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Comments on "A Two-Stage Representation of DFT and Its Applications"

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Abstract—This correspondence contains comments on and several corrections to a recently published TRANSACTIONS paper.

In the above paper,¹ Ersoy developed a two-stage representation in terms of preprocessing and postprocessing of DFT by vector transformation of sines and cosines into new basis functions using Mobius inversion of number theory. This comment points out first that the inversion Mobius transform pair, (A.3) and (A.4), used¹ are valid only when f is a positive rational number [1, p. 208]. Thus, (A.6) should read

$$X_c(f) = \frac{1}{4f} \sum_{\substack{m=1 \\ \text{odd}}}^{\infty} \frac{\mu_m}{m} \left(\sum_{n=-\infty}^{\infty} \left(2x\left(\frac{n}{mf}\right) - x\left(\frac{n}{2mf}\right) \right) \right) \quad (\text{A.6})$$

and $f > 0$. Second, (2.11) should read

$$n' = 0, 1, \dots, M_1 - 1. \quad (2.11)$$

This range is very important because it determines the size of the circular correlation in the postprocessing matrix equation. The cor-

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rectness of this new range has been verified by computer simulation.

Third, the two-stage representation form can be applied directly to the computation of discrete Hartley transform (DHT) [3]. Interestingly, since the elements in the preprocessing matrix of DHT are 0, 1, -1, 2, and -2, only shift, addition, and subtraction operations are involved for the preprocessing stage of DHT. It can be shown that the postprocessing matrix of DHT is also in a block-diagonal form, with each block being a circular correlation matrix. For example, with $N = 8$ and using the same notation defined,¹ the preprocessing and postprocessing matrix equations can be obtained respectively as follows:

$$\begin{aligned} \begin{bmatrix} h(0) \\ h(4) \\ h(2) \\ h(1) \\ h(5) \\ h(6) \\ h(7) \\ h(3) \end{bmatrix} &= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 2 & 1 & 0 & -1 & -2 & -1 & 0 \\ 1 & -2 & 1 & 0 & -1 & 2 & -1 & 0 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 0 & -1 & -2 & -1 & 0 & 1 & 2 \\ 1 & 0 & -1 & 2 & -1 & 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \\ x(6) \\ x(7) \end{bmatrix} \\ \begin{bmatrix} X(0) \\ X(4) \\ X(2) \\ X(1) \\ X(5) \\ X(6) \\ X(7) \\ X(3) \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b(1, 8) & b(5, 8) & 0 & 0 & 0 \\ 0 & 0 & 0 & b(5, 8) & b(1, 8) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & b(1, 8) & b(5, 8) \\ 0 & 0 & 0 & 0 & 0 & 0 & b(5, 8) & b(1, 8) \end{bmatrix} \\ \begin{bmatrix} h(0) \\ h(4) \\ h(2) \\ h(1) \\ h(5) \\ h(6) \\ h(7) \\ h(3) \end{bmatrix} & \end{aligned}$$

Finally, some typing errors¹ are listed below.

1) With the substituting of $l = m_1 n$ modulo N , (2.3) should read

$$h(l) = \sum_{k=0}^{N-1} x(k) \left[\mu \left(\frac{lk}{N} + \frac{1}{4} \right) - j \mu \left(\frac{lk}{N} \right) \right]. \quad (2.3)$$

2) The index i used in (2.7) and (2.12) should be replaced by n' .

3) The term $b(4, 16)$ used in the lowest block of (2.13) should be replaced by $b(9, 16)$.

4) Equation (A.16) should read

$$b(m_1, N) = P(m_1(l), N) - P(m_2(N-l), N). \quad (\text{A.16})$$

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