行政院國家科學委員會專題研究計畫成果報告

小波轉換於多載波系統之應用
Discrete Multitone Modulation Using Wavelet
計畫編號：NSC 89-2213-E-002-063
執行期限：88年8月1日至89年7月31日
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1 中文摘要

離散多載波系統(DMT)爲數據傳輸的有效技術. DMT系統爲一種將通道分割成次頻道的轉化多工器. 這種方法的效能與接收濾波器之頻率選擇性有關. 我們建議使用濾波器組或小波來實現具有好的頻率選擇性的DMT系統. 在此成果報告中,我們將設計具有好的頻率選擇性及能完全消除ISI的濾波器組轉化多工器.
關鍵詞：轉化多工器,多調變系統,多載波系統
Abstract: The discrete multitone modulation (DMT) transceivers have been shown to be a very useful technique for data transmission over frequency selective channels. The DMT scheme is realized by a transceiver that divides the channel into subbands. The efficiency of the scheme depends on the frequency selectivity of the receiving filters. The filterbank (or wavelet) transceiver system, has been proposed as an implementation of DMT transceiver that has better frequency band separation. In this report, we show how to design filterbank transceivers that have good frequency selectivity and at the same time cancel ISI completely.
Keywords: transmultiplexer, multitone modulation.

2 緣由與目的

The discrete multitone modulation (DMT) is now a widely used technique for high speed transmission over channels such as digital subscriber loops [1]-[2]. In the DMT scheme, the channel is divided into subbands, each with a different frequency band. The transmission power and bits are judiciously allocated according to the SNR (signal to noise ratio) in each band [2]. This is similar to the water pouring scheme for discrete transmission channels. The realization of the DMT scheme relies on the design of a transceiver that effectively divides the channel into subbands of different frequency bands. Band separation is of particular importance when the channel is highly frequency selective and the SNRs of different frequency bands exhibit large differences.

The DFT based DMT system has been proposed as a practical implementation of DMT system [1]. Very good transmission rate can be accomplished. In the DFT based systems, the transmitter and receiver consists of DFT filters, which have limited frequency selectivity. Narrowband noise could induce serious impairment due to the poor stopband of the receiving filters. For better frequency band separation, Sandberg and Tzannes [3] proposed the so called DWMT (discrete wavelet multitone) system, in which perfect reconstruction filter banks are used as the transceiver. The transmitting and receiving filters have excellent frequency separation property inherited from good filter bank designs. However, when the channel is not ideal, filterbank transceivers obtained from perfect reconstruction filter banks do not have ISI free property. Performance evaluation conducted in [4] shows that the resulting ISI can seriously degrade the system performance. To reduce the amount of ISI, inter- and intra-subband equalization are performed on the receiver outputs in [3][5]. So far, there is no methods for designing ISI free filterbank transceivers over frequency selective channels.

In this report we will develop design methods for FIR filterbank transceiver with ISI free property using polyphase approach. We will use over-interpolated filter banks to introduce redundancy, which enabling us to cancel ISI completely. Two methods will be proposed for designing FIR transceivers with zero ISI.

3 結果與討論:

Filterbank Transceivers with ISI Free Property. Consider Fig. 1, where an M-subband filterbank transceiver is shown. The channel is represented by an FIR filter $P(z)$ and an additive noise $e(n)$. The matrix $\mathbf{G}(z)$ and $\mathbf{S}(z)$ are respectively the polyphase matrices of the transmitter and receiver. Using polyphase representation, we can decompose the channel as

$$P(z) = P_0(z^N) + P_1(z^N)z^{-1} + \cdots + P_{N-1}(z^N)z^{-N+1}. \quad (1)$$

Applying the polyphase identity from the multirate theory the $N \times N$ system from $\mathbf{y}(n)$ to $\widehat{\mathbf{y}}(n)$ in Fig. 1 is in fact an LTI system $\mathbf{C}(z)$. The transfer matrix $\mathbf{C}(z)$ is pseudo-circulant with the first column given by,

$$\begin{pmatrix} P_0(z) & P_1(z) & \cdots & P_{N-1}(z) \end{pmatrix}^T. \quad (2)$$

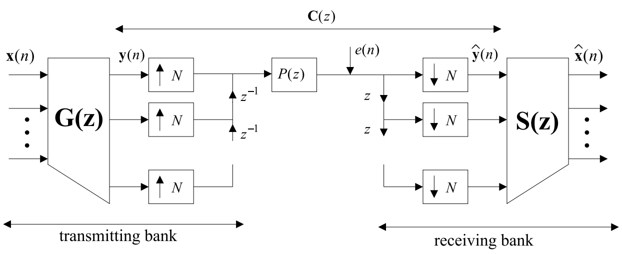

圖 1: The polyphase representation of the transmitter and receiver in a filterbank transceiver.

