COLOR PATTERN RECOGNITION BY QUATERNION CORRELATION

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ABSTRACT

It is popular to use the conventional correlation for pattern recognition. But when using the conventional correlation, the pattern should be gray-level pattern. In this paper, we will discuss how to use discrete quaternion correlation (DQCR) for the application of color pattern recognition. With the algorithm introduced here, we can detect the objects that have the same shape, color, and brightness with the reference pattern. Besides, we can also detect (a) the objects with the same shape, color, but different brightness, (b) the objects with the same shape, brightness, but different color, and (c) the objects just have the same shape as the reference. Our algorithm can classify the objects into 5 classes due to whether their shape, brightness, and color match those of the reference pattern. Besides, by our algorithm, the difference of brightness and color can also be calculated at the same time.

1. INTRODUCTION

Quaternion [1][2] is the generalization of complex number. Complex number has 2 components: real and imaginary parts. Quaternion, however, has 4 components: real, *i*-, *j*-, and *k*-parts:

$$q = q_r + q_i \cdot i + q_j \cdot j + q_k \cdot k, \qquad (1)$$

and *i*, *j*, *k* obey the rules as below:

$$i^2 = j^2 = k^2 = -1, \quad i \cdot j = k, \quad j \cdot k = i, \quad k \cdot i = j,$$
$$j \cdot i = -k, \quad k \cdot j = -i, \quad i \cdot k = -j. \qquad (2)$$

Quaternion can be used for the color image analysis. In Eq. (1), we can use q_i, q_j, q_k to represent the R, G, B values of a pixel in color image, and set q_r as 0.

Based on the concept of quaternion, the discrete quaternion correlation (DQCR) [3][4] have been developed. It is defined as:

- **Discrete quaternion correlation (DQCR):**

$$b(m,n) = \sum_{\tau=0}^{M-1} \sum_{\eta=0}^{N-1} f(\tau,\eta) \cdot \overline{h(\tau-m,\eta-n)}. \qquad (3)$$

The subtraction operation in Eq. (5) is circular subtraction operation, and '$\overline{}$' means the quaternion conjugation operation:

$$\overline{q} = q_r - q_i \cdot i - q_j \cdot j - q_k \cdot k. \qquad (4)$$

The fast algorithm of DQCR has been derived [4][5], and we can just use six 2-D FFTs to implement the DQCR [5]. With the aid of fast algorithm, using DQCR for the application of image processing becomes more efficient.

Conventional correlation can be used for the application of pattern recognition. Similarly, