行政院國家科學委員會補助專題研究計畫成果報告
(完整報告)

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應用特徵信號空間的可適性干擾信號消除技術之研究

Research on Eigenspace-Based Adaptive Interference Cancellation

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計畫類別：■個別型計畫　　□整合型計畫

計畫編號：NSC89－2213－E－002－133－ (I)

NSC90－2213－E－002－043－ (II)

執行期間： 89 年 8 月 1 日至 91 年 7 月 31 日

計畫主持人： 李枝宏

共同主持人：

計畫參與人員：徐忠廷, 林益生, 卓經綸, 賴正男

王俊惟, 薛自哲, 楊元豪, 蔡佩勳

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執行單位： 國立臺灣大學電信工程學研究所

中 華 民 國 91 年 9 月 16 日

行政院國家科學委員會補助專題研究計畫成果報告
(完整報告)

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Research on Eigenspace-Based Adaptive Interference Cancellation (I)

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計畫類別：■個別型計畫　□整合型計畫
計畫編號：NSC ~~89~~ 90－2213－E－002－~~133~~ 043
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中華民國91年9月16日

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PART 1 : RESEARCH ON EIGENSPACE-BASED ADAPTIVE INTERFERENCE CANCELLATION (1/2)

ABSTRACT

This report presents the research achievement for the first-year NSC research project. Eigenspace-based interference cancellers (EICs) possess the advantages of providing maximal suppression of interference with fast convergence over conventional adaptive beamformers. However, the performance and sensitivity to steering angle error of EICs have not been analyzed due to the use of a signal blocking matrix. We first present a technique to construct a positive definite matrix based on the signal blocking matrix and then use this matrix to compensate the effect of the signal blocking matrix on the sensor noise received by an EIC. Therefore, the interference subspace required for finding the optimal weight vector can be obtained using conventional eigenvalue decomposition (EVD). Moreover, the performance and sensitivity to the steering angle error of the EIC can be analyzed. Simulation examples are provided for confirming the theoretical results.

Keywords: Adaptive Antennas, Electromagnetic Interference.

CHAPTER ONE INTRODUCTION

Techniques for achieving the purpose of maximizing the rejection of interference regardless of the interference-to-noise ratio (INR) when processing array data by using adaptive interference cancellers have been reported in [1-6]. Notable among them is the one of [4] where an eigenanalysis interference canceller (EIC) with fast convergence speed using a uniform linear array (ULA) was presented. The optimal weight vector is computed by maximizing the output signal-to-background noise ratio subject to a constraint of orthogonality to the interference subspace (IS). The IS is obtained through the generalized eigenvalue decomposition (GEVD) of the correlation matrix of the data vector at the output of an appropriately-designed blocking processor which blocks the desired signal from the received data vector.

Due to the use of a signal blocking matrix, the noise component contained in the blocked data vector is no longer spatially white. Therefore, finding the IS for computing the optimal weight vector generally requires a complicated GEVD. Moreover, it is not an easy task to analyze the performance and sensitivity to steering angle error of an EIC. In the literature, there are thus practically no papers considering the analysis of the performance and sensitivity of an EIC. In this research report, we present a technique to construct a positive definite matrix from the signal blocking matrix of an EIC. The effect of the signal blocking matrix on the spatially white noise component received by the EIC is then eliminated by adding the matrix to the correlation matrix of the blocked data vector. This results in that the IS required for computing the optimal weight vector of an EIC can be obtained by performing conventional EVD instead of any complicated GEVD. Moreover, it is shown that the EIC's performance and sensitivity to steering angle error can be analyzed based on the proposed technique. We present the analysis of the performance of the EIC in terms of the expectation of the output signal-to-interference plus noise ratio (SINR). As to the analysis of sensitivity to steering angle error, the theoretical results show that the EIC's performance is considerably deteriorated even if there is a small steering angle error. Increasing the order of the signal blocking matrix can alleviate this performance degradation. Moreover, the breakdown thresholds for the EIC's performance in the presence of two interferers with two extreme correlation cases are derived, respectively.

This report is organized as follows. Chapter Two briefly describes the principle of a conventional EIC. Chapter Three presents the technique for constructing a positive definite matrix to eliminate the effect of the signal blocking matrix on the received data vector. Based on the proposed technique, the analysis of the EIC's statistical performance is presented in Chapter Four. We evaluate the EIC's sensitivity to steering angle error in Chapter Five. The performance breakdown thresholds are also derived for the cases of two interferers with two extreme correlation situations. Simulation examples for illustration and confirmation are included in Chapter Six. Finally, Chapter Seven concludes the report.

CHAPTER TWO THE PRINCIPLE OF A CONVENTIONAL EIGENSPACE-BASED INTERFERENCE CANCELLER

Consider an M-sensor linear array with interelement spacing equal to d illuminated by P narrow-band signal sources from the distinct direction angles θ_i, $i = 1, 2, \ldots, P$. Let the response of the mth sensor to a signal with unit amplitude and a direction angle θ_i be given by $\exp(j(m-1)u_i)$, where $j = \sqrt{-1}$, $u_i = 2\pi d \sin(\theta_i)/\lambda$, and λ is the wavelength of the signal sources. The received signal at the mth sensor can be expressed as

$$x_m(t) = \sum_{i=1}^{P} s_i(t) e^{j(m-1)u_i} + n_m(t), \tag{1}$$

where $s_i(t)$ denotes the complex amplitude of the ith signal impinging on the array with direction angle θ_i, $n_m(t)$ the spatially white sensor noise with power π_n received by the mth sensor. Both the signal and sensor noise are assumed to be independent and zero-mean stationary Gaussian random processes. In vector form, the received data vector is given by

$$\boldsymbol{x}(t) = \sum_{i=1}^{P} \boldsymbol{a}(u_i) s_i(t) + \boldsymbol{n}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t), \tag{2}$$

where the response vector of the ith signal $\boldsymbol{a}(u_i) = [1 \;\; \exp(ju_i) \ldots \exp(j(M-1)u_i)]^T$, the noise vector $\boldsymbol{n}(t)$=$[n_1(t) \; n_2(t) \; \ldots \; n_M(t)]^T$, the signal source vector $\boldsymbol{s}(t) = [s_1(t) \; s_2(t) \; \ldots \; s_P(t)]^T$, and the response matrix of the signal sources $\boldsymbol{A}$=$[\boldsymbol{a}(u_1) \; \boldsymbol{a}(u_2) \; \ldots \; \boldsymbol{a}(u_P)]$. The superscript T denotes the transpose operation.

Assume the direction angle of the desired signal is θ_1. Based on the principle of the EIC presented in [4], a blocking matrix $\boldsymbol{B}$ is appropriately designed and used as the block processor in order to block the desired signal from the received data vector. Let the order of $\boldsymbol{B}$ be q and a $(q+1)$x1 vector $\boldsymbol{b} = [b_0 \; b_1 \; \ldots \; b_q]^T$ be defined according to the following manner

$$(z - e^{ju_1})^q = \sum_{r=0}^{q} (-1)^{q-r} \binom{q}{r} e^{j(q-r)u_1} z^r = \sum_{r=0}^{q} b_r^* z^r, \tag{3}$$

where the superscript $*$ denotes the complex conjugate. Construct an Mx1 vector $\tilde{\boldsymbol{b}} = [\boldsymbol{b}^T \; \mathbf{o}_{M-q-1}]^T$, where $\mathbf{o}_{M-q-1}$ is a 1x$(M-q-1)$ row vector with zero elements. Thus the signal blocking matrix with order q can be expressed as

$$\boldsymbol{B} = [\, \tilde{\boldsymbol{b}} \;\; \tilde{\boldsymbol{I}}_M\tilde{\boldsymbol{b}} \;\; \tilde{\boldsymbol{I}}_M^2\tilde{\boldsymbol{b}} \;\; \cdots \;\; \tilde{\boldsymbol{I}}_M^{M-q-1}\tilde{\boldsymbol{b}}\,], \tag{4}$$

where $\tilde{\boldsymbol{I}}_M = [\boldsymbol{u}_2 \;\; \boldsymbol{u}_3 \;\; \ldots \;\; \boldsymbol{u}_M \;\; \boldsymbol{u}_1]$ represents a cyclic-shifting matrix with element $\boldsymbol{u}_i$ given by the ith column vector of an MxM identity matrix. Based on $\boldsymbol{a}(u_i) = [1 \;\; \exp(ju_i) \;\ldots\; \exp(j(M-1)u_i)]^T$ and (27), we have

$$\boldsymbol{B}^H\boldsymbol{a}(u_i) = \nu_i\boldsymbol{a}_q(u_i), \tag{5}$$

where $\boldsymbol{a}_q(u_i) = [1 \; \exp(ju_i) \;\ldots\; \exp(j(M-q-1)u_i)]^T$ and the blocking factor ν_i associated with the ith signal source is given by

$$\nu_i = \sum_{r=0}^{q} b_r^* e^{jru_i} = (e^{ju_i} - e^{ju_1})^q \;\; . \tag{6}$$

The data vector at the output of the signal blocking matrix $\boldsymbol{B}$ is then given by

$$\boldsymbol{x}_b(t) = \boldsymbol{B}^H\boldsymbol{x}(t) = \boldsymbol{A}_b\boldsymbol{D}_b\boldsymbol{s}(t) + \boldsymbol{B}^H\boldsymbol{n}(t) = \boldsymbol{A}_b\boldsymbol{s}_b(t) + \boldsymbol{n}_b(t), \tag{7}$$

where $\boldsymbol{A}_b = [\boldsymbol{a}_q(u_1)\; \boldsymbol{a}_q(u_2) \ldots \boldsymbol{a}_q(u_P)]$ is the corresponding response matrix and $\boldsymbol{D}_b = \text{diag}\{\nu_1, \nu_2, \ldots, \nu_P\}$. Since $\nu_1 = 0$, (7) can be rewritten as

$$\boldsymbol{x}_b(t) = \boldsymbol{A}_I\boldsymbol{D}_I\boldsymbol{s}_I(t) + \boldsymbol{n}_b(t), \tag{8}$$

where $\boldsymbol{A}_I = [\boldsymbol{a}_q(u_2)\; \boldsymbol{a}_q(u_3) \cdots \boldsymbol{a}_q(u_P)]$, $\boldsymbol{D}_I = \text{diag}\{\nu_2, \nu_3, \ldots, \nu_P\}$, and $\boldsymbol{s}_I(t)$=$[s_2(t)\; s_3(t) \ldots s_P(t)]^T$. It follows from (8) that the ensemble correlation matrix of $\boldsymbol{x}_b(t)$ is given by

$$\boldsymbol{R} = E\{\boldsymbol{x}_b(t)\boldsymbol{x}_b(t)^H\} = \boldsymbol{A}_I\boldsymbol{\Psi}_I\boldsymbol{A}_I^H + \pi_n\boldsymbol{B}^H\boldsymbol{B}, \tag{9}$$

where $\boldsymbol{\Psi}_I = E\{\boldsymbol{D}_I\boldsymbol{s}_I(t)\boldsymbol{s}_I(t)^H\boldsymbol{D}_I^H\} = \boldsymbol{D}_I\boldsymbol{\Phi}_I\boldsymbol{D}_I^H$ with $\boldsymbol{\Phi}_I = E\{\boldsymbol{s}_I(t)\boldsymbol{s}_I(t)^H\}$. Equation (9) reveals that finding the required IS from $\boldsymbol{R}$ must perform the GEVD of $\boldsymbol{R}$ due to that the noise component $\boldsymbol{n}_b(t)$ of $\boldsymbol{x}_b(t)$ is no longer spatially white and has correlation matrix $\pi_n\boldsymbol{B}^H\boldsymbol{B}$. After performing the GEVD of (9), we have the following relationship for the resulting generalized eigenvectors (g-vectors) $\boldsymbol{g}_i$ and generalized eigenvalues (g-values) λ_i

$$\boldsymbol{R}\boldsymbol{g}_i = \lambda_i\boldsymbol{B}^H\boldsymbol{B}\boldsymbol{g}_i, \tag{10}$$

where $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{P-1} > \lambda_P = \ldots = \lambda_{M-q} = \pi_n$. Using the g-vectors $\boldsymbol{g}_i$, we construct two matrices as follows : $\boldsymbol{G}_{\mathrm{I}} = [\boldsymbol{g}_1, \boldsymbol{g}_2, \ldots, \boldsymbol{g}_{P-1}]$ and $\boldsymbol{G}_{\mathrm{R}} = [\boldsymbol{g}_P, \boldsymbol{g}_{P+1}, \ldots, \boldsymbol{g}_{M-q}]$. Then, it is easy to show that the matrix $\boldsymbol{A}_{\mathrm{I}}$ and $\boldsymbol{B}^H\boldsymbol{B}\boldsymbol{G}_{\mathrm{I}}$ span the same subspace which is orthogonal to the subspace spanned by $\boldsymbol{G}_{\mathrm{R}}$, i.e., range$\{\boldsymbol{A}_{\mathrm{I}}\}$ = range$\{\boldsymbol{B}^H\boldsymbol{B}\boldsymbol{G}_{\mathrm{I}}\} \perp$ range$\{\boldsymbol{G}_{\mathrm{R}}\}$.

As presented in [4], the criterion of an optimal EIC with reduced aperture size $M-q$ is to maximize the output signal-to-background noise ratio subject to a constraint of orthogonality to the IS. If $\boldsymbol{A}_{\mathrm{I}}$ is known, the criterion is given by

$$\text{Maximize } \frac{|\boldsymbol{w}^H\boldsymbol{a}_q(u_1)|^2}{\boldsymbol{w}^H\boldsymbol{w}} \quad \text{subject to } \boldsymbol{w}^H\boldsymbol{A}_{\mathrm{I}} = 0, \tag{11}$$

and the optimal weight vector is given by

$$\boldsymbol{w}_o = (\boldsymbol{I} - \boldsymbol{A}_{\mathrm{I}}(\boldsymbol{A}_{\mathrm{I}}^H\boldsymbol{A}_{\mathrm{I}})^{-1}\boldsymbol{A}_{\mathrm{I}}^H)\boldsymbol{a}_q(u_1), \tag{12}$$

where $\boldsymbol{I}$ denotes the $(M-q)$x$(M-q)$ identity matrix. In practice, $\boldsymbol{A}_{\mathrm{I}}$ is unknown and $\boldsymbol{w}^H\boldsymbol{A}_{\mathrm{I}} = 0$ can be replaced by $\boldsymbol{w}^H\boldsymbol{B}^H\boldsymbol{B}\boldsymbol{G}_{\mathrm{I}} = 0$ since range$\{\boldsymbol{A}_{\mathrm{I}}\}$ = range$\{\boldsymbol{B}^H\boldsymbol{B}\boldsymbol{G}_{\mathrm{I}}\}$. Accordingly, the optimal weight vector $\boldsymbol{w}_o$ becomes

$$\boldsymbol{w}_o = (\boldsymbol{I} - (\boldsymbol{B}^H\boldsymbol{B}\boldsymbol{G}_{\mathrm{I}})[(\boldsymbol{B}^H\boldsymbol{B}\boldsymbol{G}_{\mathrm{I}})^H(\boldsymbol{B}^H\boldsymbol{B}\boldsymbol{G}_{\mathrm{I}})]^{-1}(\boldsymbol{B}^H\boldsymbol{B}\boldsymbol{G}_{\mathrm{I}})^H)\boldsymbol{a}_q(u_1). \tag{13}$$

From (9), (10), and (13), we note that to evaluate the performance and sensitivity to steering angle of the EIC is very difficult and, hence, there are practically no papers considering this problem.

CHAPTER THREE AN EIC USING THE PROPOSED TECHNIQUE

In this chapter, we present an EIC based on a proposed technique to alleviate the difficulty described above. From (9), it is obvious that the difficulty is induced due to the effect of the signal blocking matrix $\boldsymbol{B}$ on the received sensor noise. To eliminate this effect, a technique is developed as follows. For the sake of simplicity, the notation $HT\{c_1, c_2, \ldots, c_m\}$ is used to denote an mxm Hermitian and Toeplitz matrix with its first row given by the row vector $[c_1\ c_2 \ldots c_m]$. Using (27) and (28), it can be shown that $\boldsymbol{B}^H\boldsymbol{B}$ is given by

$$\boldsymbol{B}^H\boldsymbol{B} = HT\{\epsilon_0, \epsilon_1, \ldots, \epsilon_{M-q-1}\}, \text{ with } \epsilon_i = \sum_{r=0}^{q-i} b^*_{r+i} b_r. \tag{14}$$

Next, we construct an $(M-q+i)$x1 vector as follows

$$\boldsymbol{f}(d,i) = [1\ \ \mathbf{o}_{i-1}\ \ j^d\ \ \mathbf{o}_{M-q-1}]^T, \tag{15}$$

where $i = 1, 2, \ldots, M-q-1$ and d is an integer. From (15), an $(M-q+i)$x$(M-q)$ matrix is constructed as follows

$$\boldsymbol{F}(d,i) = [\boldsymbol{f}(d,i)\ \ \tilde{\boldsymbol{I}}_{M-q+i}\boldsymbol{f}(d,i)\ \ \tilde{\boldsymbol{I}}^2_{M-q+i}\boldsymbol{f}(d,i)\ \ \ldots\ \ \tilde{\boldsymbol{I}}^{M-q-1}_{M-q+i}\boldsymbol{f}(d,i)], \tag{16}$$

where $\tilde{\boldsymbol{I}}_{M-q+i}$ is the $(M-q+i)$x$(M-q+i)$ cyclic-shifting matrix. Using (15) and (16), we have

$$\boldsymbol{\Gamma}(d,i) = \boldsymbol{F}(d,i)^H\boldsymbol{F}(d,i) = HT\{2,\ \mathbf{o}_{i-1},\ (-j)^d,\ \mathbf{o}_{M-q-1-i}\}. \tag{17}$$

From (44), it is obvious that $\boldsymbol{\Gamma}(d,i)$ is positive definite, Hermitian, and Toeplitz. Moreover, it is easy to show that

$$\begin{aligned}&|Re\{\epsilon_i\}|\boldsymbol{\Gamma}(2\text{sgn}(Re\{\epsilon_i\}), i) + |Im\{\epsilon_i\}|\boldsymbol{\Gamma}(2\text{sgn}(Im\{\epsilon_i\}) - 1, i)\\ &\quad = HT\{2(|Re\{\epsilon_i\}| + |Im\{\epsilon_i\}|),\ \mathbf{o}_{i-1},\ -\epsilon_i,\ \mathbf{o}_{M-q-1-i}\},\end{aligned} \tag{18}$$

for $i = 1, 2, \ldots, M-q-1$, where $Re\{x\}$ and $Im\{x\}$ denote the real and imaginary parts of x, respectively. $\text{sgn}(x) = 1$ if $x > 0$, and $= 0$, otherwise. Finally, we construct a positive definite matrix as follows

$$\boldsymbol{\Omega} = \sum_{i=1}^{M-q-1} (|Re\{\epsilon_i\}|\boldsymbol{\Gamma}(2\text{sgn}(Re\{\epsilon_i\}), i) + |Im\{\epsilon_i\}|\boldsymbol{\Gamma}(2\text{sgn}(Im\{\epsilon_i\}) - 1, i)) \ . \tag{19}$$

Summing (14) and (19) thus yields a diagonal matrix as follows

$$\boldsymbol{B}^H\boldsymbol{B} + \boldsymbol{\Omega} = \left(\epsilon_0 + 2\sum_{i=1}^{M-q-1}\left(|Re\{\epsilon_i\}| + |Im\{\epsilon_i\}|\right)\right)\boldsymbol{I} = \kappa\boldsymbol{I}, \tag{20}$$

where κ denotes the proportional constant.

Based on (20), the effect of the signal blocking matrix on the received sensor noise can be eliminated by taking the following matrix

$$\boldsymbol{R}_w = \boldsymbol{R} + \pi_n\boldsymbol{\Omega} = \boldsymbol{A}_I\boldsymbol{\Psi}_I\boldsymbol{A}_I^H + \kappa\pi_n\boldsymbol{I}, \tag{21}$$

as a correlation matrix to replace the original correlation matrix $\boldsymbol{R}$. Accordingly, performing the EVD on $\boldsymbol{R}_w$ yields

$$\boldsymbol{R}_w\boldsymbol{e}_i = \gamma_i\boldsymbol{e}_i, \tag{22}$$

where $\gamma_1 \geq \gamma_1 \geq \cdots \geq \gamma_{P-1} > \gamma_P = \cdots = \gamma_{M-q} = \kappa\pi_n$. Let the matrices $\boldsymbol{E}_I = [\boldsymbol{e}_1\ \boldsymbol{e}_2 \cdots \boldsymbol{e}_{P-1}]$ and $\boldsymbol{E}_R = [\boldsymbol{e}_P\ \boldsymbol{e}_{P+1} \cdots \boldsymbol{e}_{M-q}]$. Then we can easily show that $[\boldsymbol{E}_I\ \boldsymbol{E}_R]^H[\boldsymbol{E}_I\ \boldsymbol{E}_R] = \boldsymbol{I}$ and

$$\text{range}\{\boldsymbol{E}_I\} = \text{range}\{\boldsymbol{A}_I\} \perp \text{range}\{\boldsymbol{E}_R\}. \tag{23}$$

It follows from (54) and (23) that the optimal weight vector for the EIC based on the criterion shown in (11) can also be written as

$$\boldsymbol{w}_o = (\boldsymbol{I} - \boldsymbol{E}_I\boldsymbol{E}_I^H)\boldsymbol{a}_q(u_1) = \boldsymbol{E}_R\boldsymbol{E}_R^H\boldsymbol{a}_q(u_1). \tag{24}$$

CHAPTER FOUR ANALYSIS OF THE STATISTICAL PERFORMANCE

Consider the output SINR of an EIC with optimal weight vector $\boldsymbol{w}_o$. Following the derivations presented in [7], it is easy to show that the output signal power is given by

$$p_s = \pi_1 |\boldsymbol{w}_o^H \boldsymbol{a}_q(u_1)|^2, \tag{25}$$

where π_1 denotes the input power of the desired signal, the array output power due to the interference is given by

$$p_i = \boldsymbol{w}_o^H \boldsymbol{A}_I \boldsymbol{\Phi}_I \boldsymbol{A}_I^H \boldsymbol{w}_o, \tag{26}$$

and the corresponding output noise power is given by

$$p_n = \pi_n \boldsymbol{w}_o^H \boldsymbol{w}_o. \tag{27}$$

From (25) to (27), the output SINR of the EIC is thus given by

$$SINR_o = \frac{p_s}{p_i + p_n} = \frac{\pi_1 |\boldsymbol{w}_o^H \boldsymbol{a}_q(u_1)|^2}{\boldsymbol{w}_o^H \boldsymbol{A}_I \boldsymbol{\Phi}_I \boldsymbol{A}_I^H \boldsymbol{w}_o + \pi_n \boldsymbol{w}_o^H \boldsymbol{w}_o}. \tag{28}$$

In practice, the number of signal sources P, the background noise power π_n, and the ensemble correlation matrix $\boldsymbol{R}$ required for implementing the EIC are not available and usually estimated from the received data snapshots. Using the first K data snapshots, we obtain the estimate P for the number of signal sources based on the AIC or MDL criterion presented by [11]. Moreover, implementing the AIC or MDL criterion requires performing the EVD of the corresponding data correlation matrix. Therefore, π_n can be estimated by utilizing the eigenvalue method of [12] during the same estimation process. Let the estimated value be denoted as $\widehat{\pi}_n$. Then,the next L data snapshots are used to compute the sample correlation matrix $\hat{\boldsymbol{R}}$ as follows

$$\widehat{\boldsymbol{R}} = \frac{1}{L} \sum_{l=1}^{L} \boldsymbol{x}_b(t_l) \boldsymbol{x}_b(t_l)^H. \tag{29}$$

to replace $\boldsymbol{R}$, where $\boldsymbol{x}_b(t_l)$ is the data vector taken at the time instant t_l. The correlation matrix $\boldsymbol{R}_w$ of (21) is then replaced by

$$\widehat{\boldsymbol{R}}_w = \widehat{\boldsymbol{R}} + \widehat{\pi}_n \boldsymbol{\Omega}. \tag{30}$$

Note that $\widehat{\pi}_n$ and $\widehat{\boldsymbol{R}}$ are independent in this case. Accordingly, (54) becomes

$$\widehat{\boldsymbol{R}}_w \widehat{\boldsymbol{e}}_i = \widehat{\gamma}_i \widehat{\boldsymbol{e}}_i, \tag{31}$$

where $\widehat{\gamma}_1 \geq \widehat{\gamma}_2 \geq \cdots \geq \widehat{\gamma}_{M-q}$ are the eigenvalues computed based on $\widehat{\boldsymbol{R}}_w$, $\widehat{\boldsymbol{e}}_1$, $\widehat{\boldsymbol{e}}_2$, ..., $\widehat{\boldsymbol{e}}_{M-q}$, are the corresponding eigenvectors. Next, let the matrices $\widehat{\boldsymbol{E}}_{\mathrm{I}} = [\ \widehat{\boldsymbol{e}}_1, \widehat{\boldsymbol{e}}_2, \ldots \widehat{\boldsymbol{e}}_{P-1}]$ and $\widehat{\boldsymbol{E}}_{\mathrm{R}} = [\ \widehat{\boldsymbol{e}}_P, \widehat{\boldsymbol{e}}_{P+1}, \ldots, \widehat{\boldsymbol{e}}_{M-q}]$. Then the optimal weight vector of the EIC under the L finite samples is given by

$$\widehat{\boldsymbol{w}}_o = (\boldsymbol{I} - \widehat{\boldsymbol{E}}_{\mathrm{I}} \widehat{\boldsymbol{E}}_{\mathrm{I}}^H) \boldsymbol{a}_q(u_1) = \widehat{\boldsymbol{E}}_{\mathrm{R}} \widehat{\boldsymbol{E}}_{\mathrm{R}}^H \boldsymbol{a}_q(u_1). \tag{32}$$

Based on the first-order perturbation technique presented in [8] for analysis, it is shown in Appendix that the expectation of the EIC's output SINR using finite data snapshots is approximately given by

$$E\{\widehat{SINR}_o\} \approx SINR_o(1 - \frac{1}{L} Tr\{\boldsymbol{\Phi}_{\mathrm{I}} \boldsymbol{\Psi}_{\mathrm{I}}^{-1}\} \frac{\boldsymbol{w}_o^H \boldsymbol{B}^H \boldsymbol{B} \boldsymbol{w}_o}{\boldsymbol{w}_o^H \boldsymbol{w}_o}) \tag{33}$$

if the input INR is high enough. Assume that the $P-1$ interferers are uncorrelated. Then, $\boldsymbol{\Phi}_{\mathrm{I}} = \mathrm{diag}\{\pi_2, \pi_2, \ldots, \pi_P\}$ and $\boldsymbol{\Psi}_{\mathrm{I}} = \mathrm{diag}\{|\nu_2|^2 \pi_2, |\nu_3|^2 \pi_3, \ldots, |\nu_P|^2 \pi_P\}$, where $\pi_i = E\{|s_i(t)|^2\}$ denotes the input power of the ith signal source. Hence, (33) becomes

$$E\{\widehat{SINR}_o\} \approx SINR_o(1 - \frac{1}{L} \left(\sum_{i=2}^{P} |\nu_i|^{-2} \right) \frac{\boldsymbol{w}_o^H \boldsymbol{B}^H \boldsymbol{B} \boldsymbol{w}_o}{\boldsymbol{w}_o^H \boldsymbol{w}_o}). \tag{34}$$

Equation (33) reveals that the expectation of the output SINR is bounded in the range of

$$[SINR_o \left(1 - \frac{1}{L} Tr\{\boldsymbol{\Phi}_{\mathrm{I}} \boldsymbol{\Psi}_{\mathrm{I}}^{-1}\} \lambda_{max}\{\boldsymbol{B}^H \boldsymbol{B}\}\right), SINR_o \left(1 - \frac{1}{L} Tr\{\boldsymbol{\Phi}_{\mathrm{I}} \boldsymbol{\Psi}_{\mathrm{I}}^{-1}\} \lambda_{min}\{\boldsymbol{B}^H \boldsymbol{B}\}\right)] \tag{35}$$

since $(\boldsymbol{w}_o^H \boldsymbol{B}^H \boldsymbol{B} \boldsymbol{w}_o)/(\boldsymbol{w}_o^H \boldsymbol{w}_o)$ is bounded by the minimal eigenvalue $\lambda_{min}\{\boldsymbol{B}^H \boldsymbol{B}\}$ and the maximal eigenvalue $\lambda_{max}\{\boldsymbol{B}^H \boldsymbol{B}\}$ of $\boldsymbol{B}^H \boldsymbol{B}$. It follows from (20) that $\lambda_{max}\{\boldsymbol{B}^H \boldsymbol{B}\} < \kappa$. Moreover, we note from (35) that the lower bound becomes $SINR_o(1 - \kappa(P-1)/L)$ if the $(P-1)$ interferers are uncorrelated and $|u_i - u_1|$ are not less than $\pi/3$ for $i = 2, 3, \ldots, P$. For example, the value of κ is equal to 4 for $u_1 = 0$ and $q = 1$. Thus, the expectation of the output SINR of the EIC converges with a rate at least equal to $1 - 4(P-1)/L$ when $|u_i|$ are not less than $\pi/3$ for $i = 2, 3, \ldots, P$. In the following, we consider two special cases to further simplify the result of (34) under the assumption of $M \geq 2q$.

4.1 Single Interferer

Here, let u_1=0 and $q = 1$. Define the function

$$g_{M-1}(u) = \frac{\sin((M-1)u/2)}{(M-1)\sin(u/2)}. \tag{36}$$

After performing some necessary algebraic manipulations, we can obtain

$$\boldsymbol{w}_o^H \boldsymbol{w}_o = (M-1)(1 - g_{M-1}^2(u_2)), \tag{37}$$

$$\boldsymbol{w}_o^H \boldsymbol{B}^H \boldsymbol{B} \boldsymbol{w}_o = 2 + g_{M-1}^2(u_2)\left[2 + 4(M-2)\sin^2(\frac{u_2}{2})\right] - 4g_{M-1}^2(u_2)\cos(\frac{M-2}{2}u_2), \tag{38}$$

and

$$|\nu_2|^{-2} = \left(4\sin^2(\frac{u_2}{2})\right)^{-1}. \tag{39}$$

4.2 Multiple Interferers

In this case, we assume that the interferers and the desired signal are located so that $\boldsymbol{a}_q(u_1)^H \boldsymbol{a}_q(u_i)$ is approximately equal to zero for $i = 2, 3, \ldots, P$. Then, the optimal weight vector given by (24) is approximately equal to $\boldsymbol{a}_q(u_1)$. Hence, we have from (27) and (28) that

$$\boldsymbol{w}_o^H \boldsymbol{w}_o \approx \boldsymbol{a}_q(u_1)^H \boldsymbol{a}_q(u_1) = M - q \tag{40}$$

and

$$\boldsymbol{w}_o^H \boldsymbol{B}^H \boldsymbol{B} \boldsymbol{w}_o \approx \boldsymbol{a}_q(u_1)^H \boldsymbol{B}^H \boldsymbol{B} \boldsymbol{a}_q(u_1) = \sum_{i=0}^{q-1} \left(|\sum_{r=0}^{i} b_{i-r} e^{jru_1}|^2 + |\sum_{r=0}^{i} b_{q-r} e^{jru_1}|^2 \right). \tag{41}$$

Further, substituting the b_r of (27) into (41) yields

$$\boldsymbol{w}_o^H \boldsymbol{B}^H \boldsymbol{B} \boldsymbol{w}_o \approx \boldsymbol{a}_q(u_1)^H \boldsymbol{B}^H \boldsymbol{B} \boldsymbol{a}_q(u_1) = 2\sum_{i=0}^{q-1} \left[\sum_{r=0}^{i} (-1)^r \binom{q}{r}\right]^2. \tag{42}$$

CHAPTER FIVE ANALYSIS OF SENSITIVITY TO STEERING ANGLE ERROR

In practice, the signal blocking matrix is designed according to a steering angle θ_0, i.e., u_1 in (27) and (6) is replaced by $u_0 = 2\pi d \sin(\theta_0)/\lambda$. For the case of correct steering, we have $\theta_0 = \theta_1$ and $\nu_1 = 0$. Now consider that there is a small mismatch between θ_0 and θ_1. Using the first-order approximation, the blocking factor $|\nu_1|$ is then approximately given by

$$|\nu_1| = |e^{ju_1} - e^{ju_0}|^q \approx |u_1 - u_0|^q. \tag{43}$$

Equation (43) reveals that $|\nu_1|$ can not be zero since $|u_1 - u_0|$ is not equal to zero. Hence, there will be a leakage due to the desired signal in the blocked data vector. The correlation matrix of (7) is then given by

$$\boldsymbol{R} = \boldsymbol{A}_b \boldsymbol{\Psi}_b \boldsymbol{A}_b^H + \pi_n \boldsymbol{B}^H \boldsymbol{B}, \tag{44}$$

where $\boldsymbol{\Psi}_b = E\{\boldsymbol{s}_b(t)\boldsymbol{s}_b(t)^H\}$. Therefore, using the proposed technique to construct $\boldsymbol{R}_w = \boldsymbol{R} + \pi_n \boldsymbol{\Omega}$ and performing the EVD of this matrix as shown in (54) produces the first P principal eigenvalues which are greater than $\kappa\pi_n$. Moreover, the subspace spanned by the eigenvectors corresponding to these P eigenvalues is the same as that spanned by $\boldsymbol{A}_b$ if $\boldsymbol{\Psi}_b$ has full rank.

