行政院國家科學委員會專題研究計畫　成果報告

降低 TH 預編碼平均功率之一些設計

計畫類別：個別型計畫
計畫編號：NSC93-2213-E-002-110-
執行期間：93 年 08 月 01 日至 94 年 07 月 31 日
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計畫主持人：林茂昭

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行政院國家科學委員會補助專題研究計畫成果報告

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中　華　民　國　　94 年 9 月 16 日

行政院國家科學委員會專題研究計畫成果報告

降低 TH 預編碼平均功率之一些設計

Designs for Reducing the Average Power of The Tomlinson-Harashima Precoding

計畫編號：NSC 93-2213-E-002-110
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一、中文摘要

TH 預編碼(Tomlinson-Harashima precoding)是將等化器(equalizer)放在通訊系統的發射端的技術，如此可避免雜訊經過等化器而被放大；在有線通道中的應用中，對於消除符號的互相干擾(intersymbol interference，ISI)有非常好的效果。TH 預編碼系統中，由於有回授之設計，造成輸出信號大小會在某一範圍之內作均勻分佈。針對此特性，我們提出一些新型設計以得到較低之輸出平均功率。

在本研究中，我們使用包括多擬隨機序列(multiple pseudo-random sequences)與多錯開器(multiple Interleavers)等選擇性映射(selective mapping)方式，藉此挑選其中較小平均功率的方法。並整合適當的編碼技術，設計新的降低平均功率之技術。使得 TH 預編碼系統同時具備低平均功率，高傳輸率及高傳輸可靠度的優點。

關鍵詞：錯誤更正碼、預編碼、符際干擾、擬隨機序列

英文摘要 (Abstract)

Tomlinson-Harashima precoding is a technique for eliminating intersymbol interference for the channel with response known by the transmitter. By placing equalizer at the transmitter side of the communication system, the intersymbol interference can be completely eliminated without noise enhancement. Due to the feedback design of Tomlinson-Harashima (TH) precoding, the output amplitude is uniformly distributed. By observing this phenomenon, we propose various designs so that the average power of the output sequence of the TH precoding system can be reduced.

Regarding the problem of reducing the average output power of TH precoding system, there are a few related research works, such as trellis shaping. In this research, we propose to integrate error-correcting codes and selective mapping methods including multiple maximum-length sequences and multiple interleavers, so as to obtain TH precoding systems with low output power, high transmission rate and high transmission reliability.

Keywords: Error-correcting Code，Tomlinson -Harashima precoding，Intersymbol Interference, pseudo-random sequence

二 、計畫的緣由與目的(Goals)

Consider a communication sytem with *intersymbol interference* (ISI). If the channel response is known by the transmitter, the equalizer can be placed at the transmitter side of the communication system. The resultant system is the so-called Tomlinson-Harashima (TH) precoding [1,2].

The transmitter of a one-dimensional TH precoding is illustrated in Fig. 1. The equivalent discrete-time channel response $h(D)$ is assumed to be known by the transmitter. The transmitter generates a data sequence $d(D)$ whose symbols

d_k are in the PAM signal set $A = \{-(M-1), -(M-3) \ldots, -1, 1, \ldots, M-3, M-1\}$. The transmitted symbol x_k is the unique number that satisfies the two constraints

$$x_k \equiv d_k - \sum_{j\geq 1}^{L} h_j x_{k-j} \quad \text{modulo} \quad 2M;$$
$$x_k \in (-M, M]$$

In other words, the transmitter finds the unique integer z_k such that

$$x_k = d_k + 2Mz_k - \sum_{j\geq 1}^{L} h_j x_{k-j}$$

is in the interval $(-M, M]$.

In D-transform notation,

$$x(D) = d(D) + 2Mz(D) - x(D)[h(D) - 1]$$

This reduces to

$$x(D) = \frac{d(D) + 2Mz(D)}{h(D)}$$

Let $y(D) = d(D) + 2Mz(D)$. Then, $y(D) = x(D)h(D) + n(D)$ is a sequence of odd-integer modified data symbols $y_k = d_k + 2Mz_k$. The received signal is

$$r(D) = x(D)h(D) + n(D) = y(D) + n(D)$$

where each component n_k in $n(D)$ is a white Gaussian noise sequence.