Usually the interpolation ratio is chosen to be the same as the order L of $P(z)$. In this case, the N polyphases of $P(z)$ are constants and the last $N-L-1$ polyphases are zero. The matrix $\mathbf{C}(z)$ can be partitioned as an $N \times M$ constant matrix $\mathbf{C}_0$ and an $N \times L$ FIR causal matrix $\mathbf{C}_1(z)$ that is of order 1,

$$\mathbf{C}(z) = [\underbrace{\mathbf{C}_0}_{N\times(M)} \vdots \underbrace{\mathbf{C}_1(z)}_{N \times L}]. \quad (3)$$

From Fig. 1, we see that the transfer matrix $\mathbf{T}(z)$ of the overall system can be expressed as

$$\mathbf{T}(z) = \mathbf{S}(z)\mathbf{C}(z)\mathbf{G}(z). \quad (4)$$

The overall system is free from ISI if $\mathbf{T}(z)$ is the identity matrix except delays.

Consider the transmitter $\mathbf{G}(z)$ of the form,

$$\mathbf{G}(z) = \begin{pmatrix} \mathbf{G}_0(z) \\ \mathbf{0} \end{pmatrix}. \quad (5)$$

Every input block of size M goes through an $M \times M$ transfer matrix, and L zeros are inserted between every two blocks before transmission. Then we have

$$\mathbf{C}(z)\mathbf{G}(z) = \mathbf{C}_0\mathbf{G}_0(z).$$

In this case the system is ISI free if

$$\mathbf{S}(z)\mathbf{C}_0\mathbf{G}_0(z) = \mathbf{I}. \quad (6)$$

Thus, the channel dependent term becomes a constant matrix $\mathbf{C}_0$. The receiver $\mathbf{S}(z)$ can be any left inverse for $\mathbf{C}_0\mathbf{G}_0(z)$.

Design of ISI-Free Transceivers based on Orthogonal Matrices

Let us consider the case where $\mathbf{G}_0(z)$ is FIR and $\mathbf{C}_0\mathbf{G}_0(z)$ is FIR orthogonal, i.e.,

$$\left(\mathbf{C}_0\mathbf{G}_0(e^{j\omega})\right)^\dagger \left(\mathbf{C}_0\mathbf{G}_0(e^{j\omega})\right) = \mathbf{I}.$$

Such a construction has the advantage that the receiver can be simply chosen as $\mathbf{S}(z) = \widetilde{\mathbf{G}}_0(z)\mathbf{C}_0^T$. Furthermore in the case of AWGN noise source, the channel noise will not be amplified by the receiver; the average receiver output noise power is the same as the receiver input noise power. Observe that matrix $\mathbf{C}_0$ can be decomposed using SVD (singular value decomposition),

$$\mathbf{C}_0 = \mathbf{U}\begin{pmatrix} \mathbf{\Lambda} \\ \mathbf{0} \end{pmatrix}_{N \times M} \mathbf{V},$$

where $\mathbf{U}$ and $\mathbf{V}$ are respectively $N \times N$ and $M \times M$ orthogonal matrices. The matrix $\mathbf{\Lambda}$ is diagonal and

$[\mathbf{\Lambda}]^2_{k,k}$ for $k = 0, 1, \cdots, M-1$ are the eigenvalues of $\mathbf{C}_0^T\mathbf{C}_0$, which are nonzero as $\mathbf{C}_0$ has full rank. It can be shown that if $\mathbf{C}_0\mathbf{G}_0(z)$ is FIR and orthogonal, the matrix $\mathbf{G}_0(z)$ is necessarily of the form

$$\mathbf{G}_0(z) = \mathbf{V}^T\mathbf{\Lambda}^{-1}\mathbf{Q}(z), \tag{7}$$

where $\mathbf{Q}(z)$ is an arbitrary $M \times M$ FIR orthogonal matrix. Partition $\mathbf{U}$ as

$$\mathbf{U} = [\underbrace{\mathbf{U}_0}_{N\times M}\ \underbrace{\mathbf{U}_1}_{N\times L}]. \tag{8}$$

