行政院國家科學委員會專題研究計畫 期中進度報告

多媒體影音高階處理、傳輸及設計--子計畫四:雙極正交矩陣之理論、建構及其於多媒體傳輸之應用(2/3)

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行政院國家科學委員會專題研究計畫成果報告

多媒體影音高階處理、傳輸及設計
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1 中文摘要:

這是三年期計畫的第二年。在前一年計畫中，我們提出多種雙極正交(APU)矩陣的建構方法。在本計畫中，我們研究APU矩陣於OFDM系統之應用。我們分析用APU矩陣當前置碼的OFDM系統之效能。實驗結果證明了此類矩陣前置碼可以大大的提升OFDM系統之效能。

關鍵詞：收發器，雙極正交矩陣，OFDM系統

Abstract: This is the second year of a three-year project. In the first year, we have introduced a new class of matrices called Antipodal ParaUnitary (APU) matrices and many methods have been proposed to construct these matrices. In this report, we apply the APU matrices as a precoder to OFDM systems. We will analyzed the performance of APU precoded OFDM systems. Simulation results show that by using an APU matrix as the precoder, the performance of the OFDM system can be significantly improved.

2 前言

Linearly precoded OFDM systems have been studied by a number of researchers [2] [3] [4]. Of particular interest is the OFDM system with a DFT precoding matrix. Such a system was shown to be the same as the so-called single carrier with frequency domain equalizer (SC-FDE) system [5]. In [2], it was shown that the SC-FDE system has the maximum diversity gain among all linearly precoded OFDM systems. In [3], BER minimized precoder for OFDM system was considered. For high SNR transmission, the SC-FDE system is optimal in the sense that it minimizes the bit error rate among OFDM systems with any orthogonal precoding matrix. In these studies, the precoders are constant matrices independent of z. In this report, we study precoded OFDM systems with APU precoding matrices.

3 研究方法

Fig. 1 shows the block diagram of a precoded OFDM system. In a precoded OFDM transmitter, the kth input block $\mathbf{s}(k)$, consisting of M modulation symbols such as QAM symbols, is first passed through an M by M precoding matrix $\mathbf{T}(z)$. The output of $\mathbf{T}(z)$ is given by

$$\mathbf{u}(k) = \sum_{i=0}^{N-1} \mathbf{T}_i \mathbf{s}(k-i). \tag{1}$$

In this report, the precoding matrix $\mathbf{T}(z)$ is chosen as a *normalized* APU matrix so that:

$$\tilde{\mathbf{T}}(z)\mathbf{T}(z) = \mathbf{I}_M.$$

That means, all the entries are scaled by $1\sqrt{MN}$. In practice, this normalization constant can be absorbed into the signal power of modulation symbols. After taking the M-point IDFT of the vector $\mathbf{u}(k)$, we get the $M \times 1$ vector:

$$\mathbf{x}(k) = \mathbf{W}^H \mathbf{u}(k),$$

where $\mathbf{W}$ is the DFT matrix with $[\mathbf{W}]_{kl} = exp(-j2\pi kl/M)$. Note that, unlike the conventional block transmission system, the transmitted block $\mathbf{x}(k)$ contains information of the input blocks $\mathbf{s}(k)$, $\mathbf{s}(k-1), \ldots, \mathbf{s}(k-N+1)$. Before $\mathbf{x}(k)$ is transmitted, a cyclic prefix (CP) of length L is added. In this report, we assume that the channel is slowly varying so that for each OFDM block, the channel response does not vary. We model the combined effect of DAC, transmit filter, channel, receive filter and ADC as an equivalent discrete time system with $c(n,k)$ denoting the nth tap of the impulse response when the kth block is sent. We also assume that the CP length L is large enough so that the length of the equivalent channel is $\leq (L+1)$. That is for all k, $c(n,k) = 0$ whenever $n > L+1$. The channel noise $\nu(n)$ is assumed to be AWGN with variance N_0. At the receiver end, to remove the inter block interference, the first L samples of the received block that correspond to the cyclic prefix are discarded. We obtain the $M \times 1$ vector $\mathbf{r}(k)$. Taking the DFT of $\mathbf{r}(k)$, we get

$$\mathbf{y}(k) = \mathbf{W}\mathbf{r} = \mathbf{C}(k)\mathbf{u}(k) + \boldsymbol{\nu}(k), \tag{2}$$

where $\mathbf{C}(k)$ is an $M \times M$ diagonal matrix whose (ℓ,ℓ)-th entry is given by the DFT coefficient of the channel impulse response $c(n,k)$:

$$C_\ell(k) = \sum_{n=0}^{L+1} c(n,k) e^{-j2\pi n\ell/M}. \tag{3}$$

The noise vector $\boldsymbol{\nu}(k)$ is an AWGN vector. Assume that the channel does not have spectral null so that $\mathbf{C}(k)$ is invertible. After multiplying the diagonal matrix $\mathbf{C}^{-1}(k)$, we get

$$\widehat{\mathbf{u}}(k) = \mathbf{u}(k) + \mathbf{C}^{-1}(k)\boldsymbol{\nu}(k). \tag{4}$$

In the absence of channel noise, the vector $\hat{\mathbf{u}}(k) = \mathbf{u}(k)$ for all k. When the precoding matrix $\mathbf{T}(z)$ is PU, we can get a zero forcing receiver by taking $\tilde{\mathbf{T}}(z)$ as the decoding matrix[1], as indicated in Fig. 1. Note that when we take $\mathbf{T}(z) = \tilde{\mathbf{T}}(z) = \mathbf{I}_M$, the system in Fig. 1 reduces to the conventional uncoded OFDM system. It should be emphasized that even though the precoded OFDM system has an overlapping-block transmitter, the channel impulse response $c(n, k)$ can be different for different block indices k and the system in Fig. 1 still has the zero-forcing property.

Noise Analysis Define $\boldsymbol{\beta}(k)$ as

$$\boldsymbol{\beta}(k) = \widehat{\mathbf{u}}(k) - \mathbf{u}(k) = \mathbf{C}^{-1}(k)\boldsymbol{\nu}(k).$$

The autocorrelation matrices of $\boldsymbol{\beta}(k)$ are given by

$$\mathcal{R}_\beta(k, \ell) = N_0\delta(\ell)\mathbf{C}^{-1}(k)\mathbf{C}^{-H}(k), \qquad (5)$$

where N_0 is the variance of the channel noise $\nu(n)$. Because $\mathbf{C}^{-1}(k)$ are diagonal matrices, we see from the above equation that $\boldsymbol{\beta}(k)$ is also an AWGN vector but each entry has a different variance.

Define the output noise vector $\mathbf{e}(k) = \widehat{\mathbf{s}}(k) - \mathbf{s}(k)$. Then it can be viewed as the output of $\tilde{\mathbf{T}}(z)$ with the input vector $\boldsymbol{\beta}(k)$. Therefore, we can write

$$\mathbf{e}(k) = \sum_{\ell=0}^{N-1} \mathbf{T}_\ell^H \boldsymbol{\beta}(k+\ell).$$

Using the facts that $\boldsymbol{\beta}(k)$ is an AWGN vector and $\tilde{\mathbf{T}}(z)$ is a normalized PU matrix, one can verify that its zeroth autocorrelation matrix at the kth block is given by

$$\mathcal{R}_e(k, 0) = \mathcal{E}[\mathbf{e}(k)\mathbf{e}^H(k)] = \sum_{\ell=0}^{N-1} \mathbf{T}_\ell^H \mathcal{R}_\beta(k+\ell, 0)\mathbf{T}_\ell,$$

where $\mathcal{R}_\beta(i, 0)$ is the zeroth autocorrelation matrix of $\boldsymbol{\beta}(i)$ given in (5). Note that $\mathcal{R}_\beta(i, 0)$ is a diagonal matrix. Looking at the ith diagonal term of $\mathcal{R}_e(k, 0)$, we can write the noise variance at ith subband (when the kth block is being processed) as

$$\sigma^2_{i,\mathbf{T}}(k) = \frac{1}{N}\sum_{\ell=0}^{N-1}\Big[\frac{1}{M}\sum_{n=0}^{M-1}\frac{N_0}{|C_n(k+\ell)|^2}\Big], \qquad (6)$$

where we have used (5) and the fact that all the entries of $\mathbf{T}_\ell$ have magnitude equal to $1/\sqrt{MN}$. The quantity $\sigma^2_{i,\mathbf{T}}(k)$ is independent of i; all subbands have the same noise variance! Moreover the decoding matrix $\tilde{\mathbf{T}}(z)$ has an averaging effect on the channel gains over a time period of N blocks. Note that we do not make any assumption about the APU matrix $\mathbf{T}(z)$. Any APU precoding matrix can achieve (6). From (6), we also see that the performance of precoded OFDM with a zero-forcing receiver degrades significantly when one or some of the channel gains are small. The noise variances in all subbands will be very large over a period of N blocks. To solve this problem, a minimum mean-square-error (MMSE) receiver is needed and will be derived in the next subreport.

Comparison with the uncoded OFDM and SC-FDE systems

[1] For convenience, we use a noncausal decoding matrix. A causal receiver can be easily obtained by cascading enough delays.

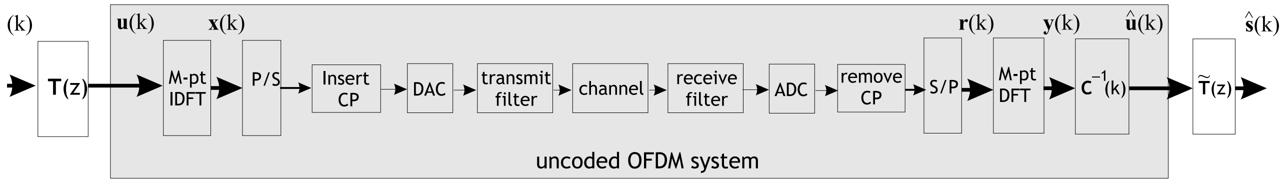

圖 1: An OFDM system with APU precoding matrix $\mathbf{T}(z)$.

When we take $\mathbf{T}(z) = \mathbf{I}_M$, the system in Fig. 1 becomes the conventional uncoded OFDM system. In this case $\mathbf{u}(k) = \mathbf{s}(k)$. Thus, for uncoded OFDM system, we can obtain from (4) the output noise variance at the ith subband as

$$\sigma^2_{i,ofdm}(k) = \frac{N_0}{|C_i(k)|^2}. \qquad (7)$$

The average output noise variance is

$$\sigma^2_{av,ofdm}(k) = \frac{1}{M}\sum_{i=0}^{M-1}\frac{N_0}{|C_i(k)|^2}. \qquad (8)$$

From (7), we see that $\sigma^2_{i,ofdm}(k)$ depends on both k and i and it is inversely proportional to $|C_i(k)|^2$. Recall that $|C_i(k)|$ is the channel gain at the ith frequency bin when the kth block is sent. For highly frequency selective channels, some of the gains $|C_i(k)|$ can be close to zero and the performance of the OFDM system will be affected by these spectral nulls.

If we allow the definition of APU matrices to include complex matrices whose coefficients have unit magnitude, then the DFT matrix $\mathbf{W}$ is APU. When we take $\mathbf{T}(z) = \mathbf{W}$, the system in Fig. 1 becomes the SC-FDE system [3]. By carrying out the same derivation, one can show that the noise variance of the SC-FDE system can be obtained by simply setting $N = 1$ in (6). The noise variance at the ith subband when the kth block is sent is given by

$$\sigma^2_{i,sc}(k) = \frac{1}{M}\sum_{n=0}^{M-1}\frac{N_0}{|C_n(k)|^2}. \qquad (9)$$

Observe from the above expression that $\sigma^2_{i,sc}(k)$ is independent of i. All the subbands have the same noise variance and they are equal to the average noise variance $\sigma^2_{av,ofdm}(k)$ in (8).

We can clearly see the difference between the conventional OFDM, the SC-FDE and the APU precoded OFDM systems from the three expressions in (7), (9) and (6). Because the decoding matrices $\tilde{\mathbf{T}}(z)$ (for precoded OFDM system) and $\mathbf{W}^H$ (for SC-FDE system) are PU, they have the energy (or power) conservation property [1]. The average output noise variance for the three systems is the same. However they distribute these noise variances to the subbands differently. For the conventional OFDM system, each subband can have a very different noise variance, especially when the channel is highly frequency selective. From (7), we see that subbands having small $|C_i(k)|$ will suffer from large noise variances. On the other hand, the SC-FDE system has an averaging effect in frequency domain; it averages over all subbands. When the channel has spectral nulls, all subbands will have large noise variances. To avoid this problem, an MMSE receiver is needed. Even when the channel does not have spectral nulls, for fast fading channel, the channel gains $C_i(k)$ can vary rapidly with respect to k. From (9), we see that if $\sum_i |C_i(k)|^2$ is small for some k, the whole kth block will be severely affected by noise amplification problem. The APU precoded OFDM system has an averaging effect in both frequency and time-domain; it averages over all subbands and over N OFDM blocks. Similarly, if the channel has spectral nulls, all the subbands will have large noise variances for the next N transmission blocks. Hence an MMSE receiver is needed for OFDM system with APU precoding matrices.

