Fig. 2 shows the CW light output power against current Fig. 2 shows the $20^{\circ} \mathrm{C}$. The threshold current (density) is characteristics at $20^{\circ} \mathrm{C}$. The threshold current (density) is
$57 \mathrm{~mA}\left(\simeq 3.2 \mathrm{kA} / \mathrm{cm}^{2}\right)$, which is as low as that for a 650 nm $57 \mathrm{~mA}\left(\simeq 3.2 \mathrm{kA} / \mathrm{cm}^{2}\right.$ ), which is as low as that for a 650 nm -
band laser diode with a double-beterostructure (DH structure) band laser diode with a double-heterostructure (DH structure)
[1]. We believe this to be the lowest ever value for a 630 nm [1]. We believe this to be the lowest ever value for a $630 \mathrm{~nm}-$
band AlGaInP laser diode. The maximum CW output power is $\sim 15 \mathrm{~mW}$, and this is limited by the COD (catastrophic optical damage) of this structure. The slope efficiency of a laser diode is as high as $0.4 \mathrm{~W} / \mathrm{A}$ for a 670 nm -band laser diode with a DH structure [8]. The maximum CW operating temperature is $50^{\circ} \mathrm{C}$.


Fig. 2 CW light output power against current characteristics $20^{\circ} \mathrm{C}$, uncoated

Fig. 3 shows the oscillation spectrum for laser diodes at $20^{\circ} \mathrm{C}, 8 \mathrm{~mW}$ under CW operation. The oscillation wavelength is 635.6 nm .


Fig. 3 Lasing spectrum
$20^{\circ} \mathrm{C}, \mathrm{CW}, 8 \mathrm{~mW}, \lambda_{\mathrm{L}}=635.6 \mathrm{~nm}$
Fig. 4 shows the far-field patterns of the laser diodes. Stable fundamental transverse-mode operation is obtained up to 8 mW . The beam divergence perpendicular to and parallel with the junction at a light output power of 8 mW are 30 and $8.5^{\circ}$, respectively.
In conclusion, the low-threshold 630 nm -band laser diodes have been achieved by using AlGaInP lasers with an MQW active layer. The threshold current was as low as 57 mA at $20^{\circ} \mathrm{C}$, and the oscillation wavelength was 635.6 nm at 8 mW . Stable fundamental transverse-mode operation was obtained up to 8 mW . The maximum light output power was 15 mW at $20^{\circ} \mathrm{C}$. The increased light output power of the lasers may be realised by introducing the current-blocking region near the
facets, and by forming AR and HR coatings for the front and rear facets, respectively.



Fig. 4 Far-field patterns
$20^{\circ} \mathrm{C}$
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SOLVING SORTING AND RELATED PROBLEMS BY QUADRATIC PERCEPTRONS
Y.-H. Tseng and J.-L. Wu

Indexing term: Neural networks
A quadratic perceptron has a polynomial of order 2 as its discriminant function. The Letter shows that it can be used to solve sorting, MIN, MAX (i.e. winner-take-all), and nonbinary majority problems all in constant time.

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application of neural networks to sorting problems used the Hopfield model and multilayer perceptrons for constant-time computing [1-3]. In this Letter, the perceptron networks with quadratic polynomials as discriminant functions are used to solve sorting and its related problems, namely winner-take-al and nonbinary majority. The basic idea used in constructing a neural network to solve these problems is to express the solution in logic forms and then convert them into a set of dis criminant functions. The resulting networks have low network complexity and short propagation delay, making them applic able as primitives of more complex computations.
The output function of a general perceptron can be described as
\[

$$
\begin{equation*}
z=\operatorname{sgn}_{0}[g(X)] \tag{1}
\end{equation*}
$$

\]