When the number of interferers is overestimated, the matrix $\boldsymbol{E}_I$ will contain more than $P-1$ principal eigenvectors and hence range$\{\boldsymbol{A}_b\} \subseteq$ range$\{\boldsymbol{E}_I\}$. From (24), the resulting optimal weight vector $\boldsymbol{w}_o$ is given by

$$\boldsymbol{w}_o = (\boldsymbol{I} - \boldsymbol{E}_I \boldsymbol{E}_I^H)\boldsymbol{a}_q(u_0) \tag{45}$$

which is orthogonal to the response vector $\boldsymbol{a}_q(u_1)$, i.e., $\boldsymbol{w}_o^H \boldsymbol{a}_q(u_1) = 0$ regardless of the input SNR and the value of q. As a result, the desired signal will be completely eliminated and, hence, the EIC will completely fail in this case.

Next , consider the situation where the number of interferers is exactly known and the desired signal is uncorrelated with the $P-1$ interferers. Based on (54) and (44), we have

$$\boldsymbol{R}_w = \pi_{b1} \boldsymbol{a}_q(u_1) \boldsymbol{a}_q(u_1)^H + \boldsymbol{R}_I + \kappa\pi_n \boldsymbol{I}, \tag{46}$$

where $\pi_{b1} = |\nu_1|^2\pi_1$ denotes the power of the desired signal leakage contained in the blocked data vector and $\boldsymbol{R}_I$ the correlation matrix due to the interferers. Let the $P-1$ nonzero eigenvalues and the corresponding eigenvectors of $\boldsymbol{R}_I$ are given by $\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_{P-1} > 0$ and $\boldsymbol{v}_i$, $i = 1, 2, \ldots, P-1$, respectively. Consider the situation in which the interferers are located far away from the desired signal so that $\boldsymbol{a}_q(u_1)^H\boldsymbol{a}_q(u_i) \approx 0$. The eigenvalues γ_i which greater than $\kappa\pi_n$ and the corresponding eigenvectors $\boldsymbol{e}_i$ of $\boldsymbol{R}_w$ in this case can thus be approximately given by $\gamma_i \approx \alpha_i + \kappa\pi_n$ and $\boldsymbol{e}_i \approx \boldsymbol{v}_i$, for $i = 1, 2, \ldots, P-1$, $\gamma_P \approx (M-q)\pi_{b1} + \kappa\pi_n$ and $\boldsymbol{e}_P \approx \boldsymbol{a}_q(u_1)/\sqrt{M-q}$, respectively. Thus, the matrix $\boldsymbol{E}_I$ will contain the first $P-1$ principal eigenvectors $\boldsymbol{v}_i$, $i = 1, 2, \ldots, P-1$, when $(M-q)\pi_{b1} < \alpha_{P-1}$. Consequently, range$\{\boldsymbol{E}_I\} \approx$ range$\{\boldsymbol{A}_I\}$ and hence the EIC will work normally. On the other hand, $\boldsymbol{E}_I$ will contain the normalized response vector $\boldsymbol{e}_P$ if $(M-q)\pi_{b1} > \alpha_{P-1}$. From the optimal weight vector given by (45), we can see that the desired signal will be suppressed due to the fact that $\boldsymbol{w}_o^H\boldsymbol{E}_I = 0$. As shown by (43) and the fact that π_{b1} is proportional to $|\nu_1|^2$, this difficulty could be alleviated by increasing the order q of the signal blocking matrix $\boldsymbol{B}$ if $|u_1 - u_0| < 1$. To look into the effect of $(M-q)\pi_{b1} > \alpha_{P-1}$, we proceed to investigate the performance of the EIC in the presence of two interferers for the following two cases.

A. Two Highly-Correlated and Widely-Separated Interferers

In this case, we assume that two interferers are widely separated so that $\boldsymbol{a}_q(u_2)^H\boldsymbol{a}_q(u_3) \approx 0$ and highly correlated so that the magnitude of their correlation coefficient $|\rho_{23}| \approx 1$. As shown in [10, p.52-p.55], the eigenvalues of $\boldsymbol{R}_I$ are given by

$$\alpha_1 = (M-q)(\pi_{b2} + \pi_{b3}) \text{ and } \alpha_2 = \frac{(M-q)\pi_{b2}\pi_{b3}}{\pi_{b2} + \pi_{b3}}(1 - |\rho_{2,3}|^2), \tag{47}$$

where $\pi_{b2} = |\nu_2|^2\pi_2$ and $\pi_{b3} = |\nu_3|^2\pi_3$ denote the powers of the two interference components contained in the blocked data vector. Hence, $(M-q)\pi_{b1} > \alpha_2$ results in the following performance breakdown threshold for the EIC

$$|\rho_{2,3}|^2 > 1 - \pi_{b1}(\pi_{b2}^{-1} + \pi_{b3}^{-1}). \tag{48}$$

B. Two Lowly-Correlated and Closely-Separated Interferers

Here, we assume that two interferers are closely separated so that $\boldsymbol{a}_q(u_2)^H \boldsymbol{a}_q(u_3) \approx M - q$ and lowly correlated so that their correlation coefficient $\rho_{2,3} \approx 0$. Again, following the derivation similar to Section 5.1, we obtain the eigenvalues of $\boldsymbol{R}_I$ as follows

$$\alpha_1 = (M-q)(\pi_{b2} + \pi_{b3}) \text{ and } \alpha_2 = \frac{(M-q)\pi_{b2}\pi_{b3}}{\pi_{b2} + \pi_{b3}}(1 - |g_{M-q}(u_3 - u_2)|^2). \tag{49}$$

Hence, $(M-q)\pi_{b1} < \alpha_2$ results in the following performance breakdown threshold for the EIC

$$|g_{M-q}(u_3 - u_2)|^2 > 1 - \pi_{b1}(\pi_{b2}^{-1} + \pi_{b3}^{-1}). \tag{50}$$

Furthermore, if $|u_3 - u_2|$ is small enough, it is also shown in [10, p.52-p.55] that

$$|g_{M-q}(u_3 - u_2)| \approx 1 - \frac{(M-q)^2 - 1}{24}(u_3 - u_2)^2. \tag{51}$$

From (51), the breakdown threshold of (50) becomes

$$(u_3 - u_2)^2 < \frac{12}{(M-q)^2 - 1}\pi_{b1}(\pi_{b2}^{-1} + \pi_{b3}^{-1}). \tag{52}$$

CHAPTER SIX SIMULATION EXAMPLES AND COMPARISON

In this chapter, several simulation examples for confirmation and comparison are presented. The adaptive array considered for all simulations is an M-element ULA with interelement spacing equal to half of the signal wavelength.

Example 1: Here, we illustrate the statistical performance of the EIC using an array with $M = 10$ sensor elements and a signal blocking matrix $\boldsymbol{B}$ with $q = 1$. The desired signal with input SNR = 0 dB is impinging on the array from the broadside, i.e., $\theta_1 = 0$. The first $K = 50$ data snapshots are used to estimate the source number P and the noise power π_n by the procedure described in Section III. Figure 1(a) shows the expectation of the output SINR versus the number L of snapshots without steering angle error. Three groups of curves from top to the bottom show the simulation results for the three cases, namely one 20 dB interferer with direction angle $\theta_2 = 50^o$, two uncorrelated 20 dB interferers with direction angles $\theta_2 = 50^o$ and $\theta_3 = 55^o$, and three uncorrelated 20 dB interferers with direction angles $\theta_2 = 50^o$, $\theta_3 = 55^o$, and $\theta_4 = -60^o$, respectively. For each case, the solid line represents the result computed based on (34), while the dash line represents the result computed based on the approximations shown by (40) and (42). In contrast, the curve with 'x' represents the result using the proposed EIC, whereas the curve with 'o' represents the result using the EIC of [4] based on the average of 100 independent runs. Comparing the results, we observe that the proposed EIC and the EIC of [4] have almost the same performance for this case. Moreover, these simulations confirm the statistical analysis for the proposed EIC presented in Section IV.

Next, we investigate the effect of q on the EIC's performance. Figure 1(b) shows the expectation of the output SINR versus the number L of snapshots. The desired signal with SNR = 0 dB is impinging on the array from the broadside, while two uncorrelated interferers with INR = 20 dB are impinging on the array from 50^o and -60^o, respectively. To make the effective aperture size (which is given by $M - q$) and thus the ideal output SINR the same for comparison, we consider three cases with $(M,q) = (12,3)$, $(M,q) = (11,2)$, and $(M,q) = (10,1)$, respectively. Three groups of curves from top to the bottom show the simulation results for the three cases. For each cases, the solid line represents the result computed based on (34), while the dash line represents the result computed based on the approximations shown by (40) and (42). In contrast, the curve with 'x' represents the result using the proposed EIC, whereas the curve with 'o' represents the result using the EIC of [4] based on the average of 100 independent runs. Comparing the results, we observe that the approximations shown by (40) and (42) are quite appropriate. Again, the proposed EIC and the EIC of [4] have almost the same performance for this case. Moreover, these simulations confirm the statistical analysis for the proposed EIC presented in Section IV.

Example 2: This example illustrates the effect of the angle separation between the desired signal and interference on the EIC's performance. For simplicity, we consider only one interferer with INR = 30 dB impinging on the array of 10 elements from direction angle θ_2. The desired signal with input SNR = 0 dB is impinging on the array from the broadside. The signal blocking matrix $\boldsymbol{B}$ has order $q = 1$. The first $K = 50$ data snapshots are used to estimate P and π_n. Figure 2(a) depicts the expectation of the output SINR versus the number L of snapshots. Four groups of curves from top to the bottom show the simulation results for $\theta_2 = 10^o$, 9^o, 8^o, and 7^o, respectively. For each case, the solid line represents the result computed based on the approximations shown by (37), (38), and (39). In contrast, the curve with 'x' represents the result using the proposed EIC, whereas the curve with 'o' represents the result using the EIC of [4] based on the average of 100 independent runs. We note that the EIC's performance deteriorates as θ_2 decreases. Figure 2(b) plots the value of $|\nu_2|^{-2}(\boldsymbol{w}_o^H \boldsymbol{B}^H \boldsymbol{B} \boldsymbol{w}_o)/(\boldsymbol{w}_o^H \boldsymbol{w}_o)$ computed from (37)-(39) versus θ_2. The curves from top to the bottom show the simulation results for the effective aperture size $M-1$ varying from 10 to 17. It is clear that the value of $|\nu_2|^{-2}(\boldsymbol{w}_o^H \boldsymbol{B}^H \boldsymbol{B} \boldsymbol{w}_o)/(\boldsymbol{w}_o^H \boldsymbol{w}_o)$ is a monotone decreasing function of θ_2 and, hence, the EIC's performance degrades as θ_2 decreases.

Example 3: The sensitivity to steering angle error is investigated. We use an array with 10 sensor elements and a signal blocking matrix B with order q=1. To avoid the finite sample effect, the ensemble correlation matrix $\boldsymbol{R} = E\{\boldsymbol{x}_b(t)\boldsymbol{x}_b(t)^H\}$ is used, where $\boldsymbol{x}_b(t) = \boldsymbol{B}^H\boldsymbol{x}(t)$ and $\boldsymbol{B}$ is designed based on the steering angle θ_0 which is not equal to θ_1. To measure the sensitivity of the EIC designed by using the proposed technique to the steering angle error, a robustness index (RI) is introduced as follows:

$$RI_1 = \frac{\text{Output SINR using (24) with } \boldsymbol{a}_q(u_1) \text{ replaced by } \boldsymbol{a}_q(u_0)}{\text{Output SINR using (12) with } \boldsymbol{a}_q(u_1) \text{ replaced by } \boldsymbol{a}_q(u_0)} \tag{53}$$

where $\boldsymbol{E}_I$ in (24) is obtained by performing the EVD of(54) as shown by (54). In contrast, the RI is defined as

$$RI_2 = \frac{\text{Output SINR using (13) with } \boldsymbol{a}_q(u_1) \text{ replaced by } \boldsymbol{a}_q(u_0)}{\text{Output SINR using (12) with } \boldsymbol{a}_q(u_1) \text{ replaced by } \boldsymbol{a}_q(u_0)} \tag{54}$$

for measuring the corresponding sensitivity of the EIC designed by using the technique of [4].

First, we consider the case of highly correlated interferers. The desired signal with SNR = 10 dB is impinging on the array from $\theta_1 = 2^o$. There are two interferers with INR = 0 dB and direction angles $\theta_2 = 43^o$ and $\theta_3 = -56^o$, respectively. The steering angle $\theta_0 = 0$. Figure 3(a) depicts the RI versus the magnitude of the correlation coefficient $|\rho_{2,3}|$ of these two interferers. The vertical line shows the breakdown threshold computed from (48). For comparison, the results using the EIC of [4] are also plotted. From this figure, we can see that the proposed EIC is very effective for dealing with the situation where steering angle error is encountered.

Next, we consider the case of closely separated interferers. Two uncorrelated interferers with INR = 0 dB are impinging on the array from $\theta_2 = 55^o$ and $\theta_3 = 55^o + \Delta\theta$, respectively. The desired signal with SNR = 7 dB is impinging on the array from $\theta_1 = 2^o$. The steering angle is still $\theta_0 = 0^o$. Figure 3(b) shows the RI versus $\Delta\theta$. Here, the vertical line shows the breakdown threshold computed from (52). Again, the results using the EIC of [4] are plotted for comparison. We observe that the proposed EIC is more effective than the EIC of [4] against steering angle error. Moreover, it is confirmed by Figure 3 that the breakdown thresholds shown by (48) and (52) are appropriate theoretical results.

CHAPTER SEVEN CONCLUSION

Conventional eigenspace-based interference cancellers (EICs) like the one of [4] suffer with the lack of performance and sensitivity analyses due to the fact that the noise at the output of its signal blocking matrix is nonwhite. This report has proposed an EIC and presented the analyses of its performance and sensitivity to steering angle error. We first present a technique to construct a positive definite matrix based on the signal blocking matrix and then use this matrix to compensate the effect of the signal blocking matrix on the sensor noise received by the EIC. Therefore, the interference subspace required for finding the optimal weight vector can be obtained using conventional EVD. This leads to that the performance and sensitivity to steering angle error of the EIC can be analyzed. Computer simulations have confirmed the theoretical results. Moreover, it has been shown by simulations that the performance of the proposed EIC is almost the same as that of [4] in the situations without steering angle error. However, the proposed EIC demonstrates the advantage of possessing robust capabilities against steering angle error over the EIC of [4].

APPENDIX

Here, we show the derivation of the result given by (33). Performing the eigen-decomposition for $\boldsymbol{R}_w$, we obtain

$$\boldsymbol{R}_w = \boldsymbol{E}_{\mathrm{I}}\boldsymbol{\Lambda}_{\mathrm{I}}\boldsymbol{E}_{\mathrm{I}}^H + \boldsymbol{E}_{\mathrm{R}}\boldsymbol{\Lambda}_{\mathrm{R}}\boldsymbol{E}_{\mathrm{R}}^H, \tag{A-1}$$

where $\boldsymbol{\Lambda}_{\mathrm{I}} = \mathrm{diag}\{\gamma_1, \gamma_2, \ldots, \gamma_{P-1}\}$ and $\boldsymbol{\Lambda}_{\mathrm{R}} = \mathrm{diag}\{\gamma_P, \gamma_{P+1}, \ldots, \gamma_{M-q}\} = \kappa\pi_n\boldsymbol{I}$. Similarly, for $\widehat{\boldsymbol{R}}_w$, we obtain

$$\widehat{\boldsymbol{R}}_w = \widehat{\boldsymbol{E}}_{\mathrm{I}}\widehat{\boldsymbol{\Lambda}}_{\mathrm{I}}\widehat{\boldsymbol{E}}_{\mathrm{I}}^H + \widehat{\boldsymbol{E}}_{\mathrm{R}}\widehat{\boldsymbol{\Lambda}}_{\mathrm{R}}\widehat{\boldsymbol{E}}_{\mathrm{R}}^H, \tag{A-2}$$

where $\widehat{\boldsymbol{\Lambda}}_{\mathrm{I}} = \mathrm{diag}\{\widehat{\gamma}_1, \widehat{\gamma}_2, \ldots, \widehat{\gamma}_{P-1}\}$ and $\widehat{\boldsymbol{\Lambda}}_{\mathrm{R}} = \mathrm{diag}\{\widehat{\gamma}_P, \widehat{\gamma}_{P+1}, \ldots, \widehat{\gamma}_{M-q}\}$. The deviation between $\widehat{\boldsymbol{R}}_w$ and $\boldsymbol{R}_w$ due to finite sample effect is given by

$$\Delta\boldsymbol{R}_w = \widehat{\boldsymbol{R}}_w - \boldsymbol{R}_w = \Delta\boldsymbol{R} + \Delta\pi_n\boldsymbol{\Omega}, \tag{A-3}$$

where $\Delta\boldsymbol{R} = \widehat{\boldsymbol{R}} - \boldsymbol{R}$ and $\Delta\pi_n = \widehat{\pi}_n - \pi_n$. Following the first-order perturbation analysis presented in [8], we can show that

$$\Delta\boldsymbol{E}_{\mathrm{R}} = \widehat{\boldsymbol{E}}_{\mathrm{R}} - \boldsymbol{E}_{\mathrm{R}} \approx -\boldsymbol{R}_{\mathrm{I}}^{+}\boldsymbol{\Delta}\boldsymbol{R}_w\boldsymbol{E}_{\mathrm{R}}, \tag{A-4}$$

where

$$\boldsymbol{R}_{\mathrm{I}}^{+} = \boldsymbol{E}_{\mathrm{I}}(\boldsymbol{\Lambda}_{\mathrm{I}} - \kappa\pi_n\boldsymbol{I})^{-1}\boldsymbol{E}_{\mathrm{I}}^H. \tag{A-5}$$

$\boldsymbol{R}_{\mathrm{I}}^{+}$ possesses the following properties

$$\boldsymbol{R}_{\mathrm{I}}^{+} = \boldsymbol{E}_{\mathrm{I}}(\boldsymbol{A}_{\mathrm{I}}^H\boldsymbol{E}_{\mathrm{I}})^{-1}\boldsymbol{\Psi}_{\mathrm{I}}^{-1}(\boldsymbol{A}_{\mathrm{I}}^H\boldsymbol{E}_{\mathrm{I}})^{-H}\boldsymbol{E}_{\mathrm{I}}^H \tag{A-6}$$

and

$$\boldsymbol{R}_{\mathrm{I}}^{+}\boldsymbol{R}_{\mathrm{I}}\boldsymbol{R}_{\mathrm{I}}^{+} = \boldsymbol{R}_{\mathrm{I}}^{+}. \tag{A-7}$$

By substituting (4) into (32), the optimal weight vector under finite samples can be approximated by

$$\widehat{\boldsymbol{w}}_o \approx \boldsymbol{w}_o + (\boldsymbol{E}_{\mathrm{R}}\boldsymbol{\Delta}\boldsymbol{E}_{\mathrm{R}}^H + \boldsymbol{\Delta}\boldsymbol{E}_{\mathrm{R}}\boldsymbol{E}_{\mathrm{R}}^H)\boldsymbol{a}_q(u_1). \tag{A-8}$$

It follows from (8) that the output power of the desired signal, the background noise, and the interference are given by

$$\begin{cases} \widehat{p}_s = \pi_1 |\widehat{\boldsymbol{w}}_o^H \boldsymbol{a}_q(\boldsymbol{u}_1)|^2 \approx p_s + \sum_{k=1}^{3} \Delta p_{s,k} + \text{the first order term} \\ \widehat{p}_n = \pi_n \widehat{\boldsymbol{w}}_o^H \widehat{\boldsymbol{w}}_o \approx p_n + \sum_{k=1}^{2} \Delta p_{n,k} + \text{the first order term} \\ \widehat{p}_i = \widehat{\boldsymbol{w}}_o^H \boldsymbol{A}_{\mathrm{I}} \boldsymbol{\Phi}_{\mathrm{I}} \boldsymbol{A}_{\mathrm{I}} \widehat{\boldsymbol{w}}_o \approx p_i + \Delta p_i + \text{the first order term} \end{cases}, \tag{A-9}$$

respectively, where p_s, p_n, and p_i denote the corresponding output powers without finite sample effect. Moreover,

$$\begin{cases} \Delta p_{s,1} = \Delta p_{s,2}^{*} = \pi_1 (\boldsymbol{a}_q(u_1)^H \Delta \boldsymbol{E}_{\mathrm{R}} \boldsymbol{E}_{\mathrm{R}}^H \boldsymbol{a}_q(u_1))^2 \\ \Delta p_{s,3} = 2\pi_1 (\boldsymbol{a}_q(u_1)^H \boldsymbol{E}_{\mathrm{R}} \Delta \boldsymbol{E}_{\mathrm{R}}^H \boldsymbol{a}_q(u_1))(\boldsymbol{a}_q(u_1)^H \Delta \boldsymbol{E}_{\mathrm{R}} \boldsymbol{E}_{\mathrm{R}}^H \boldsymbol{a}_q(u_1)) \end{cases}, \tag{A-10}$$

$$\begin{cases} \Delta p_{n,1} = \pi_n \boldsymbol{a}_q(u_1)^H \Delta \boldsymbol{E}_{\mathrm{R}} \Delta \boldsymbol{E}_{\mathrm{R}}^H \boldsymbol{a}_q(u_1) \\ \Delta p_{n,2} = \pi_n \boldsymbol{a}_q(u_1)^H \boldsymbol{E}_{\mathrm{R}} \Delta \boldsymbol{E}_{\mathrm{R}}^H \Delta \boldsymbol{E}_{\mathrm{R}} \boldsymbol{E}_{\mathrm{R}}^H \boldsymbol{a}_q(u_1) \end{cases}, \tag{A-11}$$

and

$$\Delta p_i = \boldsymbol{a}_q(u_1)^H \boldsymbol{E}_{\mathrm{R}} \Delta \boldsymbol{E}_{\mathrm{R}}^H \boldsymbol{A}_{\mathrm{I}} \boldsymbol{\Phi}_{\mathrm{I}} \boldsymbol{A}_{\mathrm{I}}^H \Delta \boldsymbol{E}_{\mathrm{R}} \boldsymbol{E}_{\mathrm{R}}^H \boldsymbol{a}_q(u_1) \tag{A-12}$$

denote the second-order perturbation terms, respectively. It is appropriate to neglect the output power due to the interference when the EIC works normally. Accordingly, the output SINR of the EIC can be written as

$$\widehat{SINR}_o = \frac{p_s(1 + (\widehat{p}_s - p_s)/p_s)}{p_n(1 + (\widehat{p}_n - p_n)/p_n + \widehat{p}_i/p_n)} \tag{A-13}$$

It is expected that all of the deviation terms $\widehat{p}_s - p_s$, $\widehat{p}_n - p_n$, and $\widehat{p}_i - p_i$ approach zero as the number of data snapshots increases. Using the first-order approximation when the number of data snapshots is large enough, (13) can be approximated by

$$\widehat{SINR}_o \approx SINR_o(1 + (\widehat{p}_s - p_s)/p_s - (\widehat{p}_n - p_n)/p_n - \widehat{p}_i/p_n). \tag{A-14}$$

Since the expectation for each of the first order terms in (9) is zero, the expectation of the output SINR can be approximately given by

$$E\{\widehat{SINR}_o\} \approx SINR_o(1 + E\{\Delta p_s/p_s\} - E\{\Delta p_n/p_n\} - E\{\Delta p_i/p_n\}), \tag{A-15}$$

where $\Delta p_s = \sum_{k=1}^{3} \Delta p_{s,k}$ and $\Delta p_n = \sum_{k=1}^{2} \Delta p_{n,k}$.

In the following, we compute each of the expectation terms $E\{\Delta p_s/p_s\}$, $E\{\Delta p_n/p_n\}$, and $E\{\Delta p_i/p_n\}$ in (15). As shown in (3), the deviation $\Delta \boldsymbol{R}_w$ is composed of two independent terms, i.e., $\Delta\pi_n$ and $\Delta \boldsymbol{R}$. Based on the eigenvalue method of [12] to estimate the noise power, it has been shown that

$$E\{|\Delta\pi_n|^2\} = \frac{\pi_n^2}{K(M-P)} \tag{A-16}$$

if K data snapshots are used. Next, let $\boldsymbol{R}_x = E\{\boldsymbol{x}(t)\boldsymbol{x}(t)^H\}$ and $\widehat{\boldsymbol{R}}_x = L^{-1}\sum_{l=1}^{L}\boldsymbol{x}(t_l)\boldsymbol{x}(t_l)^H$. Then we have

$$\widehat{\boldsymbol{R}} = \boldsymbol{B}^H\widehat{\boldsymbol{R}}_x\boldsymbol{B} \text{ and } \Delta\boldsymbol{R} = \boldsymbol{B}^H\Delta\boldsymbol{R}_x\boldsymbol{B}, \tag{A-17}$$

where $\Delta\boldsymbol{R}_x = \widehat{\boldsymbol{R}}_x$ - $\boldsymbol{R}_x$. Using the result presented in [9], it can be shown that $\Delta\boldsymbol{R}_x$ has the following statistical properties

$$E\{\Delta\boldsymbol{R}_x\} = 0 and E\{\boldsymbol{Q}_1^H\Delta\boldsymbol{R}_x\boldsymbol{Q}_2\boldsymbol{Q}_3^H\Delta\boldsymbol{R}_x\boldsymbol{Q}_4\} = L^{-1}Tr\{\boldsymbol{Q}_3^H\boldsymbol{R}_x\boldsymbol{Q}_2\}(\boldsymbol{Q}_1^H\boldsymbol{R}_x\boldsymbol{Q}_4), \tag{A-18}$$

where $\boldsymbol{Q}_i, i = 1,2,3,4$, are matrices with appropriate sizes. Substituting (4) and (3) into (10)-(12), and using the properties of (16) and (18), we can obtain the following expectations

$$\begin{cases} E\{\Delta p_i\}/p_n \approx E\{\Delta p_{i,a}\}/p_n + E\{\Delta p_{i,b}\}/p_n + E\{\Delta p_{i,c}\}/p_n \\ E\{\Delta p_{i,a}\}/p_n = (L\boldsymbol{w}_o^H\boldsymbol{w}_o)^{-1}Tr\{\boldsymbol{\Psi}_I^{-1}\boldsymbol{\Phi}_I\}\boldsymbol{w}_o^H\boldsymbol{B}^H\boldsymbol{B}\boldsymbol{w}_o \\ E\{\Delta p_{i,b}\}/p_n = (L\boldsymbol{w}_o^H\boldsymbol{w}_o)^{-1}(Tr\{\pi_n\boldsymbol{R}_I^+\boldsymbol{A}_I\boldsymbol{\Phi}_I\boldsymbol{A}_I^H\boldsymbol{R}_I^+\boldsymbol{B}^H\boldsymbol{B}\})\boldsymbol{w}_o^H\boldsymbol{B}^H\boldsymbol{B}\boldsymbol{w}_o \\ E\{\Delta p_{i,c}\}/p_n = (K(M-P)\boldsymbol{w}_o^H\boldsymbol{w}_o)^{-1}(\pi_n\boldsymbol{w}_o^H\boldsymbol{\Omega}\boldsymbol{R}_I^+\boldsymbol{A}_I\boldsymbol{\Phi}_I\boldsymbol{A}_I^H\boldsymbol{R}_I^+\boldsymbol{\Omega}\boldsymbol{w}_o), \end{cases} \tag{A-19}$$

$$\begin{cases} E\{\Delta p_{n,1}\}/p_n \approx E\{\Delta p_{n,1a}\}/p_n + E\{\Delta p_{n,1b}\}/p_n + E\{\Delta p_{n,1c}\}/p_n \\ E\{\Delta p_{n,1a}\}/p_n = (L\boldsymbol{w}_o^H\boldsymbol{w}_o)^{-1}Tr\{\boldsymbol{E}_R\boldsymbol{E}_R^H\boldsymbol{B}^H\boldsymbol{B}\}(\pi_n\boldsymbol{a}_q(u_1)^H\boldsymbol{R}_I^+\boldsymbol{a}_q(u_1)) \\ E\{\Delta p_{n,1b}\}/p_n = (L\boldsymbol{w}_o^H\boldsymbol{w}_o)^{-1}Tr\{\boldsymbol{E}_R\boldsymbol{E}_R^H\boldsymbol{B}^H\boldsymbol{B}\}(\pi_n^2\boldsymbol{a}_q(u_1)^H\boldsymbol{R}_I^+\boldsymbol{B}^H\boldsymbol{B}\boldsymbol{R}_I^+\boldsymbol{a}_q(u_1)) \\ E\{\Delta p_{n,1c}\}/p_n = (K(M-P)\boldsymbol{w}_o^H\boldsymbol{w}_o)^{-1}(\pi_n^2\boldsymbol{a}_q(u_1)^H\boldsymbol{R}_I^+\boldsymbol{\Omega}\boldsymbol{E}_R\boldsymbol{E}_R^H\boldsymbol{\Omega}\boldsymbol{R}_I^+\boldsymbol{a}_q(u_1)), \end{cases} \tag{A-20}$$

$$\begin{cases} E\{\Delta p_{n,2}\}/p_n \approx E\{\Delta p_{n,2a}\}/p_n + E\{\Delta p_{n,2b}\}/p_n + E\{\Delta p_{n,2c}\}/p_n \\ E\{\Delta p_{n,2a}\}/p_n = (L\boldsymbol{w}_o^H\boldsymbol{w}_o)^{-1}Tr\{\pi_n\boldsymbol{R}_I^+\}\boldsymbol{w}_o^H\boldsymbol{B}^H\boldsymbol{B}\boldsymbol{w}_o \\ E\{\Delta p_{n,2b}\}/p_n = (L\boldsymbol{w}_o^H\boldsymbol{w}_o)^{-1}Tr\{\pi_n^2\boldsymbol{R}_I^+\boldsymbol{B}^H\boldsymbol{B}\boldsymbol{R}_I^+\}\boldsymbol{w}_o^H\boldsymbol{B}^H\boldsymbol{B}\boldsymbol{w}_o \\ E\{\Delta p_{n,2c}\}/p_n = (K(M-P)\boldsymbol{w}_o^H\boldsymbol{w}_o)^{-1}(\pi_n^2\boldsymbol{w}_o^H\boldsymbol{\Omega}\boldsymbol{R}_I^+\boldsymbol{R}_I^+\boldsymbol{\Omega}\boldsymbol{w}_o), \end{cases} \tag{A-21}$$

$$\begin{cases} E\{\Delta p_{s,1}\}/p_s \approx E\{\Delta p_{s,1a}\}/p_s + E\{\Delta p_{s,1b}\}/p_s \\ E\{\Delta p_{s,1a}\}/p_s = (L(\boldsymbol{w}_o^H \boldsymbol{w}_o)^2)^{-1}(\pi_n \boldsymbol{w}_o^H \boldsymbol{B}^H \boldsymbol{B} \boldsymbol{R}_I^+ \boldsymbol{a}_q(u_1))^2 \\ E\{\Delta p_{s,1b}\}/p_s = (K(M-P)(\boldsymbol{w}_o^H \boldsymbol{w}_o)^2)^{-1}(\pi_n \boldsymbol{w}_o^H \boldsymbol{\Omega} \boldsymbol{R}_I^+ \boldsymbol{a}_q(u_1))^2, \end{cases} \tag{A-22}$$

and

$$\begin{cases} E\{\Delta p_{s,3}\}/p_s \approx E\{\Delta p_{s,3a}\}/p_s + E\{\Delta p_{s,3b}\}/p_s + E\{\Delta p_{s,3c}\}/p_s \\ E\{\Delta p_{s,3a}\}/p_s = 2(L(\boldsymbol{w}_o^H \boldsymbol{w}_o)^2)^{-1}(\pi_n \boldsymbol{a}_q(u_1)^H \boldsymbol{R}_I^+ \boldsymbol{a}_q(u_1)) \boldsymbol{w}_o^H \boldsymbol{B}^H \boldsymbol{B} \boldsymbol{w}_o \\ E\{\Delta p_{s,3b}\}/p_s = 2(L(\boldsymbol{w}_o^H \boldsymbol{w}_o)^2)^{-1}(\pi_n^2 \boldsymbol{a}_q(u_1)^H \boldsymbol{R}_I^+ \boldsymbol{B}^H \boldsymbol{B} \boldsymbol{R}_I^+ \boldsymbol{a}_q(u_1)) \boldsymbol{w}_o^H \boldsymbol{B}^H \boldsymbol{B} \boldsymbol{w}_o \\ E\{\Delta p_{s,3c}\}/p_s = 2(K(M-P)(\boldsymbol{w}_o^H \boldsymbol{w}_o)^2)^{-1} |\pi_n \boldsymbol{w}_o^H \boldsymbol{\Omega} \boldsymbol{R}_I^+ \boldsymbol{a}_q(u_1)|^2. \end{cases} \tag{A-23}$$

Without loss of generality, let $\boldsymbol{\Phi}_I = c\ \boldsymbol{\Phi}_o$ for some positive number c and positive definite matrix $\boldsymbol{\Phi}_o$. According to (6), it can be seen that $\boldsymbol{R}_I^+$ is proportional to c^{-1}. Moreover, it can be shown from (19)-(23) that each of the following terms

$$\left\{ \{E\{\Delta p_{i,b}\}/p_n, E\{\Delta p_{i,c}\}/p_n, E\{\Delta p_{n,1a}\}/p_n, E\{\Delta p_{n,2a}\}/p_n, and\ E\{\Delta p_{s,3a}\}/p_s\} \right. \tag{A-24}$$

is proportional to c^{-1} and each of the following terms

$$\begin{cases} \{E\{\Delta p_{n,1b}\}/p_n, E\{\Delta p_{n,1c}\}/p_n, E\{\Delta p_{n,2b}\}/p_n, E\{\Delta p_{n,2c}\}/p_n, E\{\Delta p_{s,1a}\}/p_s, \\ \quad E\{\Delta p_{s,1b}\}/p_s, E\{\Delta p_{s,3b}\}/p_s, and\ E\{\Delta p_{s,3c}\}/p_s\} \end{cases} \tag{A-25}$$

is proportional to c^{-2}, while only the term $E\{\Delta p_{i,a}\}/p_n$ is fixed and independent of c. To get a further simplification, consider the case that c is large enough, i.e., the input INR is high enough such that these terms in (24) and (A.25) are negligible as compared to $E\{\Delta p_{i,a}\}/p_n$. Accordingly, the expectation of the output SINR given by (A.15) can be approximated by

$$E\{\widehat{SINR}_o\} \approx SINR_o \left(1 - \frac{1}{L} Tr\{\boldsymbol{\Psi}_I \boldsymbol{\Phi}_I^{-1}\} \frac{\boldsymbol{w}_o^H \boldsymbol{B}^H \boldsymbol{B} \boldsymbol{w}_o}{\boldsymbol{w}_o^H \boldsymbol{w}_o}\right). \tag{A-26}$$

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行政院國家科學委員會補助專題研究計畫成果報告
(完整報告)

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應用特徵信號空間的可適性干擾信號消除技術之研究(II)

Research on Eigenspace-Based Adaptive Interference Cancellation (II)

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計畫類別：■個別型計畫　□整合型計畫

計畫編號：NSC90－2213－E－002－043－

執行期間： 90 年 8 月 1 日至 91 年 7 月 31 日

計畫主持人： 李枝宏

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中　華　民　國　91 年 9 月　16 日

PART 2 : RESEARCH ON EIGENSPACE-BASED ADAPTIVE INTERFERENCE CANCELLATION (2/2)

ABSTRACT

This report presents the research achievement for the second-year NSC research project. This project deals with the problem of eigenspace-based interference cancellation using a two-dimensional (2-D) rectangular array. An efficient 2-D signal blocking technique is presented to remove the desired signal from the received array data. In conjunction with the 2-D signal blocking technique, a positive definite matrix is further constructed and used to compensate the effect of the signal blocking operation on the sensor noise received by a 2-D eigenspace-based interference canceller (EIC). Therefore, the interference subspace required for computing the optimal weight vector of the designed 2-D EIC can be obtained by simply using conventional eigenvalue decomposition methods instead of any complicated generalized eigenvalue decomposition methods. The performances of the designed 2-D EIC under finite samples and steering angle error are also evaluated. The developed theoretical results are confirmed by several simulation examples.

CHAPTER ONE INTRODUCTION

Adaptive interference cancellation can be used for maximizing the rejection of interference regardless of the interference-to-noise ratio (INR) when processing array data. This goal can be efficiently achieved by utilizing eigenspace-based interference cancellers (EIC) as presented in the literature [1-6]. A common feature for these EICs is that the interference subspace (IS) spanned by the interferers must be first computed. Then, the optimal weight vector is computed by maximizing the output signal-to-noise power ratio (SNR) subject to a constraint of orthogonality to the IS.

Notable among these EIC is the one presented by [4] due to its several advantages over the others. Using a one-dimensional (1-D) uniformly linear array (ULA) and an appropriately designed signal blocking processor which blocks the desired signal from the received array data, it finds the IS through the generalized eigenvalue decomposition (GEVD) of the correlation matrix of the data vector at the output of the signal blocking processor. However, the noise component left in the blocked data vector is no longer spatially white because of using the signal blocking matrix. Hence, finding the required IS for computing the optimal weight vector inevitably resorts to a complicated GEVD. As a result, it is very difficult to evaluate the statistical performance under finite samples and the robust capability against steering angle error for the EIC. Moreover, the technique presented in [4] can not be extended to process two-dimensional (2-D) array data since its 1-D blocking scheme can not be directly applied to the 2-D case. In the literature, there are practically no papers dealing with eigenspace-based interference cancellation using 2-D adaptive arrays.

In this research report, we present the theoretical results for designing and analyzing an EIC using a 2-D adaptive array. Two 1-D blocking matrices are first designed for both row and column subarrays, respectively. Using the blocked data vectors at the output of these 1-D blocking matrices and the properties of Kronecker product for matrices, a 2-D blocking technique is developed to construct a blocked data correlation matrix R that does not contain the desired signal component for computing the IS. However, the noise component in R is no longer spatially white, which introduces more complexity in computing the IS. To eliminate this effect, a positive definite matrix Ω is created from the designed blocking scheme and $\pi_n\Omega$ is then added to R, where π_n is the background noise power. The resulting data correlation matrix $R + \pi_n\Omega$ then possesses a noise component which is spatially white. As a result, we can find an orthogonal basis matrix of the IS by performing the conventional EVD and then construct the optimal weight vector using this orthogonal basis matrix. This technique facilitates the analyses of the statistical performance under finite samples and the robust capability against steering angle error for the 2-D EIC. Theoretical results on the expectation of the output signal-to-interference plus noise ratio (SINR) are presented for showing the statistical performance of the 2-D EIC. As to the robust capability against steering angle error, it is shown that the performance of the 2-D EIC may be significantly degraded even if there is a small steering angle error. However, using the proposed 2-D blocking technique with higher order can alleviate the difficulty. The breakdown threshold for the 2-D EIC's performance due to steering angle error is also derived.

This report is organized as follows. Chapter Two presents the design of an eigenspace-based interference canceller using a 2-D rectangular array. A 2-D blocking technique is developed and the construction of a positive definite matrix for eliminating the effect of the 2-D blocking operation on the spatially white noise received by the 2-D array is proposed. Chapter Three analyzes the statistical performance under finite snapshots for the designed 2-D EIC. The performance of the designed 2-D EIC in the presence of steering angle error is evaluated in Chapter Four. Several simulation examples for illustration and confirmation of the theoretical works are provided in Chapter Five. Finally, Chapter Six gives a conclusion for this report.

CHAPTER TWO DESIGN OF A 2-D EIGENSPACE BASED INTERFERENCE CANCELLER

2.1 The 2-D Array Data Model

Consider a 2-D $\bar{M}$x$\bar{N}$ uniform rectangular array (URA) with sensors located on the X-Y plane at the positions $((m-1)\lambda/2,(n-1)\lambda/2)$ for $m=1,2,\ldots,\bar{M}$ and $n=1,2,\ldots,\bar{N}$, where λ represents the signal wavelength. Let the signal impinging on the array from the elevation angle θ and azimuth angle ϕ yield a unit magnitude response and a phase response given by $\exp\{j[(m-1)\pi u+(n-1)\pi v]\}$ at the array sensor located at $((m-1)\lambda/2,(n-1)\lambda/2)$, where $j=\sqrt{-1}$ and $(u,v)=(\sin(\theta)\cos(\phi),\ \sin(\theta)\sin(\phi))$. P narrowband signals are impinging on the URA from P distinct angles (u_i,v_i) for $i=1,2,\ldots,P$. Thus, the data received by the sensor located at $((m-1)\lambda/2,(n-1)\lambda/2)$ can be expressed as

$$\bar{x}_{m,n}(t)=\sum_{i=1}^{P}\bar{s}_i(t)\exp\{j[(m-1)\pi u_i+(n-1)\pi v_i]\}+\bar{y}_{m,n}(t), \tag{1}$$

where $\bar{s}_i(t)$ denotes the complex waveform of the signal emitted by the ith source and $\bar{y}_{m,n}(t)$ the spatially white sensor noise independent of $\bar{s}_i(t)$. Without loss of generality, assume that $\bar{s}_1(t)$ is the desired signal with direction angle (u_1,v_1) and the other $P-1$ signals are interferers. From (1), the data matrix received by the URA is given by

$$\bar{X}(t)=\sum_{i=1}^{P}[\bar{A}_c(u_i)\bar{A}_r(v_i)^T]\bar{s}_i(t)+\bar{Y}(t), \tag{2}$$

where $\bar{A}_c(u_i)=[1,\exp\{j\pi u_i\},\ldots,\exp\{j(\bar{M}-1)\pi u_i\}]^T$, $\bar{A}_r(v_i)=[1,\exp\{j\pi v_i\},\ldots,\exp\{j(\bar{N}-1)\pi v_i\}]^T$, and $\bar{Y}(t)$ is the received noise matrix. Rewriting (2) in vector form, we have

$$\mathrm{vec}\{\bar{X}(t)\}=[\bar{x}_{1,1}(t),\ldots,\bar{x}_{\bar{M},1}(t),\bar{x}_{1,2}(t),\ldots,\bar{x}_{\bar{M},2}(t),\bar{x}_{1,3}(t),\ldots,\bar{x}_{1,\bar{N}}(t),\ldots,\bar{x}_{\bar{M},\bar{N}}(t)]^T. \tag{3}$$

Using the following property of Kronecker product [7]

$<$ KP.1 $>$ $\mathrm{vec}\{Q_1Q_2Q_3^T\}=(Q_3\otimes Q_1)\mathrm{vec}\{Q_2\}$,

where Q_i are matrices with appropriate sizes, we can rewrite (3) as follows

$$\mathrm{vec}\{\bar{X}(t)\}=\sum_{i=1}^{P}\bar{A}(u_i,v_i)\bar{s}_i(t)+\mathrm{vec}\{\bar{Y}(t)\}=\bar{A}\mathrm{s}\bar{S}\mathrm{s}(t)+\mathrm{vec}\{\bar{Y}(t)\}, \tag{4}$$

where the response vector of the ith signal source $\bar{A}(u_i, v_i) = \bar{A}_r(v_i) \otimes \bar{A}_c(u_i)$, the response matrix of the P signal sources $\bar{A}s = [\bar{A}(u_1, v_1) \cdots \bar{A}(u_P, v_P)]$, and the signal source vector $\bar{S}s(t) = [\bar{s}_1(t), \ldots, \bar{s}_P(t)]^T$. The correlation matrix of vec$\{\bar{X}(t)\}$ is then given by

$$\bar{R} = E\{\text{vec}\{\bar{X}(t)\}\text{vec}\{\bar{X}(t)\}^H\} = \bar{A}s\bar{\Psi}s\bar{A}s^H + \pi_n I_{\bar{M}\bar{N}}, \tag{5}$$

where $\bar{\Psi}s = E\{\bar{S}s(t)\bar{S}s(t)^H\}$ denotes the full rank correlation matrix of the signal sources, π_n the noise power, and $I_{\bar{M}\bar{N}}$ the $\bar{M}\bar{N}$x$\bar{M}\bar{N}$ identity matrix.

2.2 The 2-D Blocking Technique

In the follwoing, we present a technique for the design of a 2-D EIC with a steering angle (u_0, v_0). Utilizing the results presented in [4] and letting the steering angle be accordant with the direction angle (u_1, v_1) of the desired signal, we can construct a blocking matrix B_c for the column subarrays of the 2-D URA such that

$$B_c^H \bar{A}_c(u_i) = d_{c,i} A_c(u_i) \ \text{ with } \ d_{c,i} = (e^{j\pi u_i} - e^{j\pi u_1})^\beta \ \text{ and } \ A_c(u_i) = J_c \bar{A}_c(u_i), \tag{6}$$

where β is the order of B_c. $J_c = [I_M \ O_{M,\beta}]$ is the row selection matrix which selects the first $M = \bar{M} - \beta$ rows of $\bar{A}_c(u_i)$, where $O_{m,n}$ is an mxn zero matrix. Similar results can be obtained for the row subarrays as follows

$$B_r^H \bar{A}_r(v_i) = d_{r,i} A_r(v_i) \ \text{ with } \ d_{r,i} = (e^{j\pi v_i} - e^{j\pi v_1})^\delta \ \text{ and } \ A_r(v_i) = J_r \bar{A}_r(v_i), \tag{7}$$

where δ is the order of B_r. $J_r = [I_N \ O_{N,\delta}]$ is the row selection matrix which selects the first $N = \bar{N} - \delta$ rows of $\bar{A}_r(v_i)$. Based on the above results, we present a 2-D blocking technique as follows.

Theorem 1 : Let the matrix R be given by

$$R = \tilde{B}_c^H \bar{R} \tilde{B}_c + \tilde{B}_r^H \bar{R} \tilde{B}_r, \ \text{ with } \ \tilde{B}_c = J_r^T \otimes B_c, \ \text{ and } \ \tilde{B}_r = B_r \otimes J_c^T. \tag{8}$$

Then R is an autocorrelation matrix of the blocked 2-D array data which contain all the interferers except the desired signal.