Then the product $\mathbf{C}_0\mathbf{G}_0(z)$ assumes the form

$$\mathbf{C}_0\mathbf{G}_0(z) = \mathbf{U}_0\mathbf{Q}(z).$$

In this case ISI free property can be obtained by choosing the receiver $\mathbf{S}(z)$ as

$$\mathbf{S}(z) = \widetilde{\mathbf{Q}}(z)\mathbf{U}_0^T.$$

However the above equation only gives one possible ISI free solution. To obtain all possible solutions, we note that the ISI free condition only requires that $\mathbf{S}(z)$ be a left inverse of $\mathbf{C}_0\mathbf{G}_0(z)$. As $\mathbf{C}_0\mathbf{G}_0(z)$ is of dimension $N \times M$, the receiver $\mathbf{S}(z)$ is not unique. In fact, we can incorporate the left null space of $\mathbf{U}_0$ and choose

$$\mathbf{S}(z) = \begin{pmatrix}\widetilde{\mathbf{Q}}(z) & \Xi(z)\end{pmatrix}\mathbf{U}^T, \tag{9}$$

where $\Xi(z)$ is an arbitrary $M \times L$ FIR transfer matrix. The flexibility can be exploited to improve the frequency selectivity of the receiving filters or to minimize the total output noise power [6].

Design of ISI-Free Transceivers based on Unimodular Matrices

The FIR unimodular matrices, unlike orthogonal matrices, do not allow factorization in general. However, a particular class of unimodular has been shown to be very useful in designing M-subband filter banks. Using polyphase matrices that belongs to this class, we can design analysis and synthesis filters with sharp transition bands and good stopband attenuation. The unimodular matrices in this class can be written as a product of lower-triangular and upper-triangular matrices of the following form

$$\mathbf{\Phi}(z)\mathbf{\Psi}(z)$$

where the matrices $\mathbf{\Phi}(z)$ and $\mathbf{\Psi}(z)$ are respectively lower triangular and upper triangular FIR matrices given by,

$$\mathbf{\Phi}(z) = \begin{pmatrix} D_0 & 0 & \cdots & 0 \\ \Phi_{1,0}(z) & D_1 & & \\ \Phi_{2,0}(z) & \Phi_{2,1}(z) & & \\ \vdots & & \ddots & \\ \Phi_{M-1,0}(z) & \Phi_{M-1,1}(z) & & D_{M-1} \end{pmatrix},$$

$$\mathbf{\Psi}(z) = \begin{pmatrix} 1 & \Psi_{0,1}(z) & \Psi_{0,2}(z) & \cdots & \Psi_{0,M-1}(z) \\ 0 & 1 & \Psi_{1,2}(z) & & \Psi_{1,M-1}(z) \\ 0 & 0 & 1 & & \\ \vdots & & & \ddots & \\ 0 & & & & 1 \end{pmatrix},$$

where D_k are constants and, $\Phi_{i,j}(z)$ and $\Psi_{i,j}(z)$ are FIR filters. It can be immediately verified that such a product matrix $\mathbf{\Phi}(z)\mathbf{\Psi}(z)$ is a unimodular matrix as $\det \mathbf{\Phi}(z) = \Pi_{k=0}^{M-1} D_k$ and $\det \mathbf{\Psi}(z) = 1$. Therefore, its inverse is also FIR.

Consider the following choice of receiver and transmitter pair that is based on the above class of unimodular matrices,

$$\begin{aligned} \mathbf{S}(z) &= \begin{pmatrix}\mathbf{\Phi}(z)\mathbf{\Psi}(z) & \Xi(z)\end{pmatrix}\mathbf{U}^T, \\ \text{and} \quad \mathbf{G}_0(z) &= \mathbf{V}^T\mathbf{\Lambda}^{-1}(\mathbf{\Phi}(z)\mathbf{\Psi}(z))^{-1}, \end{aligned} \tag{10}$$

where $\Xi(z)$ is an arbitrary $M \times L$ FIR transfer matrix. Using the partition of $\mathbf{U} = \begin{pmatrix}\mathbf{U}_0 & \mathbf{U}_1\end{pmatrix}$ in (8), the receiving filters $H_k(z)$ can be represented by

$$\begin{pmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{M-1}(z) \end{pmatrix} = \mathbf{\Phi}(z^N)\mathbf{\Psi}(z^N)\mathbf{U}_0^T\mathbf{d}(z) + \Xi(z^N)\mathbf{U}_1^T\mathbf{d}(z),$$

where $\mathbf{d}(z)$ is as given in (**??**). Let

$$\begin{pmatrix} \Theta_0(z) \\ \Theta_1(z) \\ \vdots \\ \Theta_{M-1}(z) \end{pmatrix} = \mathbf{\Psi}(z^N)\mathbf{U}_0^T\mathbf{d}(z).$$

