

# 行政院國家科學委員會專題研究計畫 期中進度報告

## 子計畫四：應用於多媒體傳輸之先進多載波調變技術(2/3)

計畫類別：整合型計畫

計畫編號：NSC92-2219-E-002-015-

執行期間：92年08月01日至93年07月31日

執行單位：國立臺灣大學電信工程學研究所

計畫主持人：馮世邁

計畫參與人員：李立翔 鐘元暉 羅宗惠 蔡曜年 劉智豪 蔣瀚霆

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中 華 民 國 93 年 6 月 10 日

附件：封面格式

# 行政院國家科學委員會補助專題研究計畫成果報告

## 智慧型音視訊和傳輸技術及多媒體應用 子計畫四： 應用於多媒體傳輸之先進多載波調變技術(2/3)

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本成果報告包括以下應繳交之附件：

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出席國際學術會議心得報告及發表之論文各一份

國際合作研究計畫國外研究報告書一份

執行單位：國立台灣大學電信工程研究所

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## 一、中文摘要

這是三年期計畫的第二年。在本計畫中，我們從理論方面比較 OFDM 系統及 SC-CP 系統的 BER 效能。我們證明了在高 SNR 時，SC-CP 系統比 OFDM 系統好，在低 SNR 時，OFDM 系統比 SC-CP 系統好。

**關鍵詞：**多載波調變, 單載波調變

### Abstract

This is the second year of a three-year project. In this project, we analytically compare the BER performance of OFDM systems and single-carrier with cyclic prefix (SC-CP) systems. We prove that for a moderately high SNR corresponding to a useful range of BER, the SC-CP system has a better performance than the OFDM system. For a very low SNR corresponding to a large BER, the OFDM system is better.

**Keywords:** multicarrier modulation, single carrier modulation.

## 二、緣由與目的

The OFDM transceiver has been applied successfully to various wireless channels [1]. Using redundant cyclic prefix, ISI can be canceled completely. The transmitter is channel independent. This feature is particularly attractive for wireless applications, where the transmitter usually does not have the channel profile and there is no bit or power allocation. In [2], the single carrier system with cyclic prefix (SC-CP) is introduced. The transmitter of the SC-CP system is an identity matrix followed

by cyclic prefix insertion, therefore channel independent. Unlike conventional single carrier system, the SC-CP system has  $M$  subchannels and ISI can be canceled using cyclic prefix. It has been shown that the SC-CP system outperforms the OFDM system for a useful range of BER [3]. The overall complexity is the same as the OFDM system. It has the additional advantage of having a very low peak-to-average power ratio. However the gain of the SC-CP system over the OFDM system has been demonstrated only through simulations. In this paper, we compare analytically the BER performance of uncoded OFDM and SC-CP systems for QPSK modulation.

## 三、結果與討論

The block diagram of the OFDM system is as shown in Fig. 1(a). Assume that the channel noise  $v(n)$  is complex AWGN with variance  $N_0$  and  $s_k$  are QPSK symbols with energy  $E_s$ . Define the SNR quantity  $g = E_s/N_0$ . The noise-to-signal ratio (NSR) of the  $k$ -th subchannel is

$$\beta(k) = \frac{1}{\gamma |P_k|^2}.$$

The average BER of the OFDM system is

$$P_{ofdm} = \frac{1}{M} \sum_{i=0}^{M-1} Q\left(\sqrt{1/\beta(i)}\right).$$

Fig. 1(b) shows the block diagram of the SC-CP system. In the SC-CP system, all the subchannels have the same SNR and it can be shown that the NSR is given by

$$\bar{\beta} = \frac{1}{M} \sum_{i=0}^{M-1} \frac{1}{\gamma |P_i|^2}.$$

For QPSK symbols, the BER of the SC\_CP system is given by

$$\mathcal{P}_{sc-cp} = Q\left(\sqrt{1/\beta}\right).$$

For the convenience of derivation, we define the function  $f(u) = Q\left(\frac{1}{\sqrt{u}}\right)$ . Then  $\mathcal{P}_{ofdm}$  and  $\mathcal{P}_{sc-cp}$  can be rewritten as

$$\mathcal{P}_{ofdm} = \frac{1}{M} \sum_{i=0}^{M-1} f(\beta(i)), \quad \text{and} \quad \mathcal{P}_{sc-cp} = f(\bar{\beta}),$$

We can find the first and second derivation of the function  $f(u)$  and show the lemma:

