行政院國家科學委員會補助專題研究計畫成果報告

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計畫類別：■個別型計畫　　□整合型計畫

計畫編號：NSC　90－2213－E－002－057－

執行期間：　90 年 8 月 1 日至 91 年 7 月 31 日

計畫主持人：　鄭士康　台大電信工程研究所

本成果報告包括以下應繳交之附件：

□赴國外出差或研習心得報告一份

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■出席國際學術會議心得報告及發表之論文各一份

□國際合作研究計畫國外研究報告書一份

執行單位：　台大電信工程研究所

中　華　民　國　91　年　7　月　31　日

階梯近似之尺域方法及
多層次次網格時域有限差分法研究

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主持人：鄭士康　　台大電信工程研究所
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中文摘要

近年來，電磁分析法已從過去頻域分析法慢慢轉移到時域分析法，如時域有限差分法（Finite Difference Time Domain Method, FDTD）即為一常用來模擬二維或三維電磁場之有效數值方法。然而一維問題如傳輸線之計算，如非線性電路、印刷電路板之暫態反應甚至色散特性之模擬，皆為一重要之課題。首先，FDTD 被廣泛應用來解決各種不同領域的電磁問題，FDTD 是一種直觀且計算簡便的數值電磁方法，但因為一般架構在直角座標系以及計算時間、記憶體消耗等限制，使 FDTD 在模擬任意外型問題甚至微小結構時，並不能模擬得很好，而次網格方法的提出正是為了解決 FDTD 方法的缺點，使 FDTD 法可以更有效率地模擬任意外型及微小結構。本演算法利用多層次分割及遞迴的觀念，再進一步加強次網格法對局部區域加強解析度的能力，滿足因為模擬區域中不同區域對不同解析度的要求，並保有 FDTD 方法的優點。至於一維傳輸線的暫態模擬，在此提出一階梯近似法(Staircase Approximation) 以解決包含色散性傳輸線與非線性負載的多段傳輸線問題。本法以脈衝函數當成時域基底並藉以分離時間與空間變數而將原偏微分形式之傳輸線方程式轉換成矩陣形式之常微分方程式，再由匹配邊界條件可以將傳輸線上之暫態反應求出。本法特色乃在於可以用傳統頻域的計算技巧解決一些難解問題，如非線性負載與色散傳輸線之混合問題，甚至色散性負載亦可同時解決。

關鍵詞：時域有限差分法、多層次分割、遞迴、次網格，非線性，色散，階梯近似

Abstract

In these recent years, electromagnetic analysis techniques put more stress on time-domain method than frequency-domain method.The Finite-Difference Time-Domian Method(FDTD) has being used more and more frequently to deal with variant kind of electromagnetic 2-D and 3D problems. As a numerical technique for solving Maxwell's equations, FDTD method is quite straightforward and friendly in mathematical point of view. However, it's not suitable for those that involve with arbitrary or small structure because of the memory comsuming and the Cartesian coordinate that FDTD method using. And it's believed that the subgridding method would be one of the right key to break the limit of FDTD method. Base on the idea of multi-level division and recursion, an new kind of algorithm is proposed here to make an enhancement on finer subgridding and smoother transition for ordinary subgridding method. For 1-D transmission line transients simulation, we propose the staircase approximation to solve multisection dispersion transmission lines with nonlinear loads. Using pulses as time-domain basis to separate time and space variables, we transform the original partial differential equations into ordinary matrix equations. Then transient responses can be obtained by matching boundary conditions. The feature of this new approach is that by frequency-domain-like technique we can solve difficult transmission line problems, including nonlinear loads, dispersion lines and the combination of the above two.

Keywords: FDTD, recursive, subgridding, staircase approximation, nonlinear, dispersion

I Introduction

A subgridding scheme divides the problem into regions with different grid sizes. For previous studies on subgridding [1]-[3], there's usually only one subgrid region. A multilevel subgridding scheme can divide grids by a small grid-size ratio between adjacent level to simplify programming and reduce the error owing to the mismatch of grid size.

On the other hand, computation of non-linearity and dispersion in circuits is important in practical applications. We propose the staircase approximation to handle this issues and get the transients directly and efficiently.

II Multilevel Subgridding Scheme

A multilevel subgridding scheme is shown in Figs. 1 and 2. Here the ordinary FDTD and subgridding method is as the core process. For example, in a 2-D TM FDTD problem, a coarse grid is divided by a factor of 3, and a second-order homogeneous traveling wave equation is used for spatial interpolation to find out the extra information at the boundary of a subgrid region. During the transition between grids of adjacent levels, linear interpolation in time and bilinear interpolation in space are used. And for updating the H fields at the boundary of a upper grid, the data are sampled at the common H field points located at the boundaries of the lower-level subgrid region.

Further, a tree-like data structure is used in which a flag is added to tell if the grid of a certain level is divided into subgrids or not. As the algorithm shown in Fig. 2, the calculation is designed to be recursive.

III Staircase Approximation

The dispersive telegraphist's equations in time domain are

$$-\frac{\partial}{\partial x}v(x,t) = R\cdot i(x,t) + \frac{\partial}{\partial t}[L * i(x,t)]$$

$$-\frac{\partial}{\partial x}i(x,t) = G\cdot v(x,t) + \frac{\partial}{\partial t}[C * v(x,t)]$$

where the star "*" represents the convolution to account for dispersion. Approximate the signals by

$$v(x,t) \approx \sum_{j=1}^{n} v_j(x)h_j(t)$$

$$i(x,t) \approx \sum_{j=1}^{n} i_j(x)h_j(t)$$

where $h_j(t)$ is the unit rectangular pulse with duration Δt. By inserting the approximation into the dispersive transmission line equations and integrating with respect to t, the original system of continuous time-dependent equations transforms to a system of discrete time-independent matrix equations

$$-\frac{d}{dx}[v] = ([R] + \frac{1}{\Delta t}[L])[i]$$

$$-\frac{d}{dx}[i] = ([G] + \frac{1}{\Delta t}[C])[v]$$

where $[v]$ and $[i]$ are column matrices and $[R]$, $[L]$, $[G]$, $[C]$ are square matrices with

$$[R] = R[I]$$

$$[G] = G[I]$$

$$[L] = [L_{Dis}][M]^{-1}$$

$$[C] = [C_{Dis}][M]^{-1}$$

Here [I] stands for identity matrix, $[L_{Dis}]$ and $[C_{Dis}]$ account for the dispersion and

$$[M] = \begin{bmatrix} 0.5 & 0 & \cdots & \cdots & 0 \\ 1 & 0.5 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0.5 & 0 \\ 1 & \cdots & \cdots & 1 & 0.5 \end{bmatrix}$$