Then, we have $H_k(z)$ given by,

$$H_0(z) = D_0\Theta_0(z) + \boldsymbol{\xi}_0^T(z^N)\mathbf{U}_1^T\mathbf{d}(z)$$

$$H_1(z) = \Phi_{1,0}(z)\Theta_0(z) + D_1\Theta_1(z) + \boldsymbol{\xi}_1^T(z^N)\mathbf{U}_1^T\mathbf{d}(z)$$

$$\vdots$$

$$\begin{aligned} H_{M-1}(z) &= \Phi_{M-1,0}(z)\Theta_0(z) + \Phi_{M-1,1}(z)\Theta_1(z) \\ &+ \cdots + D_{M-1}\Theta_{M-1}(z) + \boldsymbol{\xi}_{M-1}^T(z^N)\mathbf{U}_1^T\mathbf{d}(z), \end{aligned}$$

where $\boldsymbol{\xi}_k^T(z)$ is the k-th row of $\Xi(z)$. We can start the optimization process by designing D_0, $\Theta_0(z)$ and the 0-th row of $\Xi(z)$ to obtain $H_0(z)$. As $\Theta_0(z)$ is already determined in the design of $H_0(z)$, the filter $H_1(z)$ is designed by optimizing $\Phi_{1,0}(z)$, D_1, $\Theta_1(z)$ and $\boldsymbol{\xi}_1^T(z)$. In a similar manner we can continue on to the optimization of $H_2(z)$, $H_3(z)$, $\cdots$, and $H_{M-1}(z)$.

Design Example. *Design Using Unimodular Matrices.* The LTI channel to be used in the example is $P(z) = 1 + 0.8z^{-1}$. The order of $P(z)$ is $L = 1$. We choose $M = 8$ and $N = 9$. The transmitter and receiver are as given in (10). The matrices $\boldsymbol{\Phi}(z)$ and $\boldsymbol{\Psi}(z)$ are of order 3. The resulting the magnitude responses (dB) of the transmitting and receiving filters are shown in Fig. 2. The stopband attenuation of the receiving filters are around 22 dB.

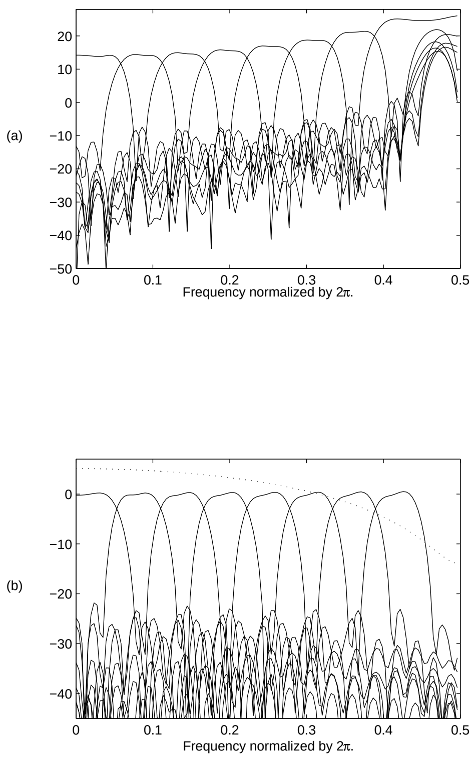

圖 2: magnitude responses (dB) of (a) the transmitting filters and (b) the receiving filters. Also shown in (b) as a dotted line is the magnitude response of the channel $P(e^{j\omega})$.

4 計畫成果自評：

The result of this project is very satisfactory. We have successfully derived ISI free discrete wavelet multitone modulation systems for frequency selective channel. ISI free DWMT systems having good receiving filters have been designed, as demonstrated in the above example. So far, several conference papers and a transaction paper have been accepted.

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