In eqn. $1, X=\left[x_{1}, x_{2}, \ldots, x_{n}\right] \in R^{n}$ is the network input; $\operatorname{sgn}_{0}$ is a dichotomy function: $\operatorname{sgn}_{0}(a)=1$ if $a>0,0$ if $a \leq 0$, and $g$, the so called discriminant function, is an $r$ th-order polynomial:

$$
\begin{equation*}
g(X)=w_{1} f_{1}(X)+w_{2} f_{2}(X)+\ldots+w_{N} f_{N}(X)+w_{0} \tag{2}
\end{equation*}
$$

where $w_{i}$ are called weights and each product term $f_{i}(X)$ is of the form

$$
\begin{equation*}
x_{k_{1}}^{n_{1}} x_{k_{2}}^{n 2} \ldots x_{k_{r}}^{n} \tag{3}
\end{equation*}
$$

$k_{1}, k_{2}, \ldots, k_{r} \in\{1, \ldots, n\}$ and $n_{1}, n_{2}, \ldots, n_{r}$ are integers. A linear perceptron has a discriminant function of order $r=1$. A perceptron with $r=2$ is called a quadratic perceptron, whose discriminant is also called a gating activation function in Ref erence 4 because the two variables in a product term can be seen as one gates the other. The above perceptron outputs binary values. Because a number can always be coded in binary form, if we allow a perceptron without using the dichotomy function, it would be more concise in representing the neural network solutions for the problems considered below.

Sorting networks: An unordered sequence $X=\left\{x_{1}, x_{2}, \ldots, x_{n}\right\}$ of real numbers can be sorted into an ordered sequence $S=$ $\left\{s_{0}, s_{1}, \ldots, s_{n-1}\right\}$ by first determining the ranks of the numbers and then outputting them according to their ranks. The rank of number $x_{i}$ is the number of elements in $S$ following $x_{i}$. It can thus be expressed as

$$
\begin{equation*}
R\left(x_{i}\right)=\sum_{j=1}^{n} \operatorname{sgn}_{0}\left(x_{i}-x_{j}\right) \tag{4}
\end{equation*}
$$

The smallest number has rank 0 , the second is of rank 1 , and so on. This rank will lead to a sorted sequence in increasing order and is thus called an increasing rank. In the same way we can define a decreasing rank to result in a decreasing sequence as follows:

$$
\begin{equation*}
R^{*}\left(x_{i}\right)=\sum_{j=1}^{n} \operatorname{sgn}_{0}\left(x_{j}-x_{i}\right) \tag{5}
\end{equation*}
$$

The above rank function results in a two-layer network: the first layer is a set of $n$ linear perceptrons each of which has only two inputs; the second layer performs linear sums of the comparison results. These are perceptrons without the dichotomy function and thus their outputs can be imme diately directed to the next layer.
The next step is to output the number with rank $k$ to the correct position. A direct translation of this operation into a logic expression takes the form

$$
\begin{equation*}
s_{k}=\sum_{i=1}^{n} \operatorname{EQ}\left[k=R\left(x_{i}\right)\right] x_{i} \tag{6}
\end{equation*}
$$

where $\mathrm{EQ}(s=t)$ is a predicate, $\mathrm{EQ}=1$ if $s=t$ and $\mathrm{EQ}=0$ if $s \neq t$. The expression is further converted into a form implementable by quadratic perceptrons:
$s_{k}=\sum_{i=1}^{n}\left\{\operatorname{sgn}_{0}\left[k+1-R\left(x_{i}\right)\right]+\operatorname{sgn}_{0}\left[R\left(x_{i}\right)-k+1\right]-1\right\} x_{i} \quad$ (7)

It can be seen that the expression in the braces is 1 if $k-1 \leq$ $R\left(x_{i}\right) \leq k+1$ and 0 if $R\left(x_{i}\right)>k+1$ or $R\left(x_{i}\right)<k-1$, exactly meets the requirement of the predicate EQ. The resulting sorting network is shown in Fig. 1.


This Figure shows the part that computes the $k$ th element in the sorted sequence
In parallel computing, an algorithm is often evaluated by a cost function defined as the product of hardware complexity and computational complexity [5]. In this sorting network, it and computational complexity [5]. In this sorting network, it
requires $O\left(n^{2}\right)$ connections but constant-time computation. requires $O\left(n^{2}\right)$ connections but constant-time computation.
Thus its cost is $O\left(n^{2}\right)$. Although it is not cost optimal, which Thus its cost is $O\left(n^{2}\right)$. Although it is not cost optimal, which
should be $O(n \log n)$, it has a simple structure where most of should be $O(n \log n)$, it $h$
the connections are local.

Winner-take-all networks: Winner-take-all is an important operation in neural networks. It is usually solved in the form of fully connected recurrent networks [6]. The disadvantages of this approach are indefinite convergence time and inaccuracy [7]. With the idea of the above sorting network, winner-take-all problems can be solved accordingly. We consider the problem in two versions: MIN, select the minimum in equence X; MAX, the maximum
The MIN network, computing the MIN operation, can be derived by choosing the increasing rank defined in eqn. 4 and the first output $s_{0}$ of the sorting network. If the index instead of the value is needed, $s_{0}$ can be modified to output the index of the minimum number:

$$
\begin{equation*}
M I N=s_{0}=\sum_{i=1}^{n} \mathrm{EQ}\left[0=R\left(x_{i}\right)\right] i \tag{8}
\end{equation*}
$$

The MAX network can be obtained similarly by choosing the decreasing rank in eqn. 5 and the first output $s_{0}$
The proposed winner-take-all network requires constant time computation and has none of the disadvantages mentioned above.

Nonbinary majority: Binary majority operation is a linearly separable problem and can be trivially solved by a linear perceptron by summing all its inputs [8]. However, the solution for the majority of nonbinary inputs is not so obvious without he help of the MAX network described above. The majority of $m$ candidates cast by $n$ voters is computed by a network which first counts the number of votes for each candidate and then feeds these numbers to the MAX network. The votes for candidate $i$ are denoted as $V_{i}$, for $i=1,2, \ldots, m$, and

$$
\begin{equation*}
V_{i}=\sum_{j=1}^{n} \mathrm{EQ}\left(i=x_{j}\right) \quad i=1,2, \ldots, m \tag{9}
\end{equation*}
$$

The set of numbers $\left\{V_{1}, V_{2}, \ldots, V_{m}\right\}$ is then fed to the MAX network to result in a correct output. The constant-time majority network can be applied to decode those majoritydecodable codes of the nonbinary case [9], for example.

Conclusion: We have presented efficient neural network solutions to sorting and related problems which are important primitives for neural networks themselves and other computation models. The main idea used in this approach is to convert the solution, expressed in logic form, into a set of discriminant functions of the perceptrons. This is quite a systematic method for constructing neural networks in solving problems because it has about the same level of complexity as writing an algorithm.

16th March 1992
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## PICOSECOND SQUARE PULSE GENERATION USING NONLINEAR FIBRE LOOP MIRROR

D. U. Noske, N. Pandit and J. R. Taylor

Indexing terms: Pulse generation, Nonlinear optics
Square pulses with controllable picosecond durations have been generated from a CW input signal in the region or $1.5 \mu \mathrm{~m}$ by using switching in a nonlinear loop mirror. The driver which can be a noise burst. Shaped pulses have also een derived using this technique. Shaped pulses have been derived using this technique.

The nonlinear ring interferometer [1] in fibre form [2] has been extensively studied with particular emphasis on ultrafast all-optical switching and has been successfully demonstrated for both silicon and nonsoliton pulse inputs, in various geometries [3, 4]. For nonsoliton pulses, intensity dependent switching in most cases gives rise to fragmentation of the switched pulse. Square pulses do not exhibit this characteristic and consequently are preferable for some applications. Here we describe a simple technique to generate square pulses or shaped pulses, based on two wavelength operation of a nonlinear loop mirror $[5,6]$. The pulses to be generated are derived from a CW input signal, driven by a pulse or noise burst derived from a laser source, such that the generated pulses are in time synchronism with the driver. In the work reported here the derived square pulses had durations of 35 ps .

A schematic diagram of the experimental arrangement is shown in Fig. 1. The pulse formation mechanism is based on the switching of a nonlinear Sagnac interferometer, which has been described previously [6-8]. Simply, in the absence of a switching pulse the transmission through the interferometer, for a low level signal (at a wavelength where the coupler has a 3 dB splitting ratio), is zero. In the presence of the switching pulse (at a wavelength where the coupler is imbalanced), through crossphase modulation, the signal experiences a net phase disturbance and is transmitted. The duration of the transmitted signal is dependent on the duration of the switching pulse and the relative group delay dispersion the signal ng pulse and the relative group delay dispersion at the signal and switching wavelengths. Where the tracking is perfect the switched pulse will have a duration equal to that of the
switching pulse. In the presence of complete walkoff, the switching pulse. In the presence of complete walkoff, the switched pulse experiences a constant phase shift over its duration (apart from the rising and falling edges), generating a square pulse, the duration of which depends on the magnitude of the dispersions and loop length. Similarly, by mixing fibres with different dispersions within the loop, various shaped pulses can be generated.


Fig. 1 Schematic diagram of experimental setup
The switching pulse was derived from the fibre-grating pair compressed output of a mode-locked Nd:YAG laser operating at $1.32 \mu \mathrm{~m}$. This generated 3 ps pulses at 100 MHz , allowing an average power of up to 100 mW in the fibre. The signal, tunable around $1.53 \mu \mathrm{~m}$, was obtained from a CW erbium fibre laser or a semiconductor laser, with maximum average powers of 30 mW and 1 mW obtained in the fibre, respectively. Signal and switching pulse were combined via a WDM and launched into the nonlinear optical loop mirror formed from a coupler with a $50: 50$ splitting ratio at $1.53 \mu \mathrm{~m}$ and $90 \cdot 10$ at $1.32 \mu \mathrm{~m}$. Various lengths of fibre with different and 90 : 10 minima were used with the loop, depening on dispersion minima were used within the loop, depending on the required output pulse shape. The switched pulse was passed through an in-line Faraday isolator (FI) and then amplified in a 15 m length of standard $\mathrm{Al}_{2} \mathrm{O}_{3}-\mathrm{GeO}_{2}-\mathrm{SiO}_{2}$ based, erbium-doped, singlemode fibre, which was pumped in a counter-propagating geometry at 980 nm via the exit WDM. Use of the amplifier outside the loop rather than inside, as in the case of so called NALMs, reduces noise problems. The amplified, generated pulse shape exited the other port of the WDM with average powers in the milliwatt range and was detected using an optical sampling oscilloscope and scanning spectrograph.
For square pulse generation, the fibre used within the loop was 20 m long with a dispersion minimum at $1.32 \mu \mathrm{~m}$. At the operating wavelengths and from the dispersive parameters of the fibre, a square pulse shape of 30 ps pulse duration was predicted (see Fig. 2a). This takes into account the finite response time of the optical sampling oscilloscope detector and the response function of the NOLM. Experimentally, a square pulse of approximately 34 ps was recorded as shown in Fig. 2b. This generated pulse was in time synchronism with the driving pulse. By modifying the fibre-grating arrangement, it is possible to generate noise bursts. By allowing total walk it is poss of the noise burst constant phase square pulses can through of the noise burst, constant phase square pulses can still be generated. Because diode laser sources can be used to
give rise to switching [8], it should be possible to use noise give rise to switching [8], it should be possible to
bursts from diode lasers to generate square pulses.
ursts from diode lasers to generate square pulses.
Replacing the fibre in the loop by a 10 m piece with a dispersion minimum at $1.45 \mu \mathrm{~m}$ and tuning the CW diode laser source, allowed perfect tracking between switching pulse and signal disturbance. This permitted the generation of


[^0]:    Introduction: Among all computational models studied over the past, sorting appears to be a vital operation included as a primitive in many computing tasks. Previous work on the