Once these matrices are determined, equations can be manipulated in a way similar to those in the time-harmonic case. Problems with linear loads thus can be solved directly, and nonlinear loads can be dealt with by a common iterative scheme

IV Numerical Examples

For verification of subgridding, a PEC cylinder with an electric current filament in free space shown in Fig. 3 is solved by the TM solver. With an alternate magnetic current filament source, the case is also solved by the TE solver. The frequency of the current source is 2.4GHz and the main grid size is 6.25mm. The test region is divided into 141 x 101 main cells with a $\lambda/20$ mesh size. At the border of the test zone, the second-order Mur's absorbing boundary condition is applied. Four different schemes were tested and compared. One scheme is with uniform coarse grid with cell size Δ=6.25mm, and one is with uniform fine grid in 1/3 main cell size. The other two schemes are with subgrids: one is with a level of subgrid, in which the subgrid size is $\Delta/3$; and the other is with two levels of subgrids as shown in Fig. 4. The size of cells in the second level of the two-level scheme is $\Delta/9$.

The curves in Fig. 5 report the steady state total field which is the response after about 27 periods around a circle of radius 12.8125cm outside the cylinder for the TM case. Comparisons of different schemes are given in Table I. The one-level subgridding scheme works well as in other reports. The improvement of this scheme in TE mode is similar to in TM mode version. In Table I., he uniform fine grid can obtain the best results, but the two-level scheme can still improve the accuracy than one-level subgrid and takes only 59.13% of the time in the uniform fine grid scheme. Moreover, for a single round of simulation, the two-level subgridding already offers 9 times the resolution of the main cell, and 3 times the resolution of the uniform fine grids within a comparable short time.

Fig. 6 shows the verification of the staircase approximation for nonlinear loads with the FDTD. Both transmission lines are lossless and are with the same parameters, $\ell = 0.5(m)$, $L = 0.5(\mu H/m)$, and $C = 0.2(nF/m)$. In addition, a matched generator excites unit rectangular pulse with duration $w = 1(n\sec)$. The nonlinear loads are described by $i = 0.01 \times v^2$ for $v > 0$ and $i = 0$ for $v \leq 0$, and shunt with capacitors $C_s = 50(pF)$. The resultant voltage signal at $x = 1.0(m)$ calculated by the staircase approximation (solid line) and the FDTD (dotted line) agrees well.

Remove the nonlinear loads and introduce the Debye dispersion to both transmission lines with parameters $\varepsilon_s = 9$, $\varepsilon_\infty = 4$ and $w_0 = 5\pi \times 10^8$. Apply the same pulse excitation and replace the internal resistor by $R_g = 150(\Omega)$. The voltage response calculated at $x = 0.5(m)$ and $x = 1.0(m)$ by the staircase approximation (solid line) and the frequency-domain transform method (dotted line) are illustrated in Fig. 7. The agreement of both curves validates the capability of the staircase approximation in dealing with dispersive transmission lines.

V Conclusion

A modified subgridding algorithm with multilevel scheme has been developed. It has been shown that it can improve the resolution of FDTD. Though we only apply it for a 2D case here, it can be easily extended for general 3D problems. For the staircase approximation, the ability to handle non-linearity and dispersion has been demonstrated. Further applications include nonlinear microwave circuit simulation and computation of transients and crosstalk of multi-conductor transmission lines.

VI Reference

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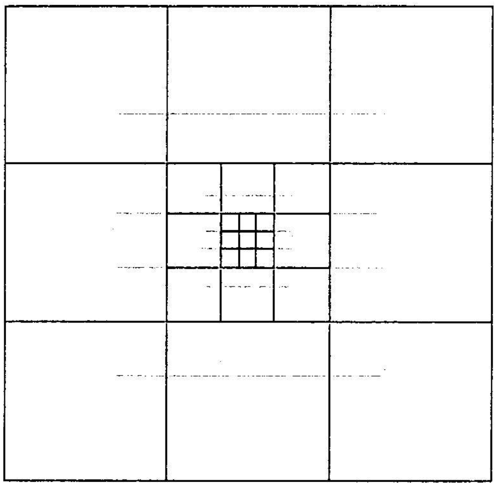