. For a general channel response, $h(D)$, the transmitted symbol x_k will be randomly distributed over the continuous interval $(-M, M]$. If $d(D)$ is precisely an i.i.d. sequence uniformly distributed over $(-M, M]$, then so is $x(D)$, with regardless of $h(D)$. If M is large, then $d(D)$ statistically approximates a continuous i.i.d uniform sequence. Therefore, $x(D)$ also statistically approximates a continuous i..i.d. uniform sequence.

The average energy of a continuous i.i.d. sequence uniformly distributed over $(-M, M]$ is

$$S_x = \frac{M^2}{3}$$

In [3], trellis shaping is used to reduce the average of the TH precoding. In this research，we propose to apply the selective mapping method using pseudo-random sequences and multi-interleavers integrated with turbo coding to reduce the average power of TH precoding. The proposed methods has the advantage over the trellis shaping in [3] for moderate to high signal-to-noise ratios.

三、研究方法與成果 (Methods and Results)

Trellis Shaping Method [3]:

Combined trellis shaping and TH precoding was first proposed by Eyuboglu and Forney[3] in 1992. The associated transmitter and receiver are shown in Fig. 2 and Fig. 3 respectively. In trellis precoding, the information sequence is mapped into a sequence $w(D)$ of two-dimensional signal points w_k in A^2, which is then modified by subtracting a sequence $a(D)$ of two-dimensional signal points a_k in the precoding lattice $\Lambda_s' = MZ^2$. The sequence $y(D) = w(D) - a(D)$ is the channel output sequence. The received sequence is

$$r(D) = x(D)h(D) + n(D) = y(D) + n(D)$$
$$= w(D) - a(D) + n(D)$$

The shaping decoder selects the shaping code sequence $c_s(D)$ from C_s and the precoding lattice sequence $a(D)$ to minimize the average power of $x(D)$. The minimization is obtained by applying Viterbi algorithm (VA) to decoding the trellis diagram of the shaping code C_s. In VA, the cumulative sums of the branch metrics are computed by

$$|x_i|^2 = \left| w_i - \sum_{j \geq 1} x_{i-j}(s)h_j - a_i \right|^2$$

where the $\{h_j\}$ are the coefficients of $h(D)$. The term $\sum_{j \geq 1} x_{i-j}(s)h_j$ represents "feedback" based upon the path history $x^{k-1}(s)$ of the state s. The term a_i is the element in MZ^2 that minimizes $|x_i|^2$.

For the signal mapper, the most significant bits represented by $z(D)$ are the *sign bits* (SB) for the I and Q dimensions of the two-dimensional signal constellation respectively as illustrated in Fig. 3. The remaining bits called the *less significant bits* (LSB) are mapped from the output of the turbo code.

Fig. 4 illustrates the receiver structure [3]. LSB are decoded first. Then LLR of SB are computed later.

In this research, we consider rate 1/2 turbo code, for which its generator of its constituent convolutional code is [37,21] and consider the 16QAM modulation with mixed labeling which is a trade-off between the the Gray labeling and the Ungerboeck labeling [4]. The ISI channel under consideration has

$h(D)$ = [1, 0.75, 0.25]

In every symbol interval, a turbo encoder G_c accepts $(k_c = 1)$ one bit, i_{1k} and produces $(n_c = 2)$ two output bits b_{0k} and b_{1k}, which select one of the four coding subsets. The input bit $(r_s = 1)$ i_{2k} is "transformed" in an inverse syndrome-former [10]

$$G_s = [1 + D^2, 1 + D + D^2]$$

$$(H_s^{-1})^T = [D, 1 + D]$$

to obtain the 2-tuple initial label t_k. This is modified by a 2-tuple c_{sk} from the shaping sequence $c_s(D) \in C_s$, selected by a Viterbi decoder, to obtain the final label $z_k = t_k \oplus c_{sk}$, which and the coded bits b_{0k} and b_{1k} are used to select a signal point $w_k \in A^2$.

The simulation results of turbo coded TH precoded system using trellis shaping with block length 1024 are shown in Fig. 5, which will be compared with the results obtained from the original turbo coded TH precoded system not using any power reduction technique and the techniques proposed in this research.

Selective Mapping Using Multiple m-sequences with Side Information :

Although the trellis shaping technique can significantly reduce the average power for TH precoded transmission, the bit error rate (BER) of trellis shaping is poorer than the original case for high signal-to-noise ratio conditions. The reason is although its LSB are protected by powerful turbo coding, the SB are not protected at all.

