In addition to these measurements at fixed substrate bias, the substrate gate can be used to modulate the channel at fixed drain/source bias V_{DS} (see Fig. 5). This is a fundamentally different meaurement to those previously described because in

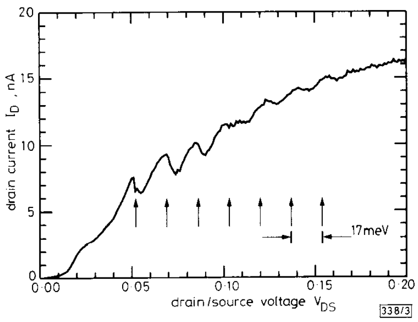

Fig. 3 *Measured lateral resonant tunnelling transistor showing nearly equal peak separations expected for harmonic oscillator potential*

4·2 K
R5023
$V_{SUB} = 2{\cdot}5$ V

the former case (with fixed substrate bias), as the source/drain bias is swept, electrons are injected from the same source sub-band, so that the spectroscopy is indicative of the quantum well eigenvalues alone. However, under fixed V_{DS}, the occupation of the 2-D sub-bands in the source and drain, as well as the quantum well resonance states, are controllable by V_{SUB}. Thus, the complicated mode mixing of quantum

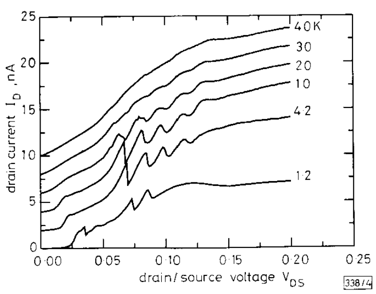

Fig. 4 *Temperature dependence of lateral resonant tunnelling transistor characteristics*

$V_{SUB} = 1{\cdot}0$ V
R5023

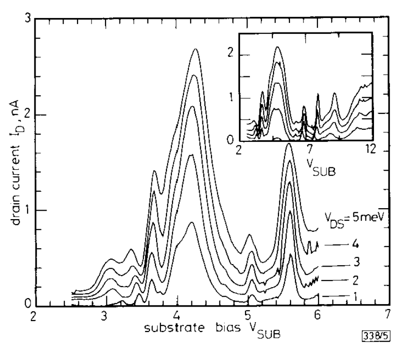

Fig. 5 *Observation of multiple negative transconductances in lateral resonant tunnelling transistor*

4·2 K
R5023

states in all three regions of the device is revealed in a rich spectrum of multiple negative transconductances. Although a quantitative explanation of the peak heights and spacings in the current curves of Fig. 5 is theoretically intractable at present, a consistent qualitative explanation can be given. As the substrate bias is increased, both the source sub-bands and the quantum well states are pulled below the Fermi level, but generally not at the same V_{SUB}. Peaks in the current occur when unoccupied states in the well align with occupied source sub-bands; minima occur when a well state is pulled below the Fermi level. As the next unoccupied well state is pulled closer to the Fermi level, it becomes aligned with a source band, and the current is once again increased. Clearly a more sophisticated theoretical treatment is needed to fully comprehend these measurements, however the first order effects are consistent with our equilibrium modelling of the structure.

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References

1 CHOU, S.-Y., ALLEE, D. R., PEASE, R. F. W., and HARRIS, J. S., JUN.: 'Observation of electron resonant tunneling in a lateral dual-gate resonant tunneling field-effect transistor', *Appl. Phys. Lett.*, 1989, **55**, pp. 176–178

2 ISMAIL, K., ANTONIADIS, D. A., and SMITH, H. I.: 'Lateral resonant tunneling in a double-barrier field-effect transistor', *Appl. Phys. Lett.*, 1989, **55**, pp. 589–591

3 KAO, Y.-C., SEABAUGH, A. C., LIU, H.-Y., KIM, T. S., REED, M. A., SAUNIER, P., BAYRAKTAROGLU, B., and DUNCAN, W. M.: 'Improved MBE growth of InGaAs–InAlAs heterostructures for high-performance device applications'. SPIE, 1989, **1144**, pp. 30–38

4 LUSCOMBE, J. H., and FRENSLEY, W. R.: 'Models for nanoelectronic devices', *Nanotechnol.*, 1990, **1**, pp. 131–140

5 BOUCHARD, A. M., LUSCOMBE, J. H., SEABAUGH, A. C., and RANDALL, J. N.: 'Lateral resonant tunneling transistors: simulation, design, and experiment'. To be published in Proc. Int. Symp. Nanostructures and Mesoscopic Devices, Santa Fe, NM, 1991

6 GRIEM, T., NATHAN, M., WICKS, G. W., HUANG, J., CAPANI, P. M., and EASTMAN, L. F.: 'High conductance and low persistent photoconductivity in $In_{0.53}Ga_{0.47}As/Al_{0.48}In_{0.52}As$ modulation-doped structures with pinchoff capabilities', *J. Vac. Sci. Technol.*, 1985, **B3**, pp. 655–656

CONVOLUTION-BASED DCT ALGORITHM

Indexing terms: Algorithms, Transforms, Convolution, Mathematical techniques, Information theory

Based on elements of number theory, a new convolution-based algorithm for computing the DCT (with power of two length) is proposed. In terms of computational counts, the proposed algorithm computes a length-N DCT (with N a power of two) using only N multiplications.

