{t+1} + (\theta - 1) \underset{t}{E} r_{m,t+1} + \underset{t}{E} r_{i,t+1} + \theta \, ((1/\sigma) - 1) \underset{t}{E} \Delta \pi_{t+1} \tag{9}$$

$$+ \frac{1}{2}[(\theta/\sigma)^2 V_{cc} + (\theta - 1)^2 V_{mm} + V_{ii} - 2(\theta/\sigma)(\theta - 1) V_{cm} - 2(\theta/\sigma) V_{ci} + 2(\theta - 1) V_{im}]$$

$$+ \frac{1}{2}\{[(\theta \, ((1/\sigma) - 1)]^2 V_{\pi\pi} - 2\theta^2 (1/\sigma)((1/\sigma) - 1) V_{\pi c} + 2\theta(\theta - 1)((1/\sigma) - 1) V_{\pi m} + 2\theta((1/\sigma) - 1) V_{\pi i}\}$$

where lowercase letters are used for logs and V_{cc} denotes $Var_t(c_{t+1})$, V_{jj} denotes $Var_t(r_{j,t+1}) \forall j = i,m$, V_{cj} denotes $Cov_t(c_{t+1}, r_{j,t+1}) \forall j = i,m$, V_{im} denotes $Cov_t(r_{i,t+1}, r_{m,t+1})$, $V_{i\pi} = Cov_t(r_{i,t+1}, r_{\pi,t+1})$, and $r_{\pi,t+1} = d\ln(\pi_{t+1}) = \frac{d\pi_{t+1}}{\pi_{t+1}}$.

Replacing the asset i by market portfolio and rearranging, we obtain the relationship between expected consumption growth and the expected return on the market portfolio:

$$\underset{t}{E} \Delta c_{t+1} = \mu_m + \sigma \underset{t}{E} r_{m,t+1} + (1 - \sigma) \underset{t}{E} r_{\pi,t+1} \tag{10}$$

where $\mu_m = \sigma \log \beta + \frac{1}{2}[(\theta/\sigma) V_{cc} + \theta\sigma V_{mm} + 2(1 - \sigma)\theta((1/\sigma) - 1) V_{\pi\pi}]$

$$- \frac{1}{2}[2\theta V_{cm} + 2\theta((1/\sigma) - 1) V_{\pi c} - 2\theta(1 - \sigma) V_{\pi m}]$$

When the second moments are conditional homoskedastic, equation (10) indicates that consumption growth is linearly related to the expected world market return and expected inflation. In addition, the coefficients of these two variables add up to one.

When we subtract the risk free version of (9) from the general version, we obtain:

$$\underset{t}{E} r_{i,t+1} - r_{f,t+1} = -\frac{V_{ii}}{2} + (\theta/\sigma)V_{ic} + (\theta - (\theta/\sigma))V_{i\pi} + (1-\theta)V_{im} \tag{11}$$

where $r_{f,t+1}$ is a log riskless nominal interest rate. If inflation is fixed or known ex-ante, this result is the same as Campbell (1993). Equation (11) shows that the expected excess log return on an asset is a linear combination of its own variance, which is produced by Jensen's inequality, and by a weighted average of three covariances. The weights on the consumption, inflation and market are θ/σ, $(\theta-(\theta/\sigma))$ and $(1-\theta)$, respectively which sum up to 1. This is one of the most important differences between Campbell's model and our random inflation model.

If the objective function is a time-separable power utility function, a real functional form of the loglinear version consumption CAPM pricing formula can be obtained:

$$\underset{t}{E} r_{i,t+1} - r_{f,t+1} = -\frac{V_{ii}}{2} + (1/\sigma)V_{ic} + (1-(1/\sigma))V_{i\pi} \tag{12}$$

The weights on the consumption and inflation are $1/\sigma$ and $(1-(1/\sigma))$, respectively that also sum up to one. However, when the coefficient of relative risk aversion $\gamma = 1$, then $\theta = 0$, and the model can be collapsed into the real functional form of the loglinear static CAPM which is the same as the nominal structure of the loglinear static CAPM

III. The Intertemporal International Asset Pricing Model

The representative agent in each country can invest his wealth in N (=L+S) assets which comprise L currencies and S equities. Currencies may be taken to be the nominal bank deposits denominated in the non-numeraire currencies. In every country, the representative agent's dynamic budget constraint under time-varying inflation can be written as:

$$\frac{W_{t+1}}{\pi_{t+1}} = R^N_{m,t+1}\frac{\pi_t}{\pi_{t+1}}\left(\frac{W_t}{\pi_t} - \frac{C^N_t}{\pi_t}\right) \tag{13}$$

where W_{t+1} is the investor's nominal wealth at time t+1. The budget constraint in equation (13) is nonlinear because of the interaction between subtraction and multiplication.