Fig. 1 Multilevel subgridding

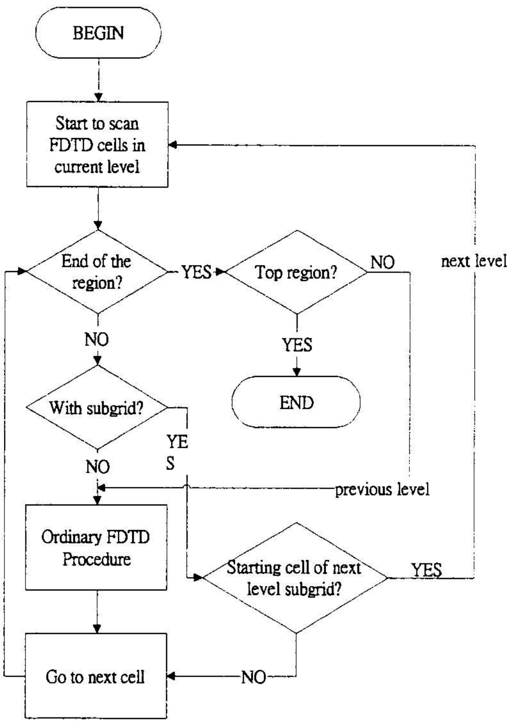

Fig. 2 The recursive algorithm for multilevel subgridding

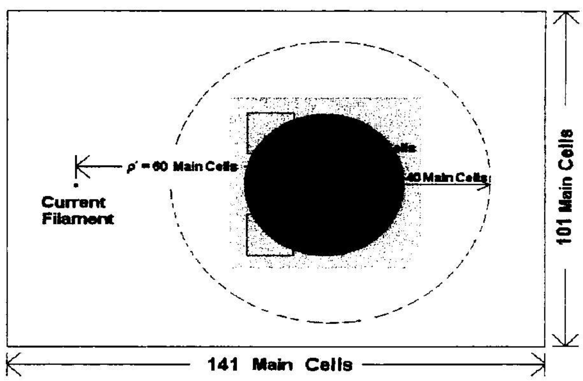

Fig. 3 A PEC cylinder with a current filament

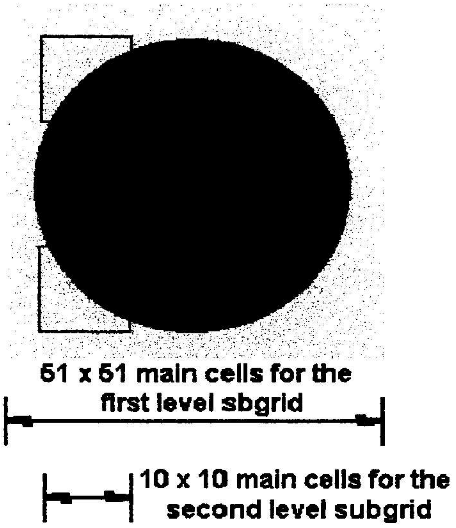

Fig. 4 Range of subgridding

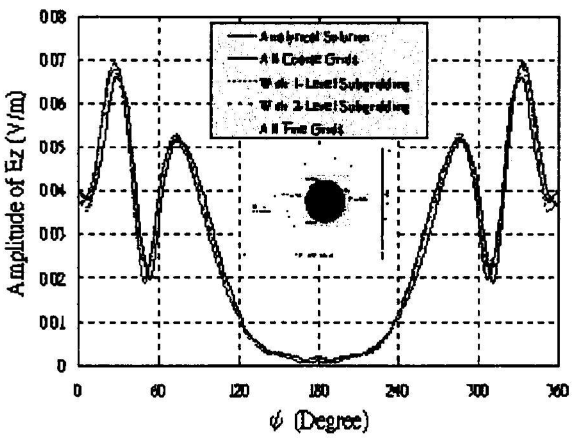

Fig. 5 Total field around a circle outside of the PEC cylinder in Fig. 3 for TM case

Scheme	Ratio of CPU time	Cell numbers	Normalized root mean square error
Uniform coarse grid	1.00	14241	1.00
1-level subgridding	5.26	37650	0.484
2-level subgridding	14.28	104628	0.438
Uniform fine grid	24.40	128169	0.445

Table I Performance of different schemes for the problem shown in Fig. 3 with electric current filament in TM mode. For every scheme 1500 coarse-grid time steps are requested.

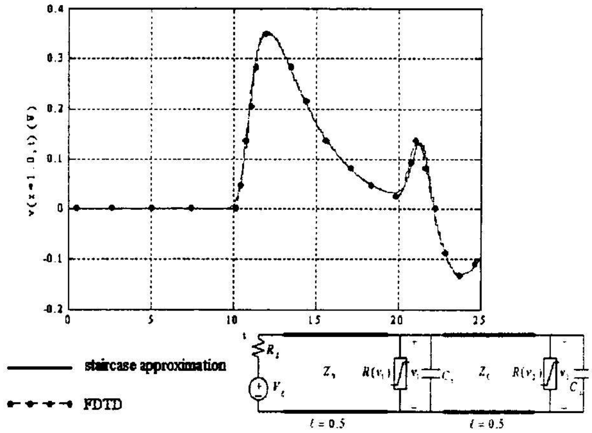

Fig.6 Verification of non-linearity with FDTD

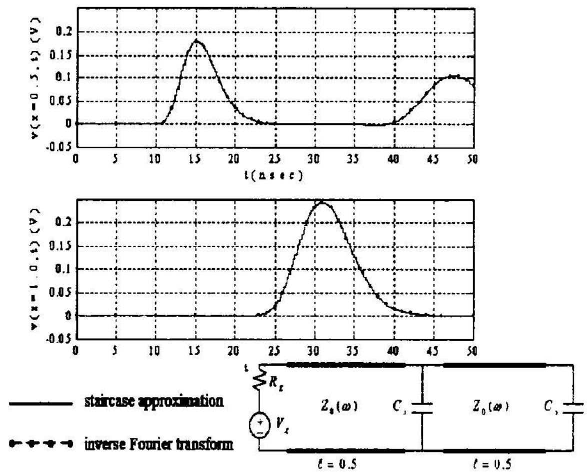

Fig.7 Verification of dispersion with phasor

參加
2002 IEEE
天線與傳播學會
國際研討會
（2002 IEEE Antennas and Propagation Society International Symposium）
報告

鄭士康
台大電機系/電信研究所
中華民國九十一年六月二十九日

一、引言

隨著電子產品快速發展，電路之傳輸線延遲問題漸受重視。敝人數年前指導學生開始此方面研究，略有心得。此次獲國科會補助，進行「階梯近似之尺域方法及多層次次網格時域有限差分法研究」，已具成果，乃指導博士班學生江怡霆將應用於傳輸線之時域階梯近似法(Staircase-Approximation Time-Domain, SATD 成果，及個人由此衍生更爲普遍化之離散電磁理論(Discrete-Time Electromagnetic Theory)，分別撰寫論文投稿 2002 年 IEEE 天線與傳播學會國際研討會(2002 IEEE AP-S International Symposium and USNC/URSI National Radio Science Meeting)，經審查通知錄取，於大會發表論文。此一研討會爲世界電磁研究人員一年一度，規模最大之盛會之一，重要論文常於此發表。此次敝人參加費用由國科會補助研究計劃之國際會議費用支出，而江同學申請教育部補助，未獲通過，僅能自費參加。