Introduction: Since the discovery of the discrete cosine transform (DCT),[1] many new algorithms for computing the DCT have been developed. These algorithms are either indirect computations using fast Fourier transforms[1–2] or direct computations using matrix factorisation (or recursive computation).[3] On the other hand, the convolution-based approach deals commonly with the prime length (prime factors) DFTs.[4] These algorithms can be optimised using the Winograd convolution algorithm,[5] or be implemented using the number theoretical transform (NTT) which needs only order N multiplications.

In this Letter, based on elements of number theory, a new convolution-based algorithm for computing the DCT (with power of two length) is proposed. In terms of computational counts, the proposed algorithm computes a length-N DCT (with N a power of two) using only N multiplications.

Useful theorems in number theory and properties of DCTs

(i) *Theorem 1:* If $n > 2$, then

$$4k + 1 \equiv 5^{\beta_1} \pmod{2^n}$$
$$4k + 3 \equiv -5^{\beta_2} \pmod{2^n}$$

where $k \in Z$ (the set of integers) and $\beta_1, \beta_2 \in Z^+$ (the set of positive integers).

(ii) *Theorem 2:* If $n > 2$, then

$$5^{2^{n-3}} \equiv 1 + 2^{n-1} \pmod{2^n}$$

The proofs of theorems 1 and 2 can be found in Reference 6.

Theorem 1 implies that there is a one-to-one mapping between the following two subsets in Z_{2^n} (the integers modulo 2^n) that is

$$\{4t + 1 \mid t = 0, 1, \ldots, 2^{n-2} - 1\} \leftrightarrow \{5^t \mid t = 0, 1, \ldots, 2^{n-2} - 1\}$$

(iii) *Corollary 1:* For the matrix of index functions

$$M = \{f[(4i + 1)(4j + 1)(\bmod 4N)]\}_{i, j = 0, 1, \ldots, N-1}$$

there exist a circular convolution matrix C and two permutation matrices P_1 and P_2, such that

$$M = P_1 C P_2$$

(iv) Proof of corollary 1: By theorem 1

$$4i + 1 \equiv 5^t \pmod{4N}$$

Therefore, we can reorder the rows and columns in M, i.e.

$$C = \{f[(5^{N-t_1} \cdot 5^{t_2})(\bmod 4N)]\}$$
$$= \{f[5^{t_2 - t_1}(\bmod 4N)]\}_{t_1, t_2 = 0, 1, \ldots, N-1}$$

Thus, C is a circular convolution matrix, and the input and output reordering processes can be achieved by two permutation matrices (say P_1 and P_2), respectively.

According to Wang,[3] there are four types of DCT definition and the computation of the four types of DCT can be reduced to the computation of the type-IV DCT. Therefore, the fast algorithms for any type of DCT depend only on the computation of the type-IV DCT.

Proposed algorithm for computing type-IV DCTs: From Reference 3, the type-IV DCT can be rewritten as

$$X(k) = \sum_{n=0}^{N-1} x(n) \cos\left[\frac{2\pi(2n + 1)(2k + 1)}{8N}\right]_{k = 0, 1, \ldots, N-1} \tag{1}$$

We prove that the work for computing N-point type-IV DCTs can be achieved by computing an N-point skew circular convolution and permutations using the following processes.

(i) *Step 1:* Extend $[C_N^{IV}]$ (the notation defined in Reference 3 is adopted in the following for simplicity) as follows:

$$Y(k) = \sum_{n=0}^{2N-1} y(n) \cos\left[\frac{2\pi(2n + 1)(2k + 1)}{8N}\right]_{k = 0, 1, \ldots, 2N-1}$$

where

$$y(n) = \begin{cases} x(n) & 0 \le n \le N - 1 \\ 0 & N \le n \le 2N - 1 \end{cases}$$

and then

$$X(k) = Y(k) \quad \text{for } k = 0, 1, \ldots, N - 1$$

(ii) *Step 2:* Reorder the input and output sequence.