Proof : Based on the fact that $\tilde{B}_c = J_r^T \otimes B_c$, $\bar{A}(u_i, v_i) = \bar{A}_r(v_i) \otimes \bar{A}_c(u_i)$, and the property of Kronecker product [7]

$$< \text{KP.2} > \quad (Q_1 \otimes Q_2)(Q_3 \otimes Q_4) = (Q_1 Q_3) \otimes (Q_2 Q_4),$$

it can be shown that

$$\tilde{B}_c^H \bar{A}(u_i, v_i) = d_{c,i} A(u_i, v_i). \tag{9}$$

where $d_{c,i}$ is given by (6) and $A(u_i, v_i) = A_r(v_i) \otimes A_c(u_i)$. Based on (5) and (9), then we have

$$\tilde{B}_c^H \bar{R} \tilde{B}_c = A_S \tilde{D}_c \bar{\Psi}_S \tilde{D}_c^H A_S^H + \pi_n \tilde{B}_c^H \tilde{B}_c, \tag{10}$$

where $A_S = [A(u_1, v_1), \ldots, A(u_P, v_P)]$ and $\tilde{D}_c = \mathrm{diag}\{d_{c,1}, \ldots, d_{c,P}\}$. Based on the fact of $d_{c,1} = 0$ and the property of KP.2, (10) can be further written as

$$\tilde{B}_c^H \bar{R} \tilde{B}_c = A_I D_c \bar{\Psi}_I D_c^H A_I^H + \pi_n (I_N \otimes (B_c^H B_c)), \tag{11}$$

where $A_I = [A(u_2, v_2), \ldots, A(u_P, v_P)]$, $D_c = \mathrm{diag}\{d_{c,2}, \ldots, d_{c,P}\}$, $\bar{\Psi}_I = E\{\bar{S}_I(t)\bar{S}_I(t)^H\}$, and $\bar{S}_I(t) = [\bar{s}_2(t), \ldots, \bar{s}_P(t)]^T$. Similar to (11), we have the following result for the row subarrays

$$\tilde{B}_r^H \bar{R} \tilde{B}_r = A_I D_r \bar{\Psi}_I D_r^H A_I^H + \pi_n ((B_r^H B_r) \otimes I_M), \tag{12}$$

where $D_r = \mathrm{diag}\{d_{r,2}, \ldots, d_{r,P}\}$. Summing (11) and (12) yields

$$R = \tilde{B}_c^H \bar{R} \tilde{B}_c + \tilde{B}_r^H \bar{R} \tilde{B}_r = A_I \Psi_I A_I^H + \pi_n \Phi, \tag{13}$$

where

$$\Psi_I = D_c \bar{\Psi}_I D_c^H + D_r \bar{\Psi}_I D_r^H, \tag{14}$$

and

$$\Phi = I_N \otimes (B_c^H B_c) + (B_r^H B_r) \otimes I_M. \tag{15}$$

Clearly, Ψ_I is a positive definite matrix if $(u_i, v_i) \neq (u_1, v_1)$ for all $i = 2, \ldots, P$. From (13) to (15), we note that R is the autocorrelation matrix of a data vector which does not contain the desired signal component. $\triangle$

2.3 The 2-D EIC Formulation

Based on the 1-D results of [4], the criterion in finding the optimal weight vector for the 2-D EIC can be defined as maximizing the output SNR subject to a constraint of orthogonality to the IS. Accordingly, we have to solve the following optimization problem

$$\text{Maximize } \frac{|W^H A(u_1, v_1)|^2}{W^H W} \quad \text{subject to } W \perp \mathrm{range}\{A_I\}, \tag{16}$$

where $A(u_1, v_1)$ serves as the steering vector. The optimal solution of (16) is given by

$$W_o = (I_{MN} - A_I(A_I^H A_I)^{-1} A_I^H) A(u_1, v_1). \tag{17}$$

Equation (17) reveals that the matrix $A_I = [A(u_2, v_2), \ldots, A(u_P, v_P)]$ due to the $P-1$ interferers must be found in order to compute W_o. However, A_I cannot be known *a priori*. Basically, one can resort to finding a basis matrix spanning range$\{A_I\}$ to solve this problem. Unfortunately, the matrix Φ given by (15) is generally not an identity matrix. Hence, we have to perform the GEVD of R. Let the generalized eigenvalues and the corresponding eigenvectors be designated as γ_i and g_i, respectively. Accordingly, we have the following expression

$$Rg_i = \gamma_i \Phi g_i, \tag{18}$$

where $\gamma_1 \geq \gamma_2 \geq \ldots \geq \gamma_{P-1} > \gamma_P = \ldots = \gamma_{MN} = \pi_n$. Let $G_I = [g_1, g_2, \ldots, g_{P-1}]$, then it can be shown that range$\{A_I\}$ = range$\{\Phi G_I\}$. Therefore, the optimal weight vector of (17) can be rewritten as

$$W_o = (I_{MN} - (\Phi G_I)((\Phi G_I)^H (\Phi G_I))^{-1} (\Phi G_I)^H) A(u_1, v_1). \tag{19}$$

From (19), we note that performing the complicated GEVD of R is inevitable for computing the optimal weight vector W_o. Moreover, evaluating the statistical performance under finite data samples and the sensitivity to steering angle error for the 2-D EIC becomes very difficult because the GEVD of R is necessary for designing the 2-D EIC.

To tackle the above two problems, in **Appendix A**, an efficient method is presented to construct such a positive definite matrix Ω that the effect of the 2-D blocking operation on the noise component of the received array data can be eliminated, i.e., $\Phi + \Omega = \sigma^2 I_{MN}$. Thus, we obtain

$$R_w = R + \pi_n \Omega = A_I \Psi_I A_I^H + \sigma^2 \pi_n I_{MN}. \tag{20}$$

Equation (20) reveals that the corresponding noise component in R_w becomes spatially white. Performing the EVD of R_w, we have the following expression

$$R_w e_i = \lambda_i e_i, \tag{21}$$

where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{P-1} > \lambda_P = \cdots = \lambda_{MN} = \sigma^2 \pi_n$. Let the matrix $E_I = [e_1 \cdots e_{P-1}]$ and the matrix $E_R = [e_P \cdots e_{MN}]$. It is easy to show that $[E_I\ E_R]^H [E_I\ E_R] = I_{MN}$ and

$$\text{range}\{E_I\} = \text{range}\{A_I\} \perp \text{range}\{E_R\}, \tag{22}$$

i.e., E_I is an orthogonal basis matrix spanning range$\{A_I\}$ and E_R is an orthogonal basis matrix spanning the complement of range$\{A_I\}$. It follows from (22) that the optimal weight vector for the 2-D EIC based on the criterion of (16) can be rewritten as

$$W_o = (I_{MN} - E_I E_I^H) A(u_1, v_1) = E_R E_R^H A(u_1, v_1). \tag{23}$$

CHAPTER THREE STATISTICAL PERFORMANCE UNDER FINITE DATA SAMPLES

In practice, the number of signal sources P, the background noise power π_n, and the ensemble correlation matrix $\bar{R}$ required for implementing the 2-D EIC are not available and usually estimated from the received data snapshots. Using the first K data snapshots, we obtain the estimate P for the number of signal sources based on the AIC or MDL criterion presented by [11]. Moreover, implementing the AIC or MDL criterion requires performing the EVD of the corresponding data correlation matrix. Therefore, π_n can be estimated by utilizing the eigenvalue method of [12] during the same estimation process. Let the estimated value be denoted as $\widehat{\pi}_n$. Then, the next L data snapshots are used to compute the sample correlation matrix $\bar{R}$ as follows

$$\widehat{\bar{R}} = \frac{1}{L}\sum_{l=1}^{L} \text{vec}\{\bar{X}(t_l)\}\text{vec}\{\bar{X}(t_l)\}^H \tag{24}$$

to replace $\bar{R}$, where $\bar{X}(t_l)$ is the data matrix received at the time instant t_l. The correlation matrix R_w of (20) is then replaced by

$$\widehat{R}_w = \widehat{R} + \widehat{\pi}_n\Omega, \tag{25}$$

where

$$\widehat{R} = \tilde{B}_c^H \widehat{\bar{R}} \tilde{B}_c + \tilde{B}_r^H \widehat{\bar{R}} \tilde{B}_r. \tag{26}$$

It is appropriate to assume that $\widehat{\pi}_n$ and $\widehat{R}$ are independent in this case. Thus, (21) becomes

$$\widehat{R}_w \widehat{e}_i = \widehat{\lambda}_i \widehat{e}_i, \tag{27}$$

where $\widehat{\lambda}_1 \geq \widehat{\lambda}_2 \geq \cdots \geq \widehat{\lambda}_{MN}$. Since the number of interferers is $P-1$, the corresponding basis matrix of IS and its complement are given by $\widehat{E}_{\mathbf{I}} = [\ \widehat{e}_1 \cdots \widehat{e}_{P-1}]$ and $\widehat{E}_{\mathbf{R}} = [\ \widehat{e}_P \cdots \widehat{e}_{MN}]$, respectively. Consequently, the optimal weight vector for the 2-D EIC under finite snapshots is given by

$$\widehat{W}_o = (I_{MN} - \widehat{E}_{\mathbf{I}}\widehat{E}_{\mathbf{I}}^H)A(u_1, v_1) = \widehat{E}_{\mathbf{R}}\widehat{E}_{\mathbf{R}}^H A(u_1, v_1). \tag{28}$$

The statistical performance of the proposed 2-D EIC under finite data samples is given by the following theorem.

Theorem 2 : For the case of input INR high enough, the expectation of the output SINR can be approximately given by

$$E\{\widehat{SINR}_o\} \approx SINR_o(1 - \frac{1}{L}FSP), \tag{29}$$

where $SINR_o$ denotes the array output SINR without the finite sample effect. FSP represents the factor of statistical performance and is given by

$$FSP = \omega^{-1}(\xi_{r,r}\omega_{r,r} + \xi_{c,c}\omega_{c,c} + \xi_{r,c}\omega_{c,r} + \xi_{c,r}\omega_{r,c}), \tag{30}$$

where $\omega = W_o^H W_o$,

$$\begin{cases} \omega_{c,c} = W_o^H \tilde{B}_c^H \tilde{B}_c W_o, & \omega_{r,r} = W_o^H \tilde{B}_r^H \tilde{B}_r W_o, \\ \omega_{c,r} = W_o^H \tilde{B}_c^H \tilde{B}_r W_o, & \omega_{r,c} = W_o^H \tilde{B}_r^H \tilde{B}_c W_o, \end{cases}, \tag{31}$$

and

$$\begin{cases} \xi_{c,c} = Tr\{\Psi_I^{-1}\bar{\Psi}_I\Psi_I^{-1}D_c\bar{\Psi}_I D_c^H\}, & \xi_{r,r} = Tr\{\Psi_I^{-1}\bar{\Psi}_I\Psi_I^{-1}D_r\bar{\Psi}_I D_r^H\}, \\ \xi_{c,r} = Tr\{\Psi_I^{-1}\bar{\Psi}_I\Psi_I^{-1}D_c\bar{\Psi}_I D_r^H\}, & \xi_{r,c} = Tr\{\Psi_I^{-1}\bar{\Psi}_I\Psi_I^{-1}D_r\bar{\Psi}_I D_c^H\}, \end{cases}. \tag{32}$$

Proof : Please see **Appendix B**.

If the interferers are uncorrelated, (32) can be further simplified as

$$\begin{cases} \xi_{c,c} = \sum_{i=2}^{P} |d_{c,i}|^2(|d_{c,i}|^2 + |d_{r,i}|^2)^{-2}, & \xi_{r,r} = \sum_{i=2}^{P} |d_{r,i}|^2(|d_{c,i}|^2 + |d_{r,i}|^2)^{-2}, \\ \xi_{c,r} = \sum_{i=2}^{P} d_{c,i}d_{r,i}^*(|d_{c,i}|^2 + |d_{r,i}|^2)^{-2}, & \xi_{r,c} = \sum_{i=2}^{P} d_{r,i}d_{c,i}^*(|d_{r,i}|^2 + |d_{c,i}|^2)^{-2}, \end{cases}. \tag{33}$$

Moreover, we have the following result.

Theorem 3 : If the angle separations between the interferers and the desired signal satisfy that $|u_i - u_1| \geq 1/3$ or $|v_i - v_1| \geq 1/3$ for $i = 2, 3, \ldots, P$, it can be shown that

$$E\{\widehat{SINR}_o\} > SINR_o(1 - (P-1)(\sigma_c + \sigma_r)^2/L), \tag{34}$$

where σ_c^2 and σ_r^2 are given by (A-11).

Proof : Please see **Appendix D**.

Theorem 3 provides a lower bound of the convergence rate for the proposed 2-D EIC under the situation considered. For example, consider the situation where the direction angle of the desired signal $(u_1, v_1) = (0, 0)$, the blocking orders $(\beta, \delta) = (1, 1)$, and the direction angles of interferers (u_i, v_i) with $|u_i| \geq 1/3$ or $|v_i| \geq 1/3$ for $i = 2, 3, \ldots, P$. Then, we have $\sigma_c^2 = \sigma_r^2 = 4$. A lower bound of the output SINR can be obtained from (34) and is given by

$$E\{\widehat{SINR}_o\} > SINR_o(1 - 16(P-1)/L). \tag{35}$$

Equation (35) shows that a satisfactory convergence speed for the designed 2-D EIC can be guaranteed in this case.

To result in a simpler version of the above theoretical results for providing an insight, we next consider a special situation where all the uncorrelated interferers are located outside the array mainlobe and the angle separations between the desired signal and the interferers are large enough so that

$$A_I(A_I^H A_I)^{-1} A_I^H A(u_1, v_1) \ll A(u_1, v_1). \tag{36}$$

Moreover, $\bar{M}$ and $\bar{N}$ are greater than 2β and 2δ, respectively. Based on these two conditions, the optimal weight vector given by (17) can be reduced to an approximation of $W_o \approx A(u_1, v_1)$ and, hence, $\omega \approx MN$, the results in (31) can be simplified as the following approximations

$$\begin{cases} \omega_{c,c} \approx 2N \sum_{i=0}^{\beta-1} \left[\sum_{k=0}^{i}(-1)^k \binom{\beta}{k}\right]^2, \quad \omega_{r,r} \approx 2M \sum_{i=0}^{\delta-1} \left[\sum_{k=0}^{i}(-1)^k \binom{\delta}{k}\right]^2 \\ \omega_{c,r} = \omega_{r,c}^* \approx e^{j(\pi u_1 - \pi v_1)}, \text{ for } (\beta,\delta) = (1,1), \text{ and } \approx 0, \text{ for } (\beta,\delta) \neq (1,1), \end{cases} . \tag{37}$$

Then, we can simply substitute (33) and (37) into (30) to obtain the corresponding *FSP*.

CHAPTER FOUR PERFORMANCE ANALYSIS UNDER STEERING ANGLE ERROR

In this case, the steering angle is not accordant with the direction angle of the desired signal, i.e., $(u_0, v_0) \neq (u_1, v_1)$. The blocking factors shown in (6) and (7) become

$$d_{c,i} = (e^{j\pi u_i} - e^{j\pi u_0})^{\beta} \text{ and } d_{r,i} = (e^{j\pi v_i} - e^{j\pi v_0})^{\delta}, \tag{38}$$

respectively. The mismatch between (u_0, v_0) and (u_1, v_1) leads to a result that the blocked data correlation matrix R given by (8) contains a leakage due to the desired signal and becomes

$$R = A_S \Psi_S A_S^H + \pi_n \Phi, \tag{39}$$

where $A_S = [A(u_1, v_1), \ldots, A(u_P, v_P)]$, $\Psi_S = \tilde{D}_c \bar{\Psi}_S \tilde{D}_c^H + \tilde{D}_r \bar{\Psi}_S \tilde{D}_r^H$, $\tilde{D}_c = \text{diag}\{d_{c,1}, \ldots, d_{c,P}\}$, and $\tilde{D}_r = \text{diag}\{d_{r,1}, \ldots, d_{r,P}\}$, respectively. Since $(d_{c,i}, d_{r,i}) \neq (0,0)$ for all $i = 1, 2, \ldots, P$, Ψ_S is a PxP positive definite matrix. Hence the matrix $R_w = R + \pi_n \Omega$ has P principal eigenvalues which are greater than $\sigma^2 \pi_n$ and the corresponding eigenvectors spans the subspace range$\{A_S\}$. The computed basis matrix E_I will contain more than $P-1$ principal eigenvectors of R_w if the number of interferers is overestimated. From (23), the optimal weight vector corresponding to this case is given by

$$W_o = (I_{MN} - E_I E_I^H) A(u_0, v_0). \tag{40}$$

This leads to the result that the 2-D EIC fails to work due to that range$\{E_I\}$ contains the vector $A(u_1, v_1)$ and the constraint of $W^H E_I = 0$.

Next, consider the situation where the number of interferers is exactly known and the desired signal is uncorrelated with the $(P-1)$ interferers. Based on (39), we have

$$R_w = R + \pi_n \Omega = \pi_1 A(u_1, v_1) A(u_1, v_1)^H + R_I + \sigma^2 \pi_n I_{MN}, \tag{41}$$

where $\pi_1 = (|d_{c,1}|^2 + |d_{r,1}|^2) E\{|\bar{s}_1(t)|^2\}$ denotes the power of the desired signal leakage in the output after the 2-D blocking operation. $R_I = A_I \Psi_I A_I$, where Ψ_I is given by (14) except that the entries of D_c and D_r are now given by (38). Let the $P-1$ nonzero eigenvalues and the corresponding eigenvectors of R_I be given by $\kappa_1 \geq \cdots \geq \kappa_{P-1} > 0$ and z_i for $i = 1, 2, \ldots, P-1$, respectively. For further simplicity, assume that the interferers are located far away from the desired signal so that $A(u_1, v_1)^H A(u_i, v_i) \approx 0$ for $i = 2, 3, \ldots, P$. Then, the eigenvalues λ_i which are greater than $\sigma^2\pi_n$ and the corresponding eigenvectors e_i of R_w can be approximated as $\lambda_i \approx \kappa_i + \sigma^2\pi_n$ and $e_i \approx z_i$ for $i = 1, 2, \ldots, P-1$, $\lambda_P \approx MN\pi_1 + \sigma^2\pi_n$ and $e_P \approx A(u_1, v_1)/\sqrt{MN}$, respectively. Note that E_I consists of the first $P-1$ principal eigenvectors of R_w. As a result, E_I consists of z_i for $i = 1, 2, \ldots, P-1$ when $MN\pi_1 < \kappa_{P-1}$. Hence, range$\{E_I\} \approx$ range$\{A_I\}$ and the 2-D EIC works normally. On the other hand, E_I contains the normalized response vector $A(u_1, v_1)/\sqrt{MN}$ if $MN\pi_1 > \kappa_{P-1}$. From the optimal weight vector given by (40), we note that the desired signal will be suppressed due to the constraint of $W_o^H E_I = 0$. As shown by (38) and the fact that π_1 is proportional to $(|d_{c,1}|^2 + |d_{r,1}|^2)$, this difficulty could be alleviated by increasing the orders β and δ if the steering angle error is small. In general, the breakdown threshold $MN\pi_1 > \kappa_{P-1}$ happens when R_I is nearly rank-deficient. To look into the effect of $MN\pi_1 > \kappa_{P-1}$, we proceed to consider the case of two uncorrelated and closely-separated interferers.