本屆會議於六月十六至二十日於美國德州聖安東尼市(San Antonio)舉行。由於六月十四日起之機票以旺季價格計算較爲昂貴，決定六月十三日星期四搭長榮班機出發，先行於加州舊金山一帶自費停留三晚，拜訪史丹佛大學友人及大學同學。復因二十二日星期六需擔任本系所四位碩士班學生畢業口試委員，故提前於二十日上午搭機直接返國，於二十一日星期五晚間返抵台北。

六月十三日下午抵達舊金山後，由大學同學葉公中接機，至其住所休息，並與葉氏夫妻及另一大學同學林世將一起晚餐，互道近況。十四日星期五前往史丹佛大學拜訪友人：敝人一九九九年時曾由國科會補助，至史丹佛大學電腦音樂與聲學研究中心(Center for Computer Researches of Music and Acoustics，CCRMA) 訪問半年，熟人不少。但因學期已經於前一天結束，故只見到幾位朋友，其中與博士班學生 Craig Sapp 相談較久，並互相展示電腦音樂方面研究成果，收穫頗豐。十五日由另一大學同學李占魁安排，與七、八位同學一起爬山餐敘，晚間改至同學陳明義家參加另一餐會，有另外七、八位同學及眷屬參加，相談甚歡；席間除獲知許多美國高科技業發展之第一手消息外，亦展示敝人學生的研究成果，獲得老友甚多改進與應用的中肯建議，獲益不淺。十五日晚即住陳明義寓所，次日由其開車送至機場，搭乘美國大陸航空班機，經約四小時飛行及一次轉機，於當地時間六時許抵達聖安東尼市，住進會議地點之凱悅飯店(Hyatt Regency)；因旅館費用昂貴，故與本系李學智教授共住一間房間。住進旅館後隨即與李教授至附近之歷史古蹟 Alamo 廣場參加大會舉辦之歡迎餐會(Reception)，與國內外電波同行友人多位交談。此次台灣各大學參加會議人數眾多，除教授外，亦有多位研究生，足見後生可畏。敝人認爲，台灣研究生較缺國際觀，政府應多鼓勵國內研究生參與國際研討會，培育下一代研發人才，此次多位研究生之參與，正是良好之典範。

二、會議經過

報到

六月十七日為正式會議之第一日(前一日為短期課程 Short Courses)，一早前往會場報到，領取資料袋及論文集光碟片。會議佔用飯店大廳(Lobby)之上的A、B 兩層，及附近的 Lasoya 會議中心，分十六個場地，同時進行。進行四天，一共有 129 個場次(Session)，參加人數預估在兩千餘人之譜。與過去參加相同會議之經驗相較，共同大會(Plenary Session)已經取消，也無開幕儀式，場次增加，容納更多的發表論文，顯見無線通訊風潮之下，天線與傳播研究之熱門程度與時俱增。

第一日

上午參加數值分析之新方法與應用(Novel Approaches and Applications of Numerical Analysis)議程，共有九篇論文發表，另一篇撤回。由發表之論文看來，仍是以積分方程式的計算為主，八篇為頻域分析，僅有一篇在時域進行計算，可見時域方法仍有很大的開拓空間。發表之論文包含積分方程與球形波模展開(Spherical Modal Decomposition)、方程式不變量測度法(Method of Equation Invariance，MEI)、特徵模態(Characteristic Modes)分析等之整合，以及一種類似時域有限體積方法(Finite Volume)之頻域計算方式，稱為細胞方法(Cell Method)。處理之問題大多為三維問題，包括汽車環境下之天線、橢圓形線圈天線、半空間(Half Space)內之散射、複雜波導等問題。看的出目前既有計算方法已能掌握三維問題，因此所解問題已漸趨實際。

下午參加議程為電磁學中的計算挑戰(Computational Challenges in Electromagnetics)。此議程由電波界名人 R. Mittra 及 T. Cwik 主持，由各應用單位提出其所面對之具挑戰性計算問題，與台下聽眾互動，個人覺得相當有意思。提出之問題較有印象者為樹叢中之坦克雷達回波之模擬、時域計算方法之整合、頻域計算方法整合、碎形天線之分析、毫米波及紅外線等之分析等等，大多為複雜環境下的電磁波行為分析。若干年前此類問題多採用幾何光學等高頻近似法分析，現今則因計算機速度的快速進步，已經達到似乎可用計算電磁理論計算之地步了。

白天議程結束後，參加大會主辦之自費遊河晚餐活動：參加人員極多，經一位年輕教授幫忙，得與另外十多位台灣來的教授及研究生、一位大陸外商公司工程師、一位在美國工作之台灣工程師、一位美國 Lawrence Livermore 國家實驗室之美國工程師等同船。聖安東尼市的優美河上風光果然名不虛傳，雖然室外氣溫

與台灣相當，卻不致感覺太熱。晚餐有沙拉、含一小塊牛排之主菜、及甜點，尚稱不錯。一面飽覽波光樹影，一面與友人暢談，一面品嚐美食，確實是一種享受。

第二日

第二日上午參加電磁學中之小波(Wavelets in Electromagnetics)議程，敝人論文時域離散電磁理論(Discrete-Time Electromagnetic Theory)爲其發表十篇論文中之最後一篇。此一議程爲特別議程，發表論文均係應議程主席 E. Tentzeris 邀請所發表。敝人論文提出時域電磁理論之全新觀點與演算方法，在各篇論文中獨樹一幟，其中之一特例用到平移不變基底(Translation-Invariant Bases)，爲小波基底之衍生。同議程其他論文則多報告小波在較傳統之動差法(Method of Moments)及時域多解析度方法(Multi-Resolution Time-Domain，MRTD)之應用，著重於動差法中矩陣稀疏度(Sparsity)之提高，或 MRTD 中吸收邊界條件(Absorption Boundary Condition)之改進與平行化處理之方法。另亦有人應用不同小波基底至實際之問題如地面穿透雷達(Ground Penetration Radar)電波於粗糙面之反射對地下目標偵測效果之分析，任意形狀之線型天線與散射體問題，及運動金屬邊界之電容效應分析等。

下午參加之議程有二，依次爲新數值方法(Novel Numerical Techniques)及時域有限差分法之應用(Applications of the FDTD Method)。其中新數值方法部分，四篇論文中有兩篇屬參數估測(Parametric Estimation)之新方法，一篇爲非線性系統中之 Karhunen-Loeve 模式，另一篇爲奇異值分解(Singular Value Decomposition，SVD)之應用：聽起來和信號與系統領域比較接近。時域有限差分法之應用方面，四篇論文中有一篇爲四階差分法及次網格(Subgrid)方法應用，可提高計算準確度，另兩篇分別爲 FDTD 在準光學接收器(Quasi-Optical Receiver)模擬及微波斷層掃描術(Microwave Resonator Tomography)之討論。最後一篇論文則提出於某些 FDTD 模擬中，如飛機或巴士客艙內之電波傳播情形，將傳輸天線之效應以其場型取代，可以避免處理複雜之天線結構，是不錯的構想。

第三日上午參加大規模計算電磁學的先進方法(Advanced Methods for Large Scale Computational Electromagnetics)議程，由這方面夙負盛名的伊利諾大學周永祖教授(W. C. Chew)及金建銘教授(J. M. Jin)主持。此一議程聽眾甚多，足見其重要性。發表之十篇論文均爲積分方程動差法或有限元素法方面，印象較深者略述如下：有一篇發展出快速方法以時域積分方程分析色散(Dispersive)物件的散射，提到各種色散性質的類型，對吾人進行中之時域計算方法，可能有用。另有一篇題爲「在小電腦解大型電磁問題」(Solving Large Electromagnetic Problems in Small Computers)的論文，吸引最多聽眾；內容仍是動差方法，運用積分方程列式，運用多層矩陣劃分演算法(Multilevel Matrix Decomposition Algorithm， MLMDA)、多層快速多極演算法(Multilevel Fast Multipole Algorithm，MLFMA)、Block ILU Preconditioner 等技巧，可以在個人電腦上解四、五十萬個基底的問題。第三篇較有印象之論文為利用高頻近似之穩相法(Stationary Phase)計算大型問題動差方法矩陣中相隔較遠之元素，其準確性及效率均較高。

下午參加議程名為：快速 Fourier 轉換為基礎之快速解法(FFT-Based Fast Solvers)。議程主席 S. Gedney 發表一篇論文，在解積分方程之動差法矩陣方程前，對積分點以特殊方式取樣，用快速 Fourier 轉換計算其疊代求解之 Preconditioner，甚為快捷。隨後幾篇論文做法相似，但用於不同問題。來自新加坡的一位作者認為他們提出了一種新方法，但若干人認為與一種稱為 Adaptive Integral Method (AIM)的方法無甚差別，略略引起一個小小的爭論。

大會最後一天，第四天仍有議程，敝人學生江怡霆之論文即排於星期四上午發表。但敝人為及時回國擔任本系所碩士班畢業口試委員，只好放棄聽講機會，趕赴機場。

三、整體印象及心得

敝人已有多年未參加此一 IEEE 天線與傳播學會國際研討會，今年再度參與，除了發現生面孔很多之外，熟識友人多半已具有相當學術成就，足見台諺「戲棚下站久就是你的」之真確。此外，參加人數亦較以往增加許多，而平行場次之增多，亦使聽眾常有分身乏術之嘆。

在探討主題方面，翻開議程手冊，即可發現天線與無線通訊技術場次增加許多，想必亦是反應無線通信熱潮。數值電磁計算場次仍多，大多可歸類於時域有限差分法、動差方法、有限元素法，而所解之問題也已多為實際三維問題，進步不可謂不大，然而基本方法上的創新則極為少見，一般均在計算之過程修改，或充分利用快速之電腦設備，以便提高計算效率。敝人目前與學生進行之時域離散電磁理論，乃是由基本理論下手，目前國際間並無類似研究，若能持續發展，數年之後，應可於國際電磁學術界占一席之地。

四、結語

感謝國科會於計劃中補助敝人參與此一會議，一方面得以發表一己之研究所得，一方面亦得以了解世界電磁研究之最新情況。明年此一會議將由國際電波學術界之傳統重鎮，美國俄亥俄州立大學主辦，預料盛況當不遜今年，希望能再有機會參加。

Discrete-Time Electromagnetic Theory*

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Abstract

A new theory for solving time-domain electromagnetic field problems is proposed. By this new theory the field is first expanded by bases in time, and the inner product of the equation residue is taken with respect to a set of testing functions to achieve a system of differential equations in space coordinates. The obtained matrix equations can then be either solved analytically, if possible, or numerically. This provides more flexibility in solution. In the special case that all matrices are Toeplitz, which is common if translation-invariant bases are adopted, the z transform is applicable. With the help of z transform theory in digital signal processing, concepts of discrete-time frequency and discrete-time traveling wave are introduced. Dispersion analysis of the central difference approximation as well as the FD-TD equation for one-dimensional wave is found easy with this theory.

Introduction

Almost all current time-domain numerical methods for electromagnetic field analysis start from dividing the problem domain into grids. This paper tries to propose another approach that discretizes in time-domain first. This approach facilitates the dispersion analysis, and is more flexible in dealing with space domain.

Time-Domain Test and Discrete-Time Representation

Define the inner product of two signals $f(t)$ and $g(t)$ as

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$\langle f(t), g(t)\rangle = \int_{-\infty}^{\infty} f(t)g(t)dt$. Select a set of testing functions $w = \{w_n(t)\}$. Hereafter real variable t and integer indices n , m, and ℓ are all from $-\infty$ to ∞. The time-domain test of a signal $f(t)$ with respect to w is then defined as a column vector $Test_w\{f(t)\} = [wf_m]$, $wf_m = \langle w_m(t), f(t)\rangle$. Choose a set of bases $h = \{h_n(t)\}$ to expand the signal $f(t)$ as $f(t) \approx \sum_{n=-\infty}^{\infty} h_n(t) f_n$. We then have a discrete-time representation (DTR) of $f(t)$ as $DTR_h\{f(t)\} = [h_n(t)]^T [f_n]$. By taking the time-domain test of the DTR of $f(t)$, we get the discrete-time transformation (DTT) of $f(t)$ as $DTT_{w,h}\{f(t)\} = [H_{mn}][f_n]$. The matrix element $H_{mn} = \langle w_m(t), h_n(t)\rangle$. It can be also proved that $DTT_{w,h}\left\{\frac{df(t)}{dt}\right\} = [\breve{H}_{mn}][f_n]$, with $\breve{H}_{mn} = \left\langle w_m(t), \frac{dh_n(t)}{dt}\right\rangle$.