Now we consider the selective mapping technique with various m-sequences (maximum length sequences) and side information to reduce the average power, which was proposed in [5]. The transmitter is shown in Fig. 6. For a rate 1/2 binary turbo code of length 1024 bits, there are 503 message bits represented by $\overline{m}$ and 9 side information bits, $\overline{s}_i$. Each side information bits $\overline{s}_i$ has a uniquely corresponding 503-bit $\bar{t}_i$, which is obtained by shortening the 511-bit m-sequence. The XOR result of the message m and $\bar{t}_i$ plus $\bar{s}_i$ is encoded by the turbo encoder to obtain 1024 code bits. Each s_i yields an average output power. The $\bar{s}_i$ with the lowest average power is selected for transmission.

In the receiver, the operations are :

1. After decoding, the data estimates at the output of the decoder are obtained. Then, the side information can be obtained through the data estimates.
2. Using the side information, the m-sequence which is added to the message bits can be regenerated. Then the desired message bits could be easily recovered by adding back the m-sequence.

We use 16-state 1/2-code-rate turbo code for which the generator is [37,21]. We also use Gray-mapped 16QAM modulation. The error performance is shown in Fig. 5.

In this method, the cost we have to pay is the increased complexity due to the searching for m-sequence that minimizes the average transmitted power and the code rate loss due to transmission of side information. In this research, we propose two selective mapping techniques not using side information for reducing the average power.

Selective Mapping Using Multiple Interleavers:

The transmitter is illustrated in Fig 7. There are L Turbo encoders and each of them has an interleaver which is different from others. We use $L=16$.

The data sequences $A_1, A_2, \cdots, A_{16}$ are arranged in $[v_0, v_1, v_0, v_2, v_0, v_1, \cdots]$ order instead of the order

Message bits	Parity bits of RSC1	Parity bits of RSC2

This arrangement can increase the variety among different patterns after TH precoding. Finally, we shall transmit the data sequence which has the smallest average power.

The receiver structure is shown in Fig. 8. Each decoder-*i* is a Turbo decoder, which has its own deterministic interleaver. For iteration time less than *r*, each turbo decoder processes its own iterative decoding. At iteration *r*, we can decide which the interleaver is actually used by the transmitter. This can be done by evaluating the sum of $L_{e,i}(c_k)$ at the output of each decoder for $i=1,\cdots,L$, and choose the largest one of them. Then the decoder with the largest sum of $L_{e,i}(c_k)$ continues to process its MAP decoding

Simulation results implemented based on *r* = 3 are given in Fig. 5. We can see that although only 16 candidates are used in the selective mapping operation using multiple interleavers without side information, the associated error performance is close to that of selective mapping using m-sequences with side information and 512 candidates.

Selective Mapping Using Multiple m-sequences without Side Information :

The concept of this technique is similar to that of selective mapping using multiple interleavers. Since varying the interleaver of a turbo coded TH precoding system will only vary the parity part of the second constituent convolutional code, we hope that varying the m-sequences for XOR operations can provide more variety of average power output and hence can further reduce the average power.

We randomly choose *L= 16* sequences from all the possible m-sequences. In the rate 1/2 case, $\bar{u}$ is the message for encoding. After turbo encoding, $\bar{v}$ is the turbo codeword. Then, we do the “bit wise XOR” operator on the binary sequence $\bar{v}$ with $\bar{t}_i$ which is the i th candidate (m-sequence). The resultant sequence $\bar{A}_i = \bar{v} \oplus \bar{t}_i$ is sent to the signal mapper. Finally, we select the sequence with smallest power to transmit. The transmitter structure is in Fig. 9. The receiver of selective mapping using multiple m-sequences without side information is similar to that of selective mapping using multiple interleavers. The associated error performances are shown in Fig. 5 and Fig. 10 respectively. We see that the selective mapping using multiple m-sequences without side information can provide a little advantage over the selective mapping using multiple interleavers.

四、結論與討論(Concluding Remarks)

We propose two selective mapping methods without side information to reduce the average transmitted power of Tomlinson-Harashima precoding system. We find that the BER performances of the proposed methods are better than the well-known trellis shaping method. Compared with the selective mapping with side information, the proposed methods have the advantage of using fewer candidates and higher coding rate.