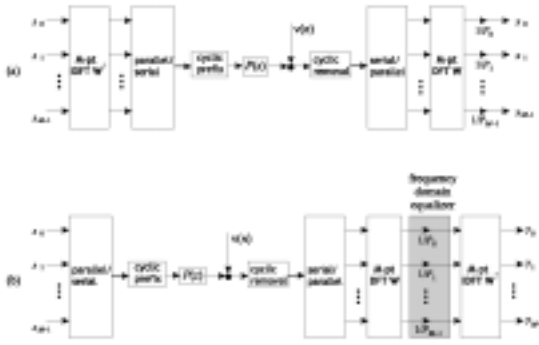


Fig. 1. OFDM and SC-CP systems

**Lemma 1:** *The function  $f(u)$  is monotone increasing and it is convex when  $u < 1/3$  and concave when  $u > 1/3$ .*

Given a set of numbers  $u_0, u_1, \dots, u_{M-1}$ , with  $0 < u_i < 1/3$ , Lemma 1 implies

$$\frac{1}{M} \sum_{i=0}^{M-1} f(u_i) \geq f\left(\frac{1}{M} \sum_{i=0}^{M-1} u_i\right).$$

Similarly, given  $u_0, u_1, \dots, u_{M-1}$  with  $u_i > 1/3$ , Lemma 1 implies

$$\frac{1}{M} \sum_{i=0}^{M-1} f(u_i) \leq f\left(\frac{1}{M} \sum_{i=0}^{M-1} u_i\right).$$

In each case, the equality holds if and only if  $u_0 = u_1 = \dots = u_{(M-1)}$ . Using the above results, we can show the following theorem:

**Theorem 1.** *The BERs of the OFDM and SC-CP systems are related by*

- (a)  $\mathcal{P}_{ofdm} \leq \mathcal{P}_{sc-cp}, \gamma \leq \gamma_0,$
- (b)  $\mathcal{P}_{ofdm} \geq \mathcal{P}_{sc-cp}, \gamma \geq \gamma_1.$

*In both cases, equalities hold if and only if  $|\mathcal{P}_0| = |\mathcal{P}_1| = \dots = |\mathcal{P}_{(M-1)}|$ .*

**Example.** The channel  $P(z)$  used in this example has 4 taps,  $0.7768 + j0.4561, -0.0667 + j0.2840, 0.1399 - j0.1592, 0.0223 + j0.2410$ . The length of cyclic prefix is 3. We compute  $\gamma_0 = 1.28$  dB and  $\gamma_1 = 13.78$  dB. Fig. 2(a) shows the performances  $\mathcal{P}_{ofdm}$  and  $\mathcal{P}_{sc-cp}$ . For  $\gamma > \gamma_1$ ,  $\mathcal{P}_{sc-cp}$  is much smaller than  $\mathcal{P}_{ofdm}$ . For  $\gamma = \gamma_1$ , Fig. 2(b) shows the subchannel BER  $\mathcal{P}_{ofdm}(k)$ . For the SC-CP system, subchannel BER is a constant as all the subchannels have identical BER. We see that only one third of subchannels in the OFDM system are worse than the SC-CP system and the other two thirds of the subchannels actually have a smaller BER. Although most of the subchannels in the OFDM system have very low BER, some of the worst subchannels have high BER, with the worst subchannel BER = 0.04. For  $\gamma > \gamma_1$ ,  $\mathcal{P}_{ofdm}$  is around 0.006 while  $\mathcal{P}_{sc-cp}$  is only around 0.0001.

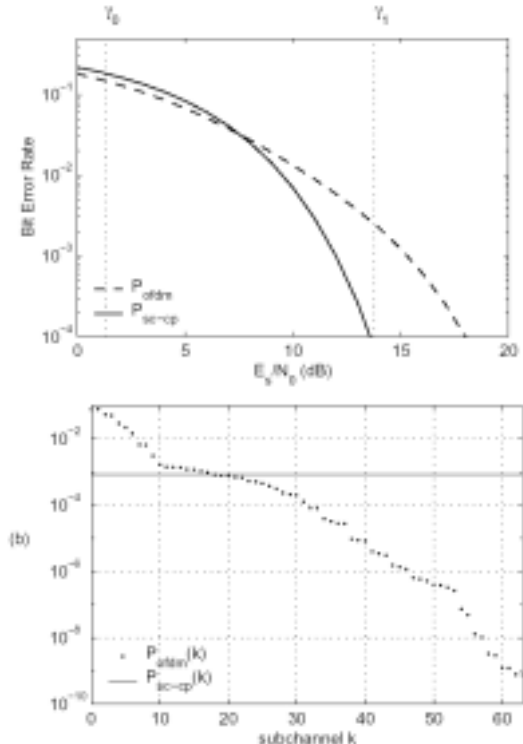


Fig. 2. Performance of OFDM and SC-CP systems.

#### 四、計畫成果自評

The result of this project has an important theoretical value. We have successfully shown that for high SNR corresponding to a moderate BER, SC-CP systems are better than OFDM systems. For very low SNR corresponding to a very large BER, OFDM systems are better.

#### 五、參考文獻

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