MMSE Receiver for Precoded OFDM Systems

We assume that the receiver removes the first L samples corresponding to the cyclic prefix so that there is no inter block interference. Given the received vector $\mathbf{r}(k)$, we want to design an MMSE receiver. As the DFT matrix $\mathbf{W}$ is invertible, there is no loss of generality if we consider the system in Fig. 2(a). It is known that when the length of the channel length is $\leq L+1$, the frequency selective channel $c(n,k)$ is converted into a set of M parallel non frequency selective subchannels. We can redraw the system in Fig. 2(a) as Fig. 2(b), where $\mathbf{C}(k)$ is a diagonal matrix whose (ℓ,ℓ)th entry is the ℓth DFT coefficient of $c(n,k)$ as defined in (3). The equivalent noise is the AWGN vector $\boldsymbol{\nu}(k)$ with the power spectral matrix $N_0\mathbf{I}_M$. In the following derivations, we assume that the transmitted signals $\mathbf{s}(k)$ satisfy

$$\mathcal{E}\{\mathbf{s}(k)\mathbf{s}^H(k-\ell)\} = E_s\delta(\ell)\mathbf{I}_M. \qquad (10)$$

In other words, the symbols are uncorrelated and have equal signal power. The fact that $\mathbf{T}(z)$ is normalized PU implies that $\mathbf{u}(k)$ also satisfies

$$\mathcal{E}\{\mathbf{u}(k)\mathbf{u}^H(k-\ell)\} = E_s\delta(\ell)\mathbf{I}_M. \qquad (11)$$

Moreover we also assume that the transmitted signals are uncorrelated to the channel noise.

Consider an MMSE receiver (possibly time-varying) with N coefficient matrices $\mathbf{Q}(k,\ell)$ for $0 \leq \ell \leq N-1$. Given the input vector $\mathbf{y}(k)$, the output of the MMSE receiver can be described as:

$$\widehat{\mathbf{s}}(k) = \sum_{\ell=0}^{N-1} \mathbf{Q}(k,\ell)\mathbf{y}(k+\ell), \qquad (12)$$

where $\mathbf{Q}(k,\ell)$ are $M \times M$ matrices. For convenience, we consider noncausal system. Our goal is to find $\mathbf{Q}(k,\ell)$ so that the following mean square error is minimized.

$$\mathcal{E}\{\big(\widehat{\mathbf{s}}(k)-\mathbf{s}(k)\big)^H\big(\widehat{\mathbf{s}}(k)-\mathbf{s}(k)\big)\}.$$

Applying the orthogonality principle, one can show that the MMSE solution is given by:

$$\mathbf{Q}(k,\ell) = \mathbf{T}_\ell^H\Lambda(k+\ell), \qquad (13)$$

where $\Lambda(i) = E_s\mathbf{C}^H(i)\Big(E_s\mathbf{C}(i)\mathbf{C}^H(i) + N_0\mathbf{I}_M\Big)^{-1}$. Note that $\Lambda(k)$ is a diagonal matrix whose nth diagonal entry is given by

$$\lambda_n(k) = \frac{C_n^*(k)}{|C_n(k)|^2 + N_0/E_s}.$$

From the above expressions, we see that the MMSE receiver can be decomposed into a time-varying diagonal matrix $\Lambda(k)$ and the time-invariant matrix $\tilde{\mathbf{T}}(z)$. When there is no noise, i.e. $N_0 = 0$, the MMSE receiver reduces to the zero-forcing receiver.

One can verify that for precoded OFDM system with an MMSE receiver, all the subbands also have the same error variance and it is given by

$$\sigma^2_{mmse}(k) = \frac{1}{MN}\sum_{\ell=0}^{N-1}\sum_{n=0}^{M-1}\underbrace{\frac{N_0}{|C_n(k+\ell)|^2 + N_0/E_s}}_{\sigma^2(n,\ell,k)} \quad (14)$$

One can clearly see from the above expression that the decoding matrix $\tilde{\mathbf{T}}(z)$ has an averaging effect in both frequency and time-domain; it averages over all subbands and over N OFDM blocks. Moreover the quantities $\sigma^2(n,\ell,k)$ are upper bounded by E_s. When some of the channel gains $|C_n(k+\ell)|$ approach zero, the error variance $\sigma^2_{mmse}(k)$ does not go to infinity. As we will see in the next report that by using an MMSE receiver, the performance of the precoded OFDM system is improved significantly.