Similarly to the previous work,[2] the above $2N$-point transform can be rewritten as

$$\tilde{Y}(k) = \sum_{n=0}^{2N-1} \tilde{y}(n) \cos\left[\frac{2\pi(4n + 1)(4k + 1)}{8N}\right] \qquad k = 0, 1, \ldots, 2N - 1 \tag{2}$$

where

$$\begin{cases} \tilde{y}(n) = y(2n) \\ \tilde{y}(2N - n - 1) = y(2n + 1) \end{cases} \qquad n = 0, 1, \ldots, N - 1$$

and

$$\begin{cases} \tilde{Y}(k) = Y(2k) = Y(2k) \\ \tilde{Y}(2N - k - 1) = Y(2k + 1) \end{cases} \qquad k = 0, 1, \ldots, N - 1$$

(iii) *Step 3:* The matrix representation of eqn. 2 is

$$G_{2N} = \left\{\cos\left[\frac{2\pi(4n + 1)(4k + 1)}{8N}\right]\right\}_{n, k = 0, 1, \ldots, 2N-1}$$

From corollary 1, it follows that the equation $G_{2N = P_{2N}} C_{2N} Q_{2N}$ holds where P_{2N} and Q_{2N} are two permutation matrices and C_{2N} is a $2N$-point circular convolution matrix and can be represented as

$$C_{2N} = \left[\cos\left(\frac{2\pi \cdot 5^{(j-i)}}{8N}\right)\right]_{i, j = 0, 1, \ldots, 2N-1}$$

(iv) *Step 4:* Because

$$\cos\left(\frac{2\pi \cdot 5^{n+N}}{8N}\right) = -\cos\left(\frac{2\pi \cdot 5^n}{8N}\right) \tag{3}$$

by theorem 2 (details are in the appendix),

$$C_{2N} = \begin{bmatrix} H_N & -H_N \\ -H_N & H_N \end{bmatrix}$$
$$= \begin{bmatrix} I_N \\ -I_N \end{bmatrix} \cdot [H_N] \cdot [I_N - I_N] \tag{4}$$

where H_N is the so called N-point skew circular convolution matrix.

By eqn. 4, it follows that the computation of C_{2N} can be achieved by calculating an N-point skew circular convolution and additional N additions/subtractions. Consider the following remarks:

(*a*) *Remark 1:* In step 1, we extend the input sequence with N zeros, therefore the N additions/subtractions in step 4 can be replaced by the 'sign change' operations.

(*b*) *Remark 2:* In step 1, we only need half of the output sequence. Therefore, the post-operations of eqn. 4 can be achieved by 'sign change' operations.

According to the above discussion, we can conclude that the computation of an N-point type-IV DCT can be achieved by an N-point skew circular convolution with some permutations and sign changes of input and output sequences. From Reference 7, an N-point skew circular convolution (or the polynomial product modulo $z^n + 1$) can be computed by means of the generalised number theoretical transform (GNTT) with only N multiplications.

Algorithms for discrete sinusoidal transforms: According to the previous works,[3,8,9] the relations between some well known discrete sinusoidal transforms (DFT, DHT (discrete Hartley transform), DCT and DST (discrete Sine transform)) are very clear, and are listed as follows:

$$DFT(N) \Rightarrow \begin{cases} DFT\left(\frac{N}{2}\right) \\ \text{two } DCT^{\mathrm{II}}\left(\frac{N}{4}\right) \end{cases} \tag{5}$$

$$DHT(N) \Rightarrow \begin{cases} DHT\left(\frac{N}{2}\right) \\ \text{two } DCT^{\mathrm{II}}\left(\frac{N}{4}\right) \end{cases} \tag{6}$$

$$DCT^{\mathrm{II}}(N) \Rightarrow \begin{cases} DCT^{\mathrm{II}}\left(\frac{N}{2}\right) \\ DCT^{\mathrm{IV}}\left(\frac{N}{2}\right) \end{cases} \tag{7}$$

$$DST^{\mathrm{II}}(N) \Rightarrow \begin{cases} DST^{\mathrm{II}}\left(\frac{N}{2}\right) \\ DCT^{\mathrm{IV}}\left(\frac{N}{2}\right) \end{cases} \tag{8}$$

Based on the discussion of the 'proposed algorithm for computing type-IV DCTs' Section, we can compute the $DCT^{\mathrm{IV}}(N)$ using N-point skew circular convolution ($SCC(N)$). Therefore, the following result can be derived by the recursive formulas of eqns. 5–8:

$$DFT(N) \Rightarrow \text{two } SCC\left(\frac{N}{4}\right), \text{ two } SCC\left(\frac{N}{8}\right), \ldots$$

$$DHT(N) \Rightarrow \text{two } SCC\left(\frac{N}{4}\right), \text{ two } SCC\left(\frac{N}{8}\right), \ldots$$

$$DCT^{\mathrm{II}}(N) \Rightarrow SCC\left(\frac{N}{2}\right), SCC\left(\frac{N}{4}\right), \ldots$$

$$DST^{\mathrm{II}}(N) \Rightarrow SCC\left(\frac{N}{2}\right), SCC\left(\frac{N}{4}\right), \ldots$$

with some interblock additions and sign changes.

