1 摘要

The transmitter output of the OFDM system is known to have large spectral sidelobes. Windowing and pulse shaping have been proposed to reduce sidelobes in the literature. When these methods are used, additional post-processing equalization is needed at the receiver. The post processing computations are often channel dependent and the coefficients need to be computed when the channel changes. In this report we consider window designs for the OFDM system. We will design windows that minimize out-of-band energy with the condition that post processing is channel independent.

Keywords: OFDM system, windowed OFDM, out-of-band energy

2 緣由與目的

The OFDM system has found applications in a wide range of wireless systems such as wireless local area network, and digital audio broadcasting [1][2]. In the conventional OFDM system the pulse shaping filter is a rectangular window. As the rectangular window has large spectral sidelobes, the transmitter output has large out-of-band energy. Many methods have been proposed to reduce sidelobes by windowing, filtering or using different pulse shaping filters. A number of non-rectangular continuous-time pulse shapes have been proposed to improve the spectral roll-off of the transmitted signal, e.g., [3]-[7]. Usually continuous-time pulse shapes are designed based on analog implementation of OFDM transmitters and these pulses usually do not admit a digital implementation [8]. Discrete-time windows that can be easily incorporated in digital implementation of OFDM transmitters have been considered in [9][10]. In [9], overlapping windows of duration longer than one OFDM symbol is proposed to reduce spectral sidelobes but significant ISI is generated even if the channel is AWGN. A post-processing equalizer is used to remove ISI. Overlapping windows that preserve the orthogonality of the OFDM system for AWGN channels are designed in [10]. If extra guard time is available, post processing can be avoided at the cost of a reduced transmission rate [11]. When there is no extra cyclic prefix, the use of windowing at the transmitter requires post processing at the receiver. In an ISI free system, if the transmitter is given, the receiver is determined accordingly. For an arbitrary window, the post processing is channel dependent in general and the post processing coefficients need to be computed when the channel changes. For wireless applications, it is important to have post processing that is channel independent.

In this report we will consider window designs for the OFDM system without using extra cyclic prefix. Windowed OFDM system with both cyclic prefix (WOFDM-CP) and zero padding (WOFDM-ZP) will be considered for frequency selective channels. We will design windows so that the transmitter output has good spectral roll-off or small out-of-band energy. For the cyclic-prefixed case, post-processing is in general channel dependent. We will consider the explicit dependency on the channel and show that the post-processing matrix can be made channel independent. In fact, the post processing matrix is channel independent if the window itself has cyclic-prefixed property. In this case, the output of the transmitter has the usual cyclic-prefixed property. Techniques that exploit cyclic prefix for synchronization can still be used. We will design windows that minimize the out-of-band energy of the transmitter output subject to the cyclic-prefixed constraint.

For the WOFDM-ZP case, we will see that the post-processing matrix depends on the window only but not on the channel. As in the WOFDM-CP case, we will design optimal windows that minimize the out-of-band energy of the transmitter outputs.

3 結果與討論:

The block diagram of the conventional OFDM system with cyclic prefix is as shown in Fig. 1. The modulation symbols to be transmitted are first blocked into M by 1 vectors, where M is the number of subchannels. The input symbols s_k are passed through an M-point IDFT, followed by the parallel to serial (P/S) operation and the insertion of cyclic prefix. The length of the cyclic prefix L is chosen to be equal to or larger than the order of the channel $C(z)$. At the receiver, the cyclic prefix is discarded and the samples are again blocked into M by 1 vectors and passed through an $M \times M$ DFT matrix $\mathbf{W}$. The scalar multipliers $1/C_k$ are also called frequency domain equalizers, where $C_0, C_1, \cdots, C_{M-1}$ are the M-point DFT of the channel impulse response c_n. The prefix is discarded at the receiver to remove inter-block ISI. The transceiver is ISI free and the receiver is a zero-forcing receiver. The only channel dependent part of the transceiver design is the set of scalars $1/C_k$, for $k = 0, 1, \cdots M-1$.