Discrete-Time Maxwell Equations

For electromagnetic field problems, select electric and magnetic bases $h^m = \{h_n^m(t)\}$, $h^e = \{h_n^e(t)\}$. Define the electric and magnetic testing functions $w^e = \{w_n^e(t)\}$, $w^m = \{w_n^m(t)\}$. Then express the electric field, magnetic flux density, and magnetic current density as $DTR_{h^e}\{\overline{\mathrm{E}}(\bar{r},t)\} = [h_n^e(t)][\overline{E}_n(\bar{r})]$, $DTR_{h^e}\{\overline{\mathrm{B}}(\bar{r},t)\} = [h_n^m(t)][\overline{B}_n(\bar{r})]$, $DTR_{h^m}\{\overline{\mathrm{M}}(\bar{r},t)\} = [h_n^m(t)][\overline{M}_n(\bar{r})]$. The Faraday equation $\nabla \times \overline{\mathrm{E}}(\bar{r},t) = -\frac{\partial \overline{\mathrm{B}}(\bar{r},t)}{\partial t} - \overline{\mathrm{M}}(\bar{r},t)$ is next transformed to a discrete-time electromotive equation $[\nabla \times \overline{E}_n(\bar{r})] = -[D_{mn}^{em}][\overline{B}_n(\bar{r})] - [X_{mn}^{em}][\overline{M}_n(\bar{r})]$, $[D_{mn}^{em}] = [H_{mn}^{ee}]^{-1}[\breve{H}_{mn}^{em}]$, $[X_{mn}^{em}] = [H_{mn}^{ee}]^{-1}[H_{mn}^{em}]$, $H_{mn}^{ee} = \langle w_m^e(t), h_n^e(t)\rangle$,

$\breve{H}_{mn}^{em} = \langle w_m^e(t), \breve{h}_n^m(t) \rangle$, $H_{mn}^{em} = \langle w_m^e(t), h_n^m(t) \rangle$, after enforcing the time-domain test of the residue (the difference of the original equation and its discrete-time representation) to be zero. Similar processes lead to other discrete-time Maxwell equations: $[\nabla \times \overline{H}_n(\overline{r})] = [D_{mn}^{me}][\overline{D}_n(\overline{r})] + [X_{mn}^{me}][\overline{J}_n(\overline{r})]$, $[\nabla \bullet \overline{D}_n(\overline{r})] = [\rho_{e,n}(\overline{r})]$, and $[\nabla \bullet \overline{B}_n(\overline{r})] = [\rho_{m,n}(\overline{r})]$. Note that the discrete-time continuity equations can be also derived from the discrete-time Maxwell equations.

For homogeneous isotropic media, the constitutive relationship of the electric flux density is $\overline{D}(\overline{r},t) = \varepsilon_\infty \overline{E}(\overline{r},t) + \int_{-\infty}^{t} \chi^e(t-\tau)\overline{E}(\overline{r},\tau)d\tau$, which is transformed to a discrete-time form as $[\overline{D}_n(\overline{r})] = [\varepsilon_{mn}][\overline{E}_n(\overline{r})]$, $[\varepsilon_{mn}] = [\varepsilon_\infty \delta_{mn}] + [H_{mn}^{me}]^{-1}\left[\left\langle w_m^m(t), \int_{-\infty}^{t} \chi^e(t-\tau)h_n^e(\tau)d\tau \right\rangle\right]$. Other discrete-time constitution relationships are $[\overline{B}_n(\overline{r})] = [\mu_{mn}][\overline{H}_n(\overline{r})]$, $[\overline{J}_n^C(\overline{r})] = [\sigma_{e,mn}][\overline{E}_n(\overline{r})]$, and $[\overline{M}_n^C(\overline{r})] = [\sigma_{m,mn}][\overline{H}_n(\overline{r})]$.

Discrete-Time One-Dimensional Wave

Consider only linearly-polarized one-dimensional problem, the source-free discrete-time Maxwell equations can be reduced to $\frac{d^2}{dx^2}[E_n(x)] - [P_{mn}^{ee}][E_n(x)] = 0$, for the electric field, where $[P_{mn}^{ee}] = ([D_{mn}^{em}][\mu_{mn}] + [X_{mn}^{em}][\sigma_{m,mn}])([D_{mn}^{me}][\varepsilon_{mn}] + [X_{mn}^{me}][\sigma_{e,mn}])$.

Assume that the bases are generated by a mother function $h_0(t)$ of unit delay Δt, i.e., $h_n(t) = h_0(t - n\Delta t)$. This set of translation-invariant bases can be regarded as a super set of wavelets, since wavelets must be both translation and scaling invariant. Assume further that all related matrices are Toeplitz matrices, which means the mn'th element depends only on m-n. The matrix equation now is written as $\frac{d^2E_m(x)}{dx^2}-\sum_n P^{ee}_{m-n}E_n(x)=0$, $P^{ee}_{mn}=P^{ee}_{m-n}$. The z transform of this equation is $\frac{d^2\mathbf{E}(x,\varsigma)}{dx^2}-\gamma_e(\varsigma)^2\mathbf{E}(x,\varsigma)=0$, $\gamma_e(\varsigma)=\sqrt{\sum_\ell P^{ee}_\ell \varsigma^{-\ell}}$, where we have changed the z variable into ς, to avoid being confused with the space z coordinate. The solution of this equation is the linear combination of fields like $\mathbf{E}^+(x,\varsigma)=F^+(\varsigma)e^{-\gamma_e(\varsigma)x}$. The inverse z transform of it leads to $E_n^+(x)=\frac{1}{2\pi}\int_{-\pi}^{\pi}F^+(e^{j\Omega})e^{-\alpha(\Omega)\frac{x}{c\Delta t}}e^{j(n\Omega-\beta(\Omega)\frac{x}{c\Delta t})}d\Omega$, where c is the light speed, Ω is the discrete-time frequency, $\alpha(\Omega)$ and $\beta(\Omega)$ are the discrete-time attenuation constant and the discrete-time wave number, respectively. If $\alpha(\Omega)$ is zero, and the discrete-time phase velocity $v_p(\Omega)=\frac{\Omega}{\beta(\Omega)}$ is a constant, we can show that $E_n^+(x_o+\ell v_p c\Delta t)=E_{n-\ell}^+(x_o)$, which represents a discrete-time traveling wave along a grid of cell size $v_p c\Delta t$.