Although the DFT is defined in the complex number system, we can still derive an algorithm using only real SCCs.

Conclusion: We have developed an algorithm which transfers the problem of N-point type-IV DCT into the problem of N-point skew circular convolution. In theory, this algorithm can achieve the lower bound of the number of multiplications according to the minimum complexity polynomial algorithms. In practice, by means of the number theoretical transform, we can compute $[C_N^{IV}]$ using only N multiplications, or we can use a filter-type structure that is very suitable for the VLSI implementation.

According to the relations between type-IV DCT and other famous transforms, we have mentioned that the other discrete sinusoidal transforms can be computed by means of the combination of some SCCs of smaller size, and possess the same advantages in both theoretical and practical as the type-IV DCT.

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References

1 AHMED, N., NATARAJAN, T., and RAO, K. R.: 'Discrete cosine transform', *IEEE Trans.*, 1974, **C-23**, pp. 89–93
2 NARASIMHA, M. J., and PETERSON, A. M.: 'On the computation of the discrete cosine transform', *IEEE Trans.*, 1978, **COM-26**, pp. 934–946
3 WANG, Z.: 'Fast algorithms for the discrete W transform and for the discrete Fourier transform', *IEEE Trans.*, 1984, **ASSP-32**, pp. 803–816
4 RADER, C. M.: 'Discrete Fourier transforms when the number of data samples is prime', *Proc. IEEE*, 1968, **56**, pp. 1107–1108
5 WINOGRAD, S.: 'Some bilinear forms whose multiplicative complexity depends on the field of constants', *Math. Syst. Theory*, 1977, **10**, pp. 169–180
6 ROSEN, K. H.: 'Elementary number theory and its application' (1988)
7 PEI, S. C., and WU, J. L.: 'Improved long convolutions using generalized number theoretic and polynomial transforms', *IEEE Trans.*, 1985, **ASSP-33**, pp. 1626–1627
8 VETTERLI, M., and NUSSBAUMER, H. J.: 'Simple FFT and DCT algorithms with reduced number of operations', *Signal Processing*, 1984, **6**, (4), pp. 267–278
9 MALVAR, H. S.: 'Fast computation of the discrete cosine transform and the discrete Hartley transform', *IEEE Trans.*, 1987, **ASSP-35**, pp. 1484–1485

Appendix: The proof of eqn. 3 is as follows:

$$\begin{aligned}\cos\left(\frac{2\pi \,.\, 5^{(n+N)}}{8N}\right) &= \cos\left[\frac{2\pi \,.\, 5^n \,.\, (1+4N)}{8N}\right] \quad \text{(by theorem 3)} \\ &= \cos\left(5^n\pi + \frac{2\pi \,.\, 5^n}{8N}\right) \\ &= \cos\left(\pi + \frac{2\pi \,.\, 5^n}{8N}\right) \\ &= -\cos\left(\frac{2\pi \,.\, 5^n}{8N}\right)\end{aligned}$$

TAPERED InP/InGaAsP WAVEGUIDE STRUCTURE FOR EFFICIENT FIBRE-CHIP COUPLING

Indexing terms: Optoelectronics, Optical waveguides, Integrated optics, Optics

A novel passive InP/InGaAsP waveguide structure for low-loss coupling of monomode fibres to semiconductor devices having waveguides with small elliptical modes has been fabricated. The device consists of a fibre-matched waveguide, a tapered waveguide structure for the necessary mode transformation, and a small spot waveguide. The transformation of the fibre mode into a mode with a spot of 2·0 μm lateral and 1·5 μm vertical extension (FWHM) is demonstrated. Nearly polarisation independent transmission losses, as low as 4·9 dB, are measured for uncoated devices with a 900 μm long tapered section.

Introduction: To achieve large alignment tolerances and high efficiencies in the coupling of a monomode fibre to an optoelectronic semiconductor chip, the coupling unit is required to perform the following fundamental task: the large circular mode guided by the fibre has to be transformed at low loss so that it matches the smaller and usually elliptic mode of the waveguide on the semiconductor chip.

Classical coupling techniques based on lenses[1] or tapered fibres[2] cannot fully meet these requirements as they conserve the circular form of the fibre mode. The losses associated with the mismatch in shape are accepted as a tradeoff for the relative ease with which such rotationally symmetric devices may be designed and fabricated. Furthermore, the efficiencies attainable with these coupling units are limited by their aberrations.[3] Finally, all these conventional techniques suffer from minute alignment tolerances representing a severe hazard to