4 結果與討論:

We carry out Monte-Carlo experiments to verify the performance of precoded OFDM systems with different precoders. The transmission channels are the modified Jakes fading channels described in [6]. In the experiments, we will use channel models with two different ratios of doppler frequency over transmission bandwidth. A larger value of r indicates that the channel is changing faster. The ratio $r = 0.0001$ corresponds to a slowly varying channel whereas $r = 0.001$ corresponds to a channel that varies 10 times faster. The number of taps of the channels is 16. The channel noise $\nu(n)$ is AWGN with variance N_0. In our simulation, we assume that the receiver knows the exact channel response. The DFT size is $M = 64$ and the length of cyclic prefix is $L = 16$. The input vector $\mathbf{s}(n)$ consists of QPSK symbols with power equal to E_s.

APU matrices of different length N will be used as the precoding matrices. When $N = 1$, the APU matrix reduces to the Hadamard matrix. It is known [3] that the OFDM system with a Hadamard precoding matrix has the same bit error rate performance as the SC-FDE system. We plot the bit error rate curves versus SNR (signal to noise ratio), which is equal to E_s/N_0. In the simulation, we do not consider MMSE receiver for the conventional OFDM system because the bit error rate performance of OFDM systems with MMSE receivers is identical to that of OFDM systems with zero-forcing receivers.

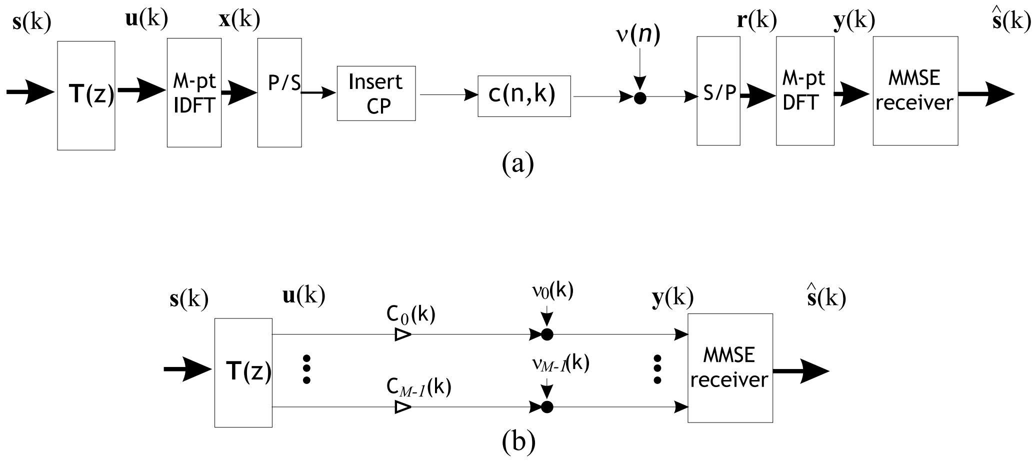

圖 2: (a) Precoded OFDM system with an MMSE receiver. (b) An equivalent system.

The results for $r = 0.0001$ are shown in Fig. 3. From the figure, we see that the performance of precoded OFDM system with a zero-forcing receiver is worse than that of the OFDM system at low SNR. This is because when the transmission encounters deep fading at some frequency bins, all the outputs of precoded OFDM receiver will be seriously affected by channel noise. On the other hand, for the OFDM system, only a portion of the outputs will be seriously affected. However when an MMSE receiver is employed, the precoded OFDM systems have a much better performance than the OFDM system. If we compare the performance of precoded OFDM systems with different precoders, we see that when the channel is slowly varying, using a longer precoding matrix does not provide much gain in performance. This is because when the channel variation in the time domain is small, averaging the performance in the time domain has little effect on the performance.

For channel that is varying 10 times faster with $r = 0.001$, the results are shown in Fig. 4. Again we see that precoded OFDM system with a zero-forcing receiver does not perform well and using an MMSE receiver can greatly improve the performance of precoded OFDM systems. Also note that the performance improves as the length of the precoding matrix N increases. As the channel is fast varying, averaging in the time domain can provide additional gain. If we compare the cases of $N = 1$ and $N = 8$, averaging over 8 blocks can yield an additional gain of more than 2 dB when the bit error rate is 10^{-5}.

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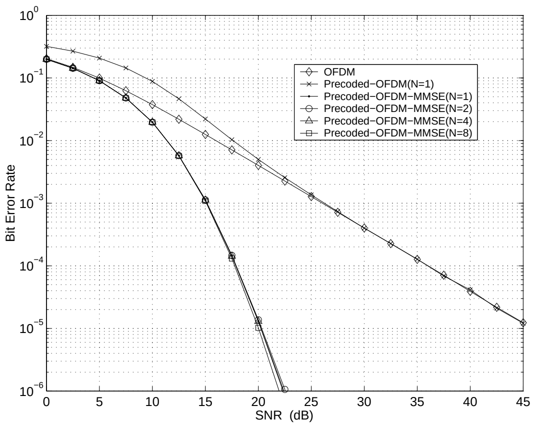

圖 3: Bit error rate performance for slowly varying channels.

[2] Z. Wang and G. B. Giannakis, "Linearly precoded or coded OFDM against wireless channel fades?," in *Proc. Third IEEE Workshop Signal Process. Adv. Wireless Commun.*, Taoyuan, Taiwan, Mar. 2001.

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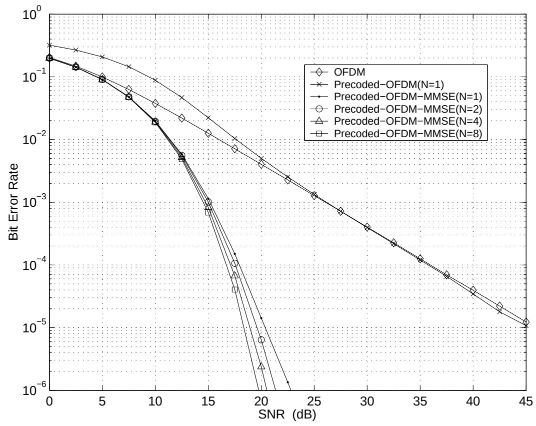

圖 4: Bit error rate performance for fast varying channels.

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5 計畫成果自評

In this project, we have successfully found an application for the APU matrices, which were proposed in the previous year. We have studied the OFDM systems with an APU matrix as precoder. The precoded systems are analyzed in detail. Moreover we have carried out numerical simulations to verify our results. The simulation showed that by using an APU precoding matrix, the output noise is averaged in both time and frequency domain. And it showed that precoded OFDM systems with MMSE receivers have a significantly better performance than the conventional OFDM system. In next year project, we are going to study the application of APU matrices for MIMO transmission system.

2006 年亞太電路與系統會議

出國報告書

馮世邁

電路與系統學會（Circuits and Systems Society）每兩年皆會在亞太地區舉辦研討會，結合所有有關電路，系統及信號處理領域的研究者集聚一堂，針對最新發展或研究做研討。每兩年的大會大多在 9--12 月間舉行。2006 年在新加坡舉行, 從 12 月 4 日至 12 月 7 日，共計 4 天。

會議第一天 12 月 4 日為專題課程(tutorials)。 12 月 5 日一早大會正式開始。本人在本次年會共有 2 篇論文發表:

1. Filterbank Framework for Multicarrier Systems with Improved Subcarrier Separation (12月5日下午發表)
2. Design of Time Domain Equalizers Incorporating Radio Frequency Interference Suppression (12月5日下午發表)

除了針對以上 2 篇論文作專題報告之外，本人還必須於 12 月 5 日下午主持一個 special session，主題是多速率信號處理及其應用。此一 special session 共有 5 篇論文，其中 2 篇來自於台灣，1 篇來自於瑞典，1 篇來自於芬蘭，1 篇來自於美國，相當國際化。近年來多速率信號處理技術逐漸被通訊領域所重視，會議中有愈來愈多人對這方面的研究有興趣，發問的人也很踴躍。

本屆亞太電路與系統研討會辦得非常成功，雖然只是一個中型區域性的研討會，但是參加的人很多，有多國際上知名的專家都來了，非常難得。主辦者 Prof. Y. C. Lim 表示，大會邀請了很多知名的專家來提出 special session，所以會議獲得這些專家的參與。本人覺得這是非常好的方法，值得國內的學者來借鏡。

每次參與會議最大感受通常是又驚又喜，驚的是因為每個領域每年都會有人提出創新性的想法或作品，充分體會到那種“不進則退”的壓力; 喜的是可以跟許多相同領域的專家互相交流, 討論一年來最新的研究成果。這一次去新加坡參加亞太電路與系統研討會，收穫良多。

攜回的資料有 CD-ROM, 各會議文件。