Dispersion Analysis

For the one-dimensional wave in free space, the pulse bases $h_n^e(t)=w_n^e(t)=\begin{cases}1, n\Delta t\le t<(n+1)\Delta t\\ 0, \text{otherwise}\end{cases}$ and $h_n^m(t)=w_n^m(t)=\begin{cases}1, (n-\frac{1}{2})\Delta t\le t<(n+\frac{1}{2})\Delta t\\ 0, \text{otherwise}\end{cases}$ are adopted. The discrete-time phase velocity is obtained as $v_p(\Omega)=\frac{\Omega}{\beta(\Omega)}=\frac{\Omega}{2}\csc(\frac{\Omega}{2})$. If the corresponding ordinary differential equation is further approximated by central finite difference in x, a difference equation identical with that used in FD-TD [1] can be derived. Its discrete-time phase velocity is deduced as $v_p=\frac{\Omega}{s\cos^{-1}(1-(1-\cos\Omega)/s^2)}$, where $s=\frac{c\Delta t}{\Delta x}$.

Reference

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Staircase-Approximation Time-Domain Method for Transient Analysis of Lossy Nonunifrom Transmission Lines Using ABCD Matrices

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Abstract

The staircase-approximation time-domain (SATD) method is applied to transient analysis of nonuniform transmission lines (NTL's) by employing cascaded uniform transmission line (UTL) sections and ABCD matrices. Numerical results to exponential nununiform transmission lines (ENTL's) are verified with analytical formulae, both in time-harmonic and SATD forms, and the CAD tool PSPICE. This method also works well even for a lossy ENTL under the excitation of trapezoidal pulse chains.

Introduction

Nonuniform transmission lines (NTL's) have a large variety of applications, including wide-band impedance matching, reduction of discontinuity effects and pulse shaping. Unfortunately, exact analysis of NTL's is usually unavailable. Although there do exist analytical solutions to some specific NTL's like [1], their implementations are quite difficult, if not impossible, which degrades their practicability. The treatment of NTL's, therefore, is mostly resorted to numerical approaches. One popular treatment, also used in this paper, is to make use of cascaded uniform transmission line (UTL) sections as well as ABCD matrices.

A new technique, the staircase-approximation time-domain (SATD) method, has been proposed and its capability of handling transients of transmission lines has been demonstrated [2][3]. Conventionally, the inverse Fourier transform (IFT) is used to obtain the transients from the frequency-domain results. This approach is inefficient when long transients or broad spectra have to be considered. The SATD is more efficient, even when loss is introduced and arbitrary waveforms are excited. In addition, our previous works show the duality between the time- harmonic and the SATD formulae. Thus, analytic forms of specific exponential nonuniform transmission lines (ENTL's) in [1] can be directly translated into the SATD forms without reformulation. For more general cases, we can still analyze the transients of an ENTL by cascaded UTL sections using the SATD technique.

Formulation

By staircase-approximation time-domain (SATD), signals are expressed as

$$v(x,t) \approx \sum_{j=0}^{n} v_j(x) h_j(t) \tag{1}$$

$$i(x,t) \approx \sum_{j=0}^{n} i_j(x) h_j(t) \tag{2}$$

where $h_j(t)$ is the unit rectangular pulse defined as

$$h_j(t) = \begin{cases} 1 & , j\Delta t \le t < (j+1)\Delta t \\ 0 & , \text{others} \end{cases} \tag{3}$$

and the original transmission line equations can be converted into matrix forms [2][3]. The equivalent ABCD matrix of a section of UTL with length ℓ in the SATD form can be derived as

$$\begin{bmatrix} [A] & [B] \\ [C] & [D] \end{bmatrix} = \begin{bmatrix} \cosh([P]\ell) & [Z_0]\sinh([P]\ell) \\ [Z_0]^{-1}\sinh([P]\ell) & \cosh([P]\ell) \end{bmatrix} \tag{4}$$

where $[P]$ is the propagation matrix and $[Z_0]$ is the characteristic impedance matrix. To deal with an NTL, we divide it into sections of UTL's using the procedures shown in Fig. 1. The equivalent ABCD matrix of an NTL thus can be obtained by

$$\begin{bmatrix} [A] & [B] \\ [C] & [D] \end{bmatrix} = \prod_{k=1}^{n} \begin{bmatrix} [A_k] & [B_k] \\ [C_k] & [D_k] \end{bmatrix} \tag{5}$$

and the original NTL circuit can be analyzed easily.