Following Campbell (1993), we linearize the budget constraint by dividing equation (13) by W_t, taking log, and then using a first-order Taylor approximation around the mean log consumption/wealth ratio $\log(C/W)$.

$$c_{t+1} - \underset{t}{E}c_{t+1} = r_{m,t+1} - \underset{t}{E}r_{m,t+1} + (1-\sigma)(\underset{t+1}{E} - \underset{t}{E})\sum_{j=1}^{\infty}\beta^j r_{m,t+1+j}$$
$$-(1-\sigma)(\underset{t+1}{E} - \underset{t}{E})\sum_{j=1}^{\infty}\beta^j r_{\pi,t+1+j} \tag{14}$$

Equation (14) implies that unexpected consumption may come from three sources. The first source is the unexpected return on invested wealth today. The second source is the expected future nominal returns that depend on the magnitude of σ. When σ is less than one, an increase (or decrease) in the expected future nominal return increases (or decreases) the unexpected consumption. Conversely, when σ is greater than one, an increase (or decrease) in the expected future nominal return decreases (or increases) the unexpected consumption. The third source is the inflation rate in investor's own country that also depends on the magnitude of σ. When σ is less than one, an increase (or decrease) in the inflation decreases (or increases) the unexpected consumption. Conversely, when σ is greater than one, an increase (or decrease) in the inflation increases (or decreases) the unexpected consumption.

Based on equation (14), the conditional covariance of any asset return with consumption can be rewritten in terms of the covariances with the return on the market and revisions in expectations of future return on the market and inflation:

$$\text{cov}_t(r_{i,t+1}, \Delta c_{t+1}) \equiv V_{ic} = V_{im} + (1-\sigma)V_{ih} - (1-\sigma)V_{ih\pi} \ , \tag{15}$$

where, $V_{ih} = Cov_t(r_{i,t+1}, (\underset{t+1}{E} - \underset{t}{E})\sum_{j=1}^{\infty} \beta^j r_{m,t+1+j})$

$$V_{ih\pi} = Cov_t(r_{i,t+1}, (\underset{t+1}{E} - \underset{t}{E})\sum_{j=1}^{\infty} \beta^j r_{\pi,t+1+j})$$

Substituting equation (15) into equation (11), we obtain an asset pricing model with random price, which is not related to consumption:

$$\underset{t}{E} r_{i,t+1} - r_{f,t+1} = -\frac{V_{ii}}{2} + \gamma V_{im} + (\gamma - 1)V_{ih} + (1-\gamma)V_{i\pi} + (1-\gamma)V_{ih\pi} \tag{16}$$

The only preference parameter that enters equation (16) is the coefficient of relative risk aversion γ. The elasticity of intertemporal substitution σ has disappeared from this asset pricing model. Equation (16) states that the expected excess log return of an asset, adjusted for a Jensen's inequality effect, is a weighted average of four covariances. The covariances are with the return on the market portfolio, with news about future returns on invested wealth, with the return from inflation, and with news about future inflation. Note that this result is different from both the international asset pricing model of Adler and Dumas (1983) and the intertemporal model of Campbell (1993). Since Adler and Dumas use Von Newmann-Morgenstern utility and assume a constant investment opportunity set, their model does not include V_{ih} or $V_{ih\pi}$. On the other hand, Campbell's intertemporal model does not include $V_{i\pi}$ and $V_{ih\pi}$ since it does not deal with the issues of inflation and currency that are of particular relevance in international asset pricing.

We now turn to the problem of aggregation across investors. It is true that different investors may use different sets of information and models to forecast future world market returns and inflation. To obtain the aggregation results, we first add the superscript l to equation (16) to indicate the optimal condition for investor l:

$$\underset{t}{E} r_{i,t+1} - r_{f,t+1} = -\frac{V_{ii}}{2} + \gamma^l V^l_{im} + (\gamma^l - 1)V^l_{ih} + (1-\gamma^l)V^l_{i\pi} + (1-\gamma^l)V^l_{ih\pi} \qquad (17)$$

Next, we multiply equation (17) by η^l, where $\eta^l = 1/\gamma^l$ and then take an average over all investors, where the weights are the investors' relative wealth.

$$\underset{t}{E} r_{i,t+1} - r_{f,t+1} = -\frac{V_{ii}}{2} + \frac{1}{\eta^m} V^m_{im} + (\frac{1}{\eta^m} - 1)\sum_l \omega^l V^l_{ih} \qquad (18)$$

$$+ (1 - \frac{1}{\eta^m})\sum_l \omega^l V^l_{i\pi} + (1 - \frac{1}{\eta^m})\sum_l \omega^l V^l_{ih\pi}$$

where, $\eta^m = (\sum_l W^l \eta^l)/(\sum_l W^l)$ and $\omega^l = \dfrac{(1-\gamma^l)W^l}{\sum_l (1-\gamma^l)W^l}$.

The drawback is that the third, fourth, and fifth terms of (18) are unobservable. In the international setting, they contain the covariances of security i with the heterogeneous investors' rate of market return forecasting, rate of inflation, and inflation forecasting. They are weighted by their wealth and by one minus their relative risk aversion. Because it is impossible to measure each individual's relative risk aversion, we cannot use the wealth and relative risk aversion weighted average rate of market return forecasting, rate of inflation, or inflation forecasting. But if all the individuals of the same country who use the same deflator and use the national wealth weighted average relative risk aversion instead of the individual ones, we have a sum of one term per each country.

Several interesting and intuitive results emerge. First, the international asset risk premium adjusted for one-half its own variance is related to its covariances with: (a) the world market portfolio, (b) the aggregate of innovation in discounted expected future world market returns from different investors across countries, (c) the aggregate of inflation from different countries, and (d) the aggregate of innovation in inflation of different investors, discounted using values for expected future inflation across different countries. The weights are $1/\eta^m$, $1/\eta^m - 1$, $1 - 1/\eta^m$, $1 - 1/\eta^m$, respectively that sum to one.

Second, the international asset is priced without referring to its covariance with consumption growth. Third, because consumption has been substituted out, the coefficient of risk tolerance, η_m, is the only preference parameter that enters the asset pricing model - equation (18). Similar results have been documented by Svensson (1989). They show that when asset returns are independently and identically distributed over time, the coefficient of intertemporal substitution is irrelevant for asset returns.

If we are willing to make further simplifying assumptions, we can obtain a more compact result. Specifically, if investors use identical current and future world market portfolio returns, we can take an average over all investors, where the weights are their relative wealth, and thus obtain a simple version of the intertemporal international asset pricing model:

$$\underset{t}{E} r_{i,t+1} - r_{f,t+1} = -\frac{V_{ii}}{2} + \gamma^m V_{im} + (\gamma^m - 1)V_{ih} + (1-\gamma^m)\sum_l \omega^l V_{i\pi}^l + (1-\gamma^m)\sum_l \omega^l V_{ih\pi}^l \quad (19)$$

where, $\gamma^m = (\sum_l W^l \gamma^l)/(\sum_l W^l)$ and $\omega^l = \dfrac{(1-\gamma^l)W^l}{\sum_l (1-\gamma^l)W^l}$.

In our model, inflation variable includes inflation risk and exchange rate risk. That is because inflation variable is priced under numeraire currency. Therefore, in addition to inflation risk that expresses inflation risk in domestic market, there exists exchange rate risk that indicates the risk of each currency relative to numeraire currency. If domestic inflation is a stable variable, the only random component in π is the relative change in the exchange rate between the numeraire currency and the investor's home currency. Then, $V_{i\pi}^{l}$ is a pure measure of the exposure of asset i to the exchange risk and $V_{ih\pi}^{l}$ is a measure of the exposure of asset i to hedge against the exchange risk of the country in which investor l resides.

Equation (19) also states that the exchange risk is different from the hedging risk. Indeed, if V_{ih} and $V_{i\pi}^{l}$ are significantly large, then their contribution to expected return depends on whether γ^{m} is different from one. This may be the reason why Dumas and Solnik (1995) argue that exchange rate risk premium may be equivalent to intertemporal risk premium. Their conjecture is, however, based on an empirical "horse race" test between an international model and an intertemporal model.

IV. Conclusions

This report develops an intertemporal IAPM. We use a loglinear approximation of the budget constraint to obtain an international asset pricing model without consumption. The model shows that the expected asset return is determined by a weighted average of market risk, market hedging risk, exchange risk, and exchange hedging risk. The weights are related only to relative risk aversion and sum up to one. Our results may be contrasted with the pioneering work of Adler and Dumas (1983). They assumed a constant investment opportunity set, thus their model could not deal with market hedging risk and exchange hedging risk. In our model, the price of market hedging risk is equal to the negative price of exchange risk.

REFERENCES

Adler, M. and B. Dumas. (1983), "International portfolio selection and corporation finance: a synthesis." *Journal of Finance* , 38, 925-984.

Campbell, John Y. (1993), "Intertemporal asset pricing without consumption data." *American Economic Review*, 83, 487-512.

Dumas, B. and B. Solnik (1995), "The world price of foreign exchange risk." *Journal of Finance* , 50, 445-479.

Merton, Robert C. (1973), "An intertemporal capital asset pricing model." *Econometrica*, 41, 867-887.

Solnik, Richard E. (1974), "An equilibrium model of the international capital market." *Journal of Economic Theory*, 8, 500-524.

Stulz, R.M., (1981), "A model of international asset pricing." *Journal of Financial Economics*, 9, 383-406.

Svensson Lars E. O. (1989), "Portfolio choice with non-expected utility in continuous time." *Economics Letters*, 30, 313-317.