Let the two uncorrelated interferers be closely separated so that $A(u_2, v_2)^H A(u_3, v_3) \approx MN$. From [8, pp.25-pp.27], we can easily show that

$$\kappa_2 = \frac{MN\pi_2\pi_3}{\pi_2 + \pi_3}(1 - |g_{c,2,3}|^2 |g_{r,2,3}|^2). \tag{42}$$

where $\pi_i = (|d_{c,i}|^2 + |d_{r,i}|^2)E\{|\bar{s}_i(t)|^2\}$ for $i = 2$ and 3. $g_{c,2,3}$ and $g_{r,2,3}$ are given by

$$\begin{cases} g_{c,2,3} = \dfrac{\sin(\pi M(u_2 - u_3)/2)}{M\sin(\pi(u_2 - u_3)/2)} e^{j\pi(M-1)(u_2-u_3)/2} \\ g_{r,2,3} = \dfrac{\sin(\pi N(v_2 - v_3)/2)}{N\sin(\pi(v_2 - v_3)/2)} e^{j\pi(N-1)(v_2-v_3)/2} \end{cases}.$$

Hence, the condition $MN\pi_1 > \kappa_2$ causing the performance failure becomes

$$1 - |g_{c,2,3}|^2 |g_{r,2,3}|^2 < \pi_1(\pi_2^{-1} + \pi_3^{-1}). \tag{43}$$

When $|u_2 - u_3|$ and $|v_2 - v_3|$ are small enough, it is also shown in [8] that

$$\begin{cases} |g_{c,2,3}|^2 \approx 1 - \frac{M^2-1}{12}\pi^2(u_2 - u_3)^2 \\ |g_{r,2,3}|^2 \approx 1 - \frac{N^2-1}{12}\pi^2(v_2 - v_3)^2 \end{cases}. \tag{44}$$

Substituting (44) into (43) and taking the first-order approximation yields the following performance breakdown threshold

$$\frac{M^2-1}{12}\pi^2(u_2-u_3)^2+\frac{N^2-1}{12}\pi^2(v_2-v_3)^2=\pi_1(\pi_2^{-1}+\pi_3^{-1}). \qquad (45)$$

CHAPTER FIVE COMPUTER SIMULATION EXAMPLES

In this section, several simulation examples for illustrating and confirming the theoretical works are presented. The 2-D array used for all simulations is a URA with $\bar{M} = 7$ and $\bar{N} = 6$. Moreover, the simulation results based on the direct GEVD of R given by (18) to obtain an IS basis matrix required for computing the optimal weight vector are also presented for comparison.

Example 1 : This example is performed to illustrate the theoretical results presented in Section III. We set $(\beta, \delta) = (1,1)$. The desired signal with input SNR = 0dB is impinging on the array from $(u_1, v_1) = (0,0)$. One interferer has input INR = 20dB. The first $K = 50$ data snapshots are used to estimate the source number P and the noise power π_n. Figure 1 plots the array output SINR in dB versus the number of snapshots L for two different interfering angles. Each simulation result is obtained by averaging 100 independent runs with independent noise samples for each run. The solid curve represents the theoretical results computed by using (29) based on (28), (30), (31), and (32). This confirms the validity of (29) given by **Theorem 2**. On the other hand, the curve with "x" represents the simulation results for the performance of the 2-D EIC designed by using the proposed technique, while the curve with "o" represents the simulation results of the 2-D EIC designed by using the direct GEVD technique. The coincidence between these two curves shows that the 2-D EIC designed by using the proposed technique provides the same performance as that directly using the complicated GEVD technique.

Comparing the results of Figure 1(a) and (b), we note that the output SINR of Figure 1(a) is smaller than that of Figure 1(b) for each number of snapshots as expected because the angle separation between the desired signal and the interferer is smaller for Figure 1(a). This phenomenon is further demonstrated in Figure 2 by plotting the *FSP* of (30) versus the interfering angle (u_2, v_2). We note that *FSP* increases and hence the performance degradation increases as (u_2, v_2) approaches $(u_1, v_1) = (0,0)$.

Example 2 : This example considers the case of multiple interferers. Again, we set (β, δ) $= (1,1)$. The desired signal with input SNR = 0dB is impinging on the array from $(u_1, v_1) = (0,0)$, while the uncorrelated interferers have the same input INR = 20dB. The first $K = 50$ data snapshots are used to estimate the source number P and the noise power π_n. Figure 3 depicts the array output SINR in dB versus the number of snapshots L for different interfering situations. Each simulation result is obtained by averaging 100 independent runs with independent noise samples for each run. The solid curve represents the theoretical results computed by using (29) based on (28), (30), (31), and (32). In contrast, the dash curve represents the theoretical results computed by using (29) based on the approximations described by (37). The dash curve almost coincides with the solid curve. This confirms the validity of the approximations given by (37). Moreover, the coincidence between the curves with "x" and "o" illustrates that the 2-D EICs designed by using the proposed technique and directly using the complicated GEVD technique have the same performance.

Example 3 : Here, we illustrate the performance of the designed 2-D EIC in the presence of steering angle error. The steering angle is $(u_o, v_o) = (0,\ 0)$. The desired signal with input SNR = 3dB is impinging on the 2-D array from $(u_1, v_1) = (0.03, 0.03)$. Two uncorrelated interferers with input INR = 3dB are impinging on the array from $(u_2, v_2) = (0.5, 0.6)$ and $(u_3, v_3) = (u_2 + \Delta u, v_2 + \Delta v)$. To evaluate the sensitivity to the angle separation $(\Delta u, \Delta v)$, we define a robustness index (RI) as follows

$$RI_p = \frac{\text{The output SINR using } W_o \text{ of (40)}}{\text{The output SINR using } W_o \text{ of (17) with } A(u_1, v_1) \text{ replaced by } A(u_0, v_0)} \tag{46}$$

for the designed 2-D EIC and

$$RI_h = \frac{\text{The output SINR using } W_o \text{ of (19) with } A(u_1, v_1) \text{ replaced by } A(u_0, v_0)}{\text{The output SINR using } W_o \text{ of (17) with } A(u_1, v_1) \text{ replaced by } A(u_0, v_0)} \tag{47}$$

for the 2-D EIC directly using the complicated GEVD technique. Figure 4(a) plots the RI versus $(\Delta u, \Delta v)$. The top curve represents the RI_p versus $(\Delta u, \Delta v)$, while the bottom curve represents the RI_h versus $(\Delta u, \Delta v)$. It shows that the proposed technique possesses the advantage of robust capability against steering angle error over the GEVD technique. Figure 4(b) depicts the curves of the breakdown threshold for $RI_p = 0.5$. The dash curve represents the breakdown threshold computed by (45), while the solid curve represents the simulation results. This figure also confirms the presented theoretical results.

CHAPTER SIX CONCLUSION

The theoretical works for the design and analysis of a 2-D eigenspace-based interference canceller (EIC) have been presented. An effective 2-D signal blocking technique is first presented to remove the desired signal from the received array data. To compensate the effect of the signal blocking operation on the sensor noise, a positive definite matrix has been constructed. Therefore, the interference subspace required for computing the optimal weight vector can be obtained by using conventional eigenvalue decomposition methods. The performances of the designed 2-D EIC under finite samples and steering angle error have been evaluated, respectively. The developed theoretical results are confirmed by several simulation examples. It has been shown that the performance of the designed 2-D EIC is the same as that of a 2-D EIC directly using a complicated GEVD technique in the situation without steering angle error. However, the proposed 2-D EIC possesses the advantage of more robust capability against steering angle error over the 2-D EIC based on the GEVD technique.

APPENDIX A

Let $\tilde{I}_m$ be an mxm cyclic-shifting matrix defined as

$$\tilde{I}_m = [\ I_{m,2}\ \ I_{m,3}\ \dots\ I_{m,m}\ \ I_{m,1}\ ], \tag{A-1}$$

where $I_{m,i}$ is the ith column vector of the mxm identity matrix. Following the results presented in [4], the blocking matrix B_c which satisfies (6) can be constructed as follows

$$B_c = [\ \tilde{b}_c\ \ \tilde{I}_{\bar{M}}\tilde{b}_c\ \ \cdots\ \ (\tilde{I}_{\bar{M}})^{M-1}\tilde{b}_c\ ], \tag{A-2}$$

where $\tilde{b}_c$ is an $\bar{M}$x1 vector given by

$$\tilde{b}_c = [\ b_{c,0},\ \dots\ , b_{c,\beta},\ 0,\ \dots\ , 0\ ]^T, \tag{A-3}$$

and $b_{c,k}$ are the coefficients satisfying

$$(z - e^{j\pi u_1})^{\beta} = \sum_{k=0}^{\beta} b^*_{c,k} z^k. \tag{A-4}$$

The subscript '$*$' denotes the complex conjugate. From (A-2) and (A-3), it can be seen that $B_c^H B_c$ is an MxM Hermitian and Toeplitz matrix. Furthermore, let $HT\{x_1, x_2, \dots, x_m\}$ denote an mxm Hermitian and Toeplitz matrix with its first row given by $[x_1, x_2, \dots, x_m]$. Then, we have

$$B_c^H B_c = HT\{\epsilon_{c,0}, \epsilon_{c,1}, \dots, \epsilon_{c,M-1}\}, \text{ with } \epsilon_{c,i} = \sum_{k=0}^{\beta-i} b^*_{c,k+i} b_{c,k}. \tag{A-5}$$

Next, we construct an $(M+i)$x1 vector as follows

$$f_c(k,i) = [\ 1\ \ O_{1,i-1}\ \ j^k\ \ O_{1,M-1}\ ]^T, \tag{A-6}$$

where $i = 1, 2, \dots, M-1$ and k is an integer. From (A-6), an $(M+i)$xM matrix is constructed as follows

$$F_c(k,i) = [\ f_c(k,i)\ \ \tilde{I}_{M+i} f_c(k,i)\ \ \tilde{I}^2_{M+i} f_c(k,i)\ \ \dots\ \ \tilde{I}^{M-1}_{M+i} f_c(k,i)\ ]. \tag{A-7}$$

Using (A-6) and (A-7), we have

$$\Gamma_c(k,i) = F_c(k,i)^H F_c(k,i) = HT\{\ 2,\ O_{1,i-1},\ (-j)^k,\ O_{M-1-i}\ \}. \tag{A-8}$$

From (A-8), we note that $\Gamma_c(k,i)$ is positive definite, Hermitian, and Toeplitz. Moreover, it is easy to show that

$$\begin{aligned} &|Re\{\epsilon_{c,i}\}|\Gamma_c(2\mathrm{sgn}(Re\{\epsilon_{c,i}\}),i) + |Im\{\epsilon_{c,i}\}|\Gamma_c(2\mathrm{sgn}(Im\{\epsilon_{c,i}\})-1,i) \\ &\qquad = HT\{\ 2(|Re\{\epsilon_{c,i}\}| + |Im\{\epsilon_{c,i}\}|),\ O_{1,i-1},\ -\epsilon_{c,i},\ O_{1,M-1-i}\ \}, \end{aligned} \tag{A-9}$$

for $i = 1, 2, \ldots, M-1$, where $Re\{x\}$ and $Im\{x\}$ denote the real and imaginary parts of x, respectively. $\mathrm{sgn}(x) = 1$ if $x \geq 0$, and $= 0$, otherwise. We then construct a positive definite matrix as follows

$$\Omega_c = \sum_{i=1}^{M-1} (|Re\{\epsilon_{c,i}\}|\Gamma_c(2\mathrm{sgn}(Re\{\epsilon_{c,i}\}),i) + |Im\{\epsilon_{c,i}\}|\Gamma_c(2\mathrm{sgn}(Im\{\epsilon_{c,i}\})-1,i)) \tag{A-10}$$

Summing (A-5) and (A-10) thus yields a diagonal matrix as follows

$$B_c^H B_c + \Omega_c = \left(\epsilon_{c,0} + 2\sum_{i=1}^{M-1} (|Re\{\epsilon_{c,i}\}| + |Im\{\epsilon_{c,i}\}|)\right) I_M = \sigma_c^2 I_M, \tag{A-11}$$

where σ_c^2 denotes the proportional constant. Following the same procedure, we can find a positive definite matrix Ω_r such that $B_r^H B_r + \Omega_r = \sigma_r^2 I_N$ for some positive σ_r^2. Finally, we form the following matrix

$$\Omega = I_N \otimes \Omega_c + \Omega_r \otimes I_M. \tag{A-12}$$

Based on (15) and the property of Kronecker product [7]

$$< \mathrm{KP.3} > (\sum_i Q_i) \otimes (\sum_k T_k) = \sum_i \sum_k (Q_i \otimes T_k)$$

for matrices Q_i and T_k with appropriate sizes, we can easily show that $\Phi + \Omega = \sigma^2 I_{MN}$, where $\sigma^2 = \sigma_c^2 + \sigma_r^2$.

APPENDIX B

Here, we show the result given by (29) in **Theorem 2**. Performing the EVD of R_w, we obtain the following expression

$$R_w = E_I \Lambda_I E_I^H + E_R \Lambda_R E_R^H, \tag{B-1}$$

where $\Lambda_I = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_{P-1}\}$ and $\Lambda_R = \text{diag}\{\lambda_P, \lambda_{P+1}, \ldots, \lambda_{MN}\} = \sigma^2 \pi_n I_{MN}$. Similarly, we have the following expression

$$\widehat{R}_w = \widehat{E}_I \widehat{\Lambda}_I \widehat{E}_I^H + \widehat{E}_R \widehat{\Lambda}_R \widehat{E}_R^H \tag{B-2}$$

from the EVD of $\widehat{R}_w$, where $\widehat{\Lambda}_I = \text{diag}\{\widehat{\lambda}_1, \widehat{\lambda}_2, \ldots, \widehat{\lambda}_{P-1}\}$ and $\widehat{\Lambda}_R = \text{diag}\{\widehat{\lambda}_P, \widehat{\lambda}_{P+1}, \ldots, \widehat{\lambda}_{MN}\}$. From (20) and (25), the deviation between $\widehat{R}_w$ and R_w due to finite sample effect can be expressed as

$$\Delta R_w = \widehat{R}_w - R_w = \Delta R + \Delta \pi_n \Omega, \tag{B-3}$$

where $\Delta \pi_n = \widehat{\pi}_n - \pi_n$ and $\Delta R = \widehat{R} - R$. Using (8) and (26), ΔR is given by

$$\Delta R = \tilde{B}_c^H \Delta \bar{R} \tilde{B}_c + \tilde{B}_r^H \Delta \bar{R} \tilde{B}_r. \tag{B-4}$$

Following the first-order perturbation analysis presented in [9], we can show that

$$\Delta E_R = \widehat{E}_R - E_R \approx -R_I^+ \Delta R_w E_R, \tag{B-5}$$

where

$$R_I^+ = E_I(\Lambda_I - \sigma^2 \pi_n I_{P-1})^{-1} E_I^H. \tag{B-6}$$

It follows from (B-6) that R_I^+ possesses the following properties

$$R_I^+ = E_I(A_I^H E_I)^{-1} \Psi_I^{-1} (A_I^H E_I)^{-H} E_I^H, \quad \text{and} \quad R_I^+ R_I R_I^+ = R_I^+. \tag{B-7}$$

Substituting (B-5) into (28) and preserving only the first-order term, we obtain the following approximation for the optimal weight vector under finite samples

$$\widehat{W}_o \approx W_o + (E_R \Delta E_R^H + \Delta E_R E_R^H) A(u_1, v_1). \tag{B-8}$$