Numerical Examples

As the first example, the transients of a lossless ENTL, as shown in Fig. 2, excited by a unit trapezoidal pulse defined in Fig. 3, is analyzed. The ENTL has parameters

$L(x)=0.5\cdot 3^{x/\ell}(\mu H/m)$ and $C(x)=0.2\cdot 3^{-x/\ell}(nF/m)$, which is equivalent to the characteristic impedance $Z_0(x)=50\cdot 3^{x/\ell}(\Omega)$. The exact solution to this ENTL can be expressed by the ABCD-matrix [1] as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}=\begin{bmatrix} e^{-\delta\ell/2}(\Gamma\cosh\Gamma\ell+\frac{\delta}{2}\sinh\Gamma\ell) & Z_{01}\gamma\, e^{\delta\ell/2}\sinh\Gamma\ell \\ \frac{\gamma\, e^{\delta\ell/2}\sinh\Gamma\ell}{Z_{01}} & e^{-\delta\ell/2}(\Gamma\cosh\Gamma\ell-\frac{\delta}{2}\sinh\Gamma\ell) \end{bmatrix} \tag{6}$$

where $\delta=(1/\ell)\ln Z_{02}/Z_{01}$ ($Z_{01}=50(\Omega)$ and $Z_{02}=150(\Omega)$ in this example), $\Gamma=\sqrt{\gamma^2+\delta^2/4}$, and $\gamma=j\beta$, the position-independent propagation constant. The corresponding ABCD matrix in SATD form can be obtained by suitable substitution [3]. Figure 4 shows the transients of the voltage at the load, including the exact solution (6) of the ENTL in both time-harmonic (phasor) and SATD forms, the numerical solutions using 20 sections of UTL's in both time-harmonic (phasor) and SATD forms and the CAD tool PSPICE. These five different approaches show nice agreement. This validates the applicability of UTL approximation for an NTL and the duality between the time-harmonic and the SATD formulation.

As the second example, we analyze the circuit in Fig. 2, but adopt another set of parameters for the ENTL that $L=0.5(\mu H/m)$ and $C(x)=0.2\cdot 9^{-x/\ell}(nF/m)$. The analytic form (6) can't be applied due to the position-dependent propagation constant. In addition, the voltage generator excites a pulse chain of unit trapezoidal pulses, shown in Fig. 3, of a $10(ps)$ period. By the approximation of 20 cascaded UTL sections, the voltage transients at the load in both lossless and lossy ($R=5(\Omega/m)$ and $G=10(S/m)$) cases are exhibited in Fig. 5 for comparison. The loss contributes to lower voltage levels, which stands to reason. All results also agree quite well.

Conclusions

The capability of handling transients of NTL's using the SATD has been demonstrated. Although only the ENTL is verified here, other NTL's could be treated likewise. This approach provides a simple but efficient way to get the transients, even for dispersive NTL's with nonlinear loads, which will be reported in our next paper.

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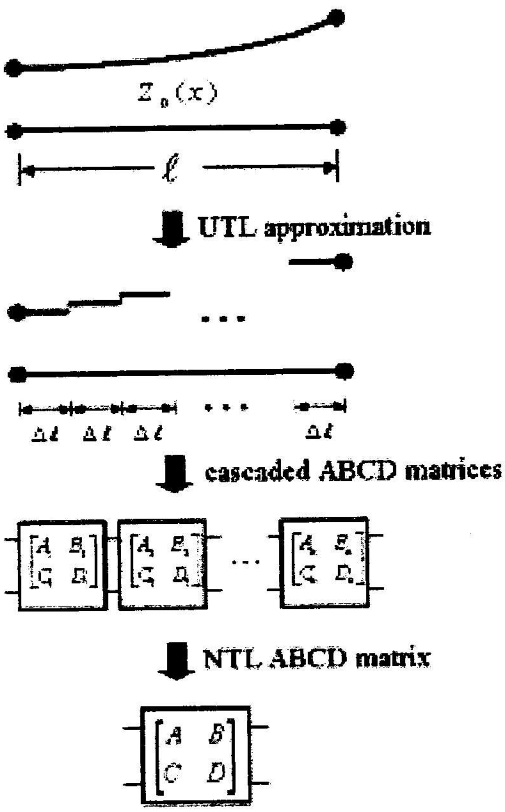

Fig. 1 Procedures to the equivalent ABCD matrix of a NTL.

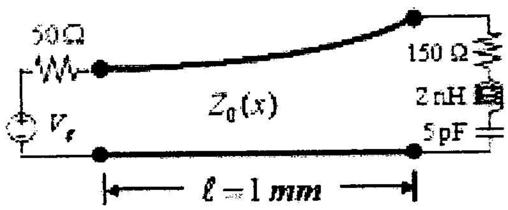

Fig. 2 A terminated NTL circuit.

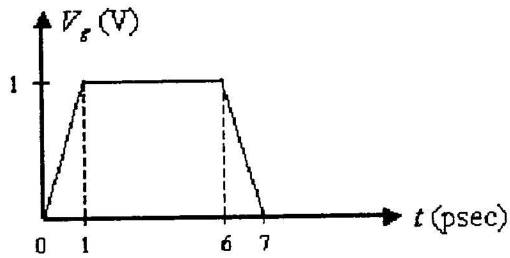

Fig. 3 Unit trapezoidal pulse.

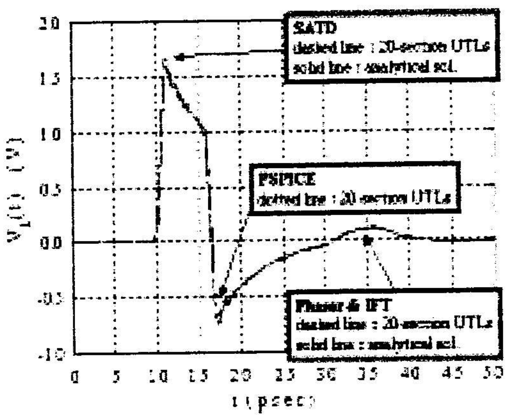

Fig. 4 Results of Example 1.

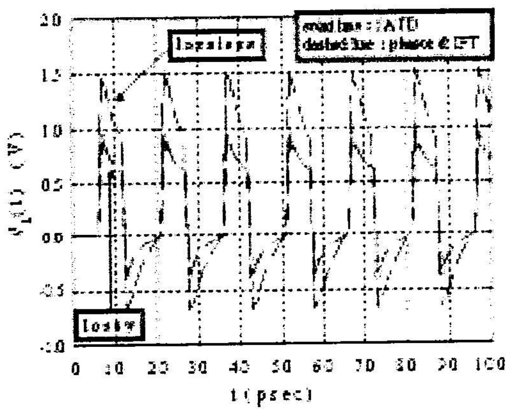

Fig. 5 Results of Example 2.