The conventional OFDM system with cyclic prefix can be redrawn as in Fig. 2. The matrices $\mathbf{G}$ and $\mathbf{S}$ shown in Fig. 2 are of dimensions $N \times M$ and $M \times N$, where $N = M + L$. They are given respectively by

$$\mathbf{G}_{cp} = \begin{pmatrix} \mathbf{0} & \mathbf{I}_L \\ & \mathbf{I}_M \end{pmatrix} \mathbf{W}^{\dagger}, \quad \text{and} \quad \mathbf{S}_{cp} = \mathbf{W} \begin{pmatrix} \mathbf{0} & \mathbf{I} \end{pmatrix}. \tag{1}$$

The matrix $\mathbf{\Lambda}$ indicated in Fig. 2 is diagonal, given by

$$\mathbf{\Lambda} = \text{diag} \begin{pmatrix} \frac{1}{C_1} & \frac{1}{C_2} & \cdots & \frac{1}{C_{M-1}} \end{pmatrix}.$$

We can obtain a WOFDM system by applying a window to each output block as shown in Fig. 3. The length of the window is the same as the block length N. The window has coefficients $d_0, d_1, \cdots, d_{N-1}$. The conventional OFDM system in Fig. 2 can be viewed as having a rectangular window with length N. Due to the non-rectangular window at the transmitter, the receiver needs an additional post processing matrix $\mathbf{P}$ to cancel intra-subchannel ISI. As there is no constraint on the matrix $\mathbf{P}$, there is no loss of generality in considering the receiver of the form shown in Fig. 3. The transmitting matrix can be written as $\mathbf{D}_{cp}\mathbf{G}_{cp}$, where $\mathbf{D}_{cp}$ is the diagonal matrix

$$\mathbf{D}_{cp} = \text{diag} \begin{pmatrix} d_0 & d_1 & \cdots & d_{N-1} \end{pmatrix}. \tag{2}$$

We partition $\mathbf{D}$ as

$$\mathbf{D}_{cp} = \begin{pmatrix} \mathbf{D}_0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{D}_2 \end{pmatrix},$$

where $\mathbf{D}_0$ and $\mathbf{D}_2$ are of dimensions $L \times L$, and $\mathbf{D}_1$ is of dimensions $(M-L) \times (M-L)$. For a given window, we now derive the condition on $\mathbf{P}$ so that the overall system is ISI free.

Lemma 1 *Consider the WOFDM system with cyclic prefix in Fig. 3. The system is ISI free if and only if the post processing matrix* $\mathbf{P}$ *is given by*

$$\mathbf{P}_{cp} = \mathbf{W} \left[\mathbf{W} \begin{pmatrix} \mathbf{D}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_2 \end{pmatrix} + \mathbf{\Lambda W} \begin{pmatrix} \mathbf{0} & \mathbf{C}_2(\mathbf{D}_0 - \mathbf{D}_2) \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \right]^{-1} \tag{3}$$

where $\mathbf{C}_2$ *is an* L *by* L *lower triangle Toeplitz matrix with the first column given by* $\begin{pmatrix} c_0 & c_1 & \cdots & c_{L-1} \end{pmatrix}^T$.

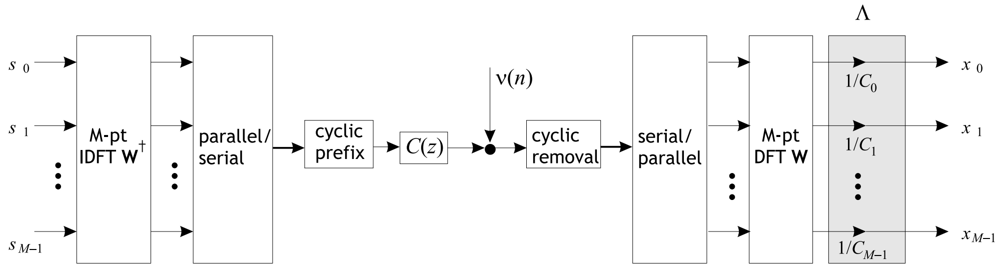

圖 1: The cyclic-prefixed OFDM system over a channel $C(z)$ with additive noise $\nu(n)$.

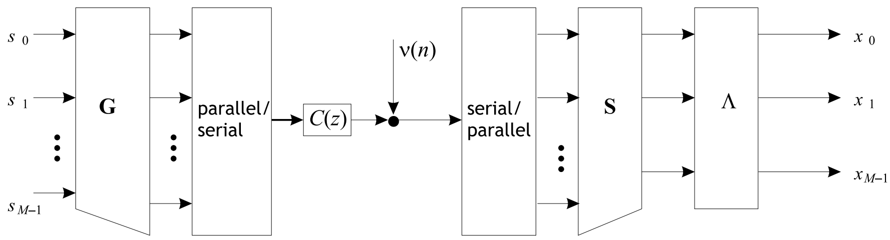

圖 2: The block based representation of the OFDM system with transmitting matrix $\mathbf{G}$ and receiving matrix $\mathbf{S}$.