Using (B-8) and the property that $\Delta E_R{}^H E_R = 0$, we can find the powers of the desired signal, the noise, and the interferers at the array output as follows

$$\begin{cases} \widehat{p}_s = \bar{\pi}_1 |\widehat{W}_o^H A(u_1, v_1)|^2 \approx p_s + \sum_{k=1}^{3} \Delta p_{s,k} + \text{the first-order terms} \\ \widehat{p}_n = \pi_n \widehat{W}_o^H \widehat{W}_o \approx p_n + \sum_{k=1}^{2} \Delta p_{n,k} + \text{the first-order terms} \\ \widehat{p}_i = \widehat{W}_o^H A_I \bar{\Psi}_I A_I \widehat{W}_o \approx p_i + \Delta p_i + \text{the first-order terms} \end{cases}, \qquad \text{(B-9)}$$

where $p_s = \bar{\pi}_1 |W_o^H A(u_1, v_1)|^2$, $p_n = \pi_n W_o^H W_o$, and $p_i = W_o^H A_I \bar{\Psi}_I A_I{}^H W_o$ represent the output powers of the desired signal, the noise, and the interferers without finite sample effect, respectively. $\bar{\pi}_1 = E\{|\bar{s}_1(t)|^2\}$ denotes the input power of the desired signal. p_i is negligible when the 2-D EIC works normally. The other terms are the second-order perturbation terms which are given by

$$\begin{cases} \Delta p_{s,1} = \Delta p_{s,2}^* = \bar{\pi}_1 (A(u_1, v_1)^H E_R \Delta E_R{}^H A(u_1, v_1))^2, \\ \Delta p_{s,3} = 2\bar{\pi}_1 |A(u_1, v_1)^H E_R \Delta E_R{}^H A(u_1, v_1)|^2, \end{cases} \qquad \text{(B-10)}$$

$$\begin{cases} \Delta p_{n,1} = \pi_n A(u_1, v_1)^H \Delta E_R \Delta E_R{}^H A(u_1, v_1), \\ \Delta p_{n,2} = \pi_n A(u_1, v_1)^H E_R \Delta E_R{}^H \Delta E_R E_R{}^H A(u_1, v_1), \end{cases} \qquad \text{(B-11)}$$

and

$$\Delta p_i = A(u_1, v_1)^H E_R \Delta E_R{}^H A_I \bar{\Psi}_I A_I{}^H \Delta E_R E_R{}^H A(u_1, v_1), \qquad \text{(B-12)}$$

respectively. Since p_i is negligible, the output SINR of the 2-D EIC can be written as

$$\widehat{SINR}_o = \frac{\widehat{p}_s}{\widehat{p}_i + \widehat{p}_n} = \frac{p_s(1 + (\widehat{p}_s - p_s)/p_s)}{p_n(1 + (\widehat{p}_n - p_n)/p_n + \widehat{p}_i/p_n)}. \qquad \text{(B-13)}$$

Consider the situation where the number of data snapshots is large enough. Utilizing the first-order approximation of $(1+x)^{-1} \approx 1 - x$ for a small x, we can obtain an approximation for (B-13) as follows

$$\widehat{SINR}_o \approx SINR_o(1 + (\widehat{p}_s - p_s)/p_s - (\widehat{p}_n - p_n)/p_n - \widehat{p}_i/p_n), \qquad \text{(B-14)}$$

where $SINR_o = p_s/p_n$ represents the output SINR without finite sample effect. Since the expectation for each of the first-order terms in (B-9) is zero, the expectation of the output SINR can be approximated from (B-14) as follows

$$E\{\widehat{SINR}_o\} \approx SINR_o(1 + \frac{E\{\Delta p_s\}}{p_s} - \frac{E\{\Delta p_n\}}{p_n} - \frac{E\{\Delta p_i\}}{p_n}), \qquad \text{(B-15)}$$

where $\Delta p_s = \sum_{k=1}^{3} \Delta p_{s,k}$ and $\Delta p_n = \sum_{k=1}^{2} \Delta p_{n,k}$.

Next, we compute the individual terms in (B-15). As shown in (B-3), the deviation ΔR_w is composed of two independent terms, i.e., $\Delta\pi_n$ and $\Delta\bar{R}$. By using the eigenvalue method of [12] to estimate the noise power, it has been shown that

$$E\{|\Delta\pi_n|^2\} = \frac{\pi_n^2}{K(MN-P)} \tag{B-16}$$

if K data snapshots are used. On the other hand, it has been shown in [10] that the deviation $\Delta\bar{R}$ due to finite sample effect has zero mean and the second-order statistical property as follows

$$E\{Q_1^H \Delta\bar{R} Q_2 Q_3^H \Delta\bar{R} Q_4\} = \frac{1}{L}\mathrm{Tr}\{Q_3^H \bar{R} Q_2\}(Q_1^H \bar{R} Q_4), \tag{B-17}$$

where Q_i are matrices with appropriate sizes. By substituting (B-5) into (B-10)-(B-12) and using the properties of (B-7),(B-16), and (B-17), the individual terms in (B-15) are computed. The results are listed in **Appendix C**. It is also shown in **Appendix C** that the term $LE\{\Delta p_{i,a}\}/p_n$ is dominant in the case of input INR high enough since all the other terms decrease as the input INR increases. Accordingly, (B-15) can be approximately expressed as

$$E\{\widehat{SINR}_o\} \approx SINR_o(1 - \frac{1}{L}FSP) \tag{B-18}$$

for input INR high enough, where the factor of statistical performance $FSP = LE\{\Delta p_{i,a}\}/p_n$ is given by (30).

APPENDIX C

To ease the presentation, we employ the subscripts $x(1)$ and $x(2)$ to replace the subscripts "r" and "c", respectively. For example, $B_{x(1)}$ and $B_{x(2)}$ represent the notations B_r and B_c, respectively. Thus, (B-4) can be rewritten as

$$\Delta R = \sum_{i=1}^{2} \tilde{B}_{x(i)}^{H} \Delta \bar{R} \tilde{B}_{x(i)}. \tag{C-1}$$

Using the above notations and performing some algebraic manipulation provides

$$\begin{cases} E\{\Delta p_i\}/p_n \approx E\{\Delta p_{i,a}\}/p_n + E\{\Delta p_{i,b}\}/p_n + E\{\Delta p_{i,c}\}/p_n \\ E\{\Delta p_{i,a}\}/p_n = (L\omega)^{-1} \sum_{i,k}(\xi_{x(k),x(i)}\omega_{x(i),x(k)}) \\ E\{\Delta p_{i,b}\}/p_n = (L\omega)^{-1} \sum_{i,k}(Tr\{\pi_n R_I^+ A_I \bar{\Psi}_I A_I^H R_I^+ \tilde{B}_{x(k)}^H \tilde{B}_{x(i)}\}\omega_{x(i),x(k)}) \\ E\{\Delta p_{i,c}\}/p_n = (K(MN-P)\omega)^{-1}(\pi_n W_o^H \Omega R_I^+ A_I \bar{\Psi}_I A_I^H R_I^+ \Omega W_o) \end{cases}, \tag{C-2}$$

$$\begin{cases} E\{\Delta p_{n,1}\}/p_n \approx E\{\Delta p_{n,1a}\}/p_n + E\{\Delta p_{n,1b}\}/p_n + E\{\Delta p_{n,1c}\}/p_n \\ E\{\Delta p_{n,1a}\}/p_n = (L\omega)^{-1} \sum_{i,k}(Tr\{E_R E_R^H \tilde{B}_{x(k)}^H \tilde{B}_{x(i)}\}\eta_{x(i),x(k)}) \\ E\{\Delta p_{n,1b}\}/p_n = (L\omega)^{-1} \sum_{i,k}(Tr\{E_R E_R^H \tilde{B}_{x(k)}^H \tilde{B}_{x(i)}\}\rho_{x(i),x(k)}) \\ E\{\Delta p_{n,1c}\}/p_n = (K(MN-P)\omega)^{-1}(\pi_n^2 A(u_1,v_1)^H R_I^+ \Omega E_R E_R^H \Omega R_I^+ A(u_1,v_1)) \end{cases}, \tag{C-3}$$

$$\begin{cases} E\{\Delta p_{n,2}\}/p_n \approx E\{\Delta p_{n,2a}\}/p_n + E\{\Delta p_{n,2b}\}/p_n + E\{\Delta p_{n,2c}\}/p_n \\ E\{\Delta p_{n,2a}\}/p_n = (L\omega)^{-1} \sum_{i,k}(Tr\{\pi_n R_I^+ A_I D_{x(k)} \bar{\Psi}_I D_{x(i)}^H A_I^H R_I^+\}\omega_{x(i),x(k)}) \\ E\{\Delta p_{n,2b}\}/p_n = (L\omega)^{-1} \sum_{i,k}(Tr\{\pi_n^2 R_I^+ \tilde{B}_{x(k)}^H \tilde{B}_{x(i)} R_I^+\}\omega_{x(i),x(k)}) \\ E\{\Delta p_{n,2c}\}/p_n = (K(MN-P)\omega)^{-1}(\pi_n^2 W_o^H \Omega R_I^+ R_I^+ \Omega W_o) \end{cases}, \tag{C-4}$$

$$\begin{cases} E\{\Delta p_{s,1}\}/p_s \approx E\{\Delta p_{s,1a}\}/p_s + E\{\Delta p_{s,1b}\}/p_s \\ E\{\Delta p_{s,1a}\}/p_s = (L\omega^2)^{-1} \sum_{i,k} \\ \quad \left[(\pi_n W_o^H \tilde{B}_{x(i)}^H \tilde{B}_{x(k)} R_I^+ A(u_1,v_1))(\pi_n W_o^H \tilde{B}_{x(k)}^H \tilde{B}_{x(i)} R_I^+ A(u_1,v_1))\right] \\ E\{\Delta p_{s,1b}\}/p_s = (K(MN-P)\omega^2)^{-1}(\pi_n W_o^H \Omega R_I^+ A(u_1,v_1))^2 \end{cases}, \tag{C-5}$$

and

$$\begin{cases} E\{\Delta p_{s,3}\}/p_s \approx E\{\Delta p_{s,3a}\}/p_s + E\{\Delta p_{s,3b}\}/p_s + E\{\Delta p_{s,3c}\}/p_s \\ E\{\Delta p_{s,3a}\}/p_s = 2(L\omega^2)^{-1} \sum_{i,k}(\eta_{x(i),x(k)}\omega_{x(k),x(i)}) \\ E\{\Delta p_{s,3b}\}/p_s = 2(L\omega^2)^{-1} \sum_{i,k}(\rho_{x(i),x(k)}\omega_{x(k),x(i)}) \\ E\{\Delta p_{s,3c}\}/p_s = 2(K(MN-P)\omega^2)^{-1}|\pi_n W_o^H \Omega R_I^+ A(u_1,v_1)|^2 \end{cases}, \tag{C-6}$$

respectively, where $\omega = W_o^H W_o$ and

$$\begin{cases} \omega_{x(i),x(k)} = W_o^H \tilde{B}_{x(i)}^H \tilde{B}_{x(k)} W_o \\ \xi_{x(k),x(i)} = Tr\{\Psi_I^{-1} \bar{\Psi}_I \Psi_I^{-1} D_{x(k)} \bar{\Psi}_I D_{x(i)}^H\} \\ \eta_{x(i),x(k)} = \pi_n A(u_1, v_1)^H R_I^+ A_I D_{x(i)} \bar{\Psi}_I D_{x(k)}^H A_I^H R_I^+ A(u_1, v_1) \\ \rho_{x(i),x(k)} = \pi_n^2 A(u_1, v_1)^H R_I^+ \tilde{B}_{x(i)}^H \tilde{B}_{x(k)} R_I^+ A(u_1, v_1) \end{cases}, \quad \text{(C-7)}$$

for $i, k = 1, 2$, respectively.

Next, let the positive definite matrix $\bar{\Psi}_I$ be expressed as $\bar{\Psi}_I = \tilde{a}\bar{\Psi}_0$ for some positive number $\tilde{a}$ and positive definite matrix $\bar{\Psi}_0$. Then it can be easily shown from (B-7) that R_I^+ is proportional to $\tilde{a}^{-1}$. From (C-2) to (C-7), it can also be shown that each of the following terms

$$E\{\Delta p_{i,b}\}/p_n, \quad E\{\Delta p_{i,c}\}/p_n, \quad E\{\Delta p_{n,1a}\}/p_n, \quad E\{\Delta p_{n,2a}\}/p_n, \quad and \quad E\{\Delta p_{s,3a}\}/p_s \quad \text{(C-8)}$$

is proportional to $\tilde{a}^{-1}$ and each of the following terms

$$E\{\Delta p_{n,1b}\}/p_n, \quad E\{\Delta p_{n,1c}\}/p_n, \quad E\{\Delta p_{n,2b}\}/p_n, \quad E\{\Delta p_{n,2c}\}/p_n, \quad E\{\Delta p_{s,1a}\}/p_s,$$
$$E\{\Delta p_{s,1b}\}/p_s, \quad E\{\Delta p_{s,3b}\}/p_s, \quad and \quad E\{\Delta p_{s,3c}\}/p_s \quad \text{(C-9)}$$

is proportional to $\tilde{a}^{-2}$, while only the term $E\{\Delta p_{i,a}\}/p_n$ is fixed and independent of $\tilde{a}$. To get a further simplification, consider the case that $\tilde{a}$ is large enough, i.e., the input INR is high enough so that these terms in (C-8) and (C-9) are negligible as compared to $E\{\Delta p_{i,a}\}/p_n$. Then, we have the result as shown by (B-18).

APPENDIX D

By using the Cauchy-Schwarz inequality that

$$|Tr\{Q_1 Q_2^H\}|^2 \le Tr\{Q_1 Q_1^H\} Tr\{Q_2 Q_2^H\}, \tag{D-1}$$

where Q_1 and Q_2 are matrices with appropriate sizes, it follows from (31) and (32) that

$$\begin{cases} |\omega_{c,r}|^2 = |\omega_{r,c}|^2 \le \omega_{c,c}\omega_{r,r} \\ |\xi_{c,r}|^2 = |\xi_{r,c}|^2 \le \xi_{c,c}\xi_{r,r} \end{cases}. \tag{D-2}$$

Based on (30) and (D-2), it can be shown that

$$FSP \le \left[(\xi_{r,r}\frac{\omega_{r,r}}{\omega})^{1/2} + (\xi_{c,c}\frac{\omega_{c,c}}{\omega})^{1/2} \right]^2. \tag{D-3}$$

Substituting (31) into (D-3), we obtain

$$FSP \le \left[(\xi_{r,r}\lambda_{max}\{\tilde{B}_r^H\tilde{B}_r\})^{1/2} + (\xi_{c,c}\lambda_{max}\{\tilde{B}_c^H\tilde{B}_c\})^{1/2} \right]^2, \tag{D-4}$$

where $\lambda_{max}\{Q\}$ denotes the maximal eigenvalue of Q. Furthermore, based on (A-11) and the property of Kronecker product [7]

$$< \text{KP.4} > \quad \det\{Q_1 - q_1 I\} = 0,\ \det\{Q_2 - q_2 I\} = 0,\ \Rightarrow\ \det\{Q_1 \otimes Q_2 - q_1 q_2 I\} = 0,$$

where $\det\{Q\}$ denotes the determinant of the matrix Q, it can be shown that

$$\begin{cases} \lambda_{max}\{\tilde{B}_c^H\tilde{B}_c\} = \lambda_{max}\{B_c^H B_c\} < \sigma_c^2 \\ \lambda_{max}\{\tilde{B}_r^H\tilde{B}_r\} = \lambda_{max}\{B_r^H B_r\} < \sigma_r^2 \end{cases}. \tag{D-5}$$

Therefore, we have

$$FSP < (\xi_{c,c}^{1/2}\sigma_c + \xi_{r,r}^{1/2}\sigma_r)^2. \tag{D-6}$$

If the interferers are uncorrelated, (33) reveals that both $\xi_{c,c}$ and $\xi_{r,r}$ are not greater than $\sum_{i=2}^{P}(|d_{c,i}|^2 + |d_{r,i}|^2)^{-1}$. Thus, the inequality in (D-6) becomes

$$FSP < \sum_{i=2}^{P}(|d_{c,i}|^2 + |d_{r,i}|^2)^{-1}(\sigma_c + \sigma_r)^2. \tag{D-7}$$

From (6), it can be shown that $|d_{c,i}|^2 > 1$ if $|u_i - u_1| > 1/3$. Similarly, we have $|d_{r,i}|^2 > 1$ if $|v_i - v_1| > 1/3$. Thus, if $|u_i - u_1| > 1/3$ or $|v_i - v_1| > 1/3$, we have $|d_{c,i}|^2 + |d_{r,i}|^2 > 1$ for $i = 2, 3, \ldots, P$. Hence, (D-7) reduces to

$$FSP < (P-1)(\sigma_c + \sigma_r)^2. \tag{D-8}$$

Finally, substituting (D-8) into (29) yields the result shown by (34).

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