五、參考文獻(References)

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[4] G. Ungerboeck, “Channel coding with multilevel/phase signals,” *IEEE Trans. Inform. Theory,* Vol. IT-28, pp.55-67, Jan. 1988.

[5] Ghia-Hsiea Lin, “Some Designs for communication systems on channels with intersymbol interference,” M.Sc. thesis. National Taiwan University, July 2001.

六、圖表（Figures and Tables）

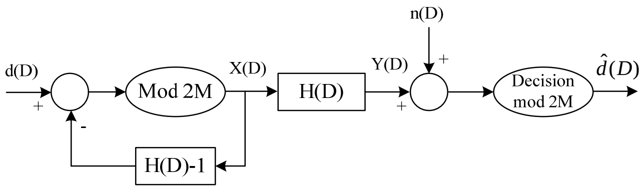

Figure 1. The structure of TH precoding for ISI channel.

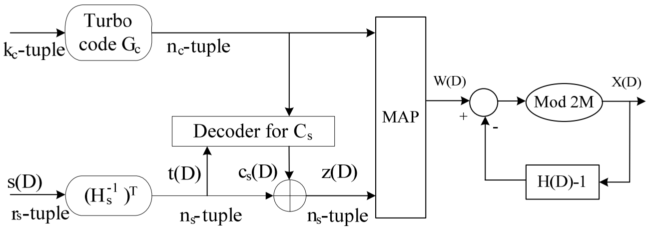

Figure 2. The transmitter of TH precoding using trellis shaping.

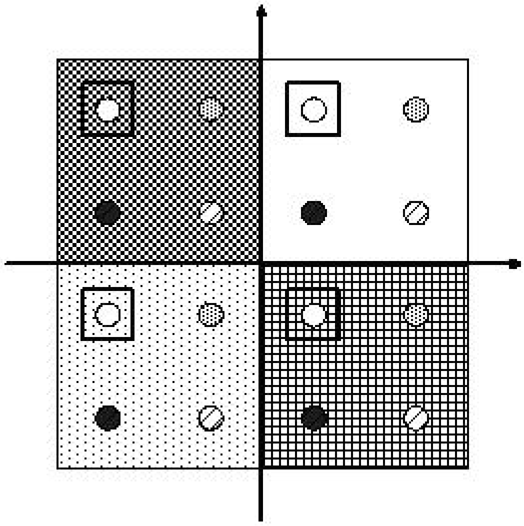

Figure 3. Initial signal set A^2

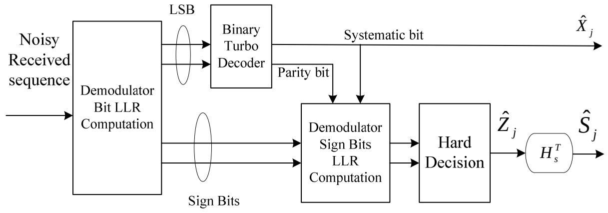

Figure 4. The receiver of TH precoding using trellis shaping

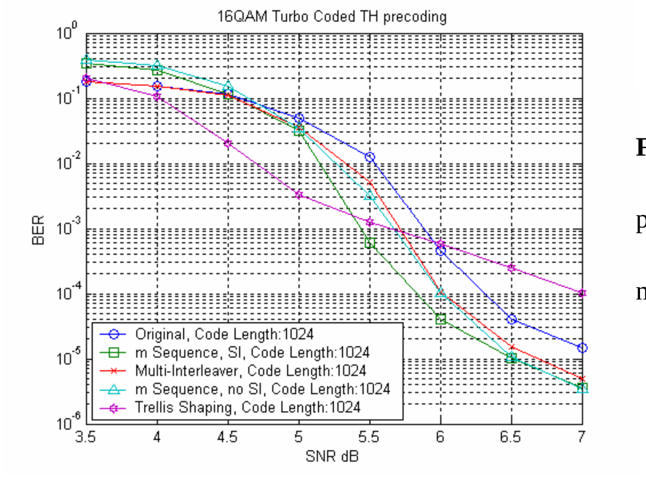

Figure 5. BER of Trellis Shaping, selective mapping using multiple m-sequences with side information (SI), selective mapping uing multiple interleavers, and selective mapping using multiple m-sequences without side information.

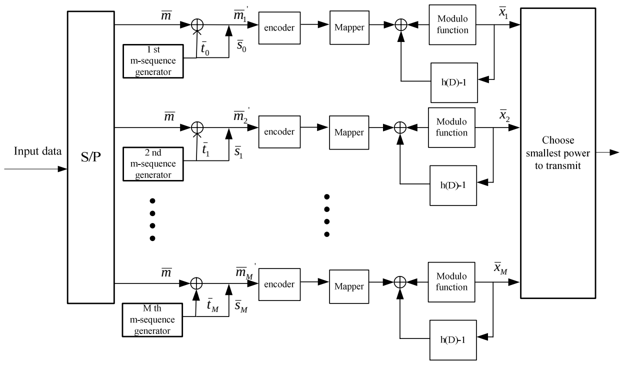

Figure 6. The transmitter of turbo coded TH precoding system employing selective mapping using multiple m-sequences with side information.

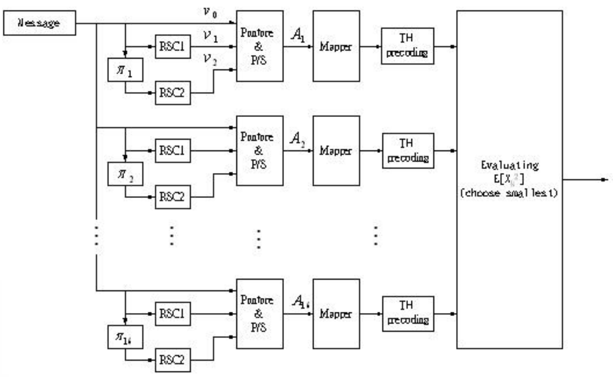

Figure 7. The transmitter of turbo coded TH precoding system employing selective mapping using multiple interleavers.

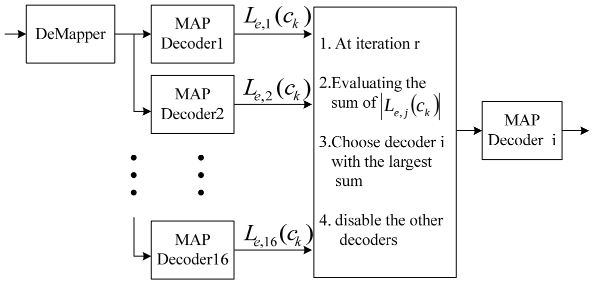

Figure 8. The receiver of turbo coded TH precoding system employing selective mapping using multiple interleavers.

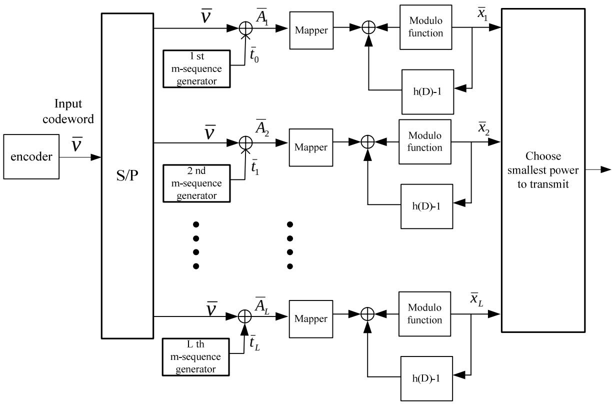

Figure 9. The transmitter of turbo coded TH precoding system employing selective mapping using multiple m-sequences without side information.

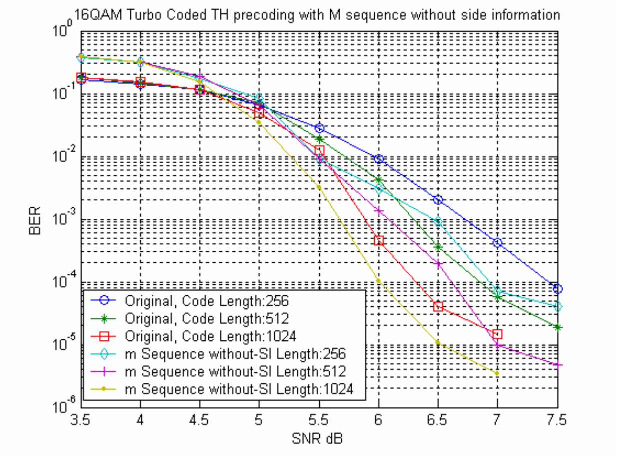

Figure 10. BER of turbo coded TH precoding system employing selective mapping using multiple m-sequences without side information (SI).