A proof can be found in [12].

From the above lemma, we see that the solution of the post processing matrix depends on the window $\mathbf{D}$ as well as the channel. This channel dependency means that $\mathbf{P}_{cp}$ needs to be updated along with other channel dependent parameters as soon as the channel changes. To remove such a dependency, we observe that $\mathbf{P}_{cp}$ is channel independent if $\mathbf{D}_0 = \mathbf{D}_2$. That is, the window itself has the cyclic-prefix property. In this case, the post processing matrix is given by

$$\mathbf{P}_{cp} = \mathbf{W}\text{diag}\begin{pmatrix} 1/d_L & 1/d_{L+1} & \cdots & 1/d_{N-1} \end{pmatrix} \mathbf{W}^\dagger. \tag{4}$$

Notice that to have a channel independent $\mathbf{P}_{cp}$ for any channel, the condition $\mathbf{D}_0 = \mathbf{D}_2$ is not only sufficient but also necessary. In section 4, we will see how to design windows that improve the spectral roll-off of the WOFDM transmitter output, subject to the cyclic prefix condition.

Zero-padding case. In an OFDM system with zero padding, for each block of size M to be transmitted, L zeros, instead of cyclic prefix, are padded. The zero-padded OFDM system can also be represented using the block based transceiver in Fig. 2. Now, the matrices $\mathbf{G}$ and $\mathbf{S}$ in Fig. 3 are given by

$$\mathbf{G}_{zp} = \begin{pmatrix} \mathbf{W}^\dagger \\ \mathbf{0} \end{pmatrix}, \quad \mathbf{S}_{zp} = \mathbf{W}\begin{pmatrix} \mathbf{I}_M & \begin{matrix} \mathbf{I}_L \\ \mathbf{0} \end{matrix} \end{pmatrix}.$$

We can obtain a WOFDM system with zero padding from Fig. 2 by setting the last L window coefficients to zero. In this case the transmitting matrix can be expressed as $\begin{pmatrix} \mathbf{D}_{zp}\mathbf{W}^\dagger \\ \mathbf{0} \end{pmatrix}$, where $\mathbf{D}_{zp}$ is an $M \times M$ diagonal matrix with $\mathbf{D}_{zp} = \text{diag}\begin{pmatrix} d_0 & d_1 & \cdots & d_{M-1} \end{pmatrix}$. The window has M coefficients, d_0, $d_1, \cdots d_{M-1}$. The window is contrained to satisfy $1/M \sum_{k=0}^{M-1} |d_k|^2 = 1$ so that it has the same transmission power as the zero-padded OFDM system with a rectangular window.

Lemma 2 *Consider the WOFDM system*

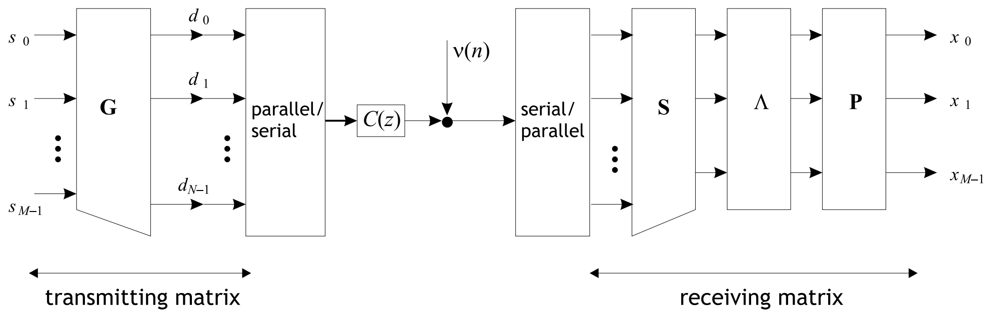

圖 3: A windowed OFDM system.

with zero padding in Fig. 3. The system is ISI free if and only if the post processing matrix **P** *is given by*

$$\mathbf{P}_{zp} = \mathbf{W}\mathbf{D}_{zp}^{-1}\mathbf{W}^{\dagger}.$$

A proof can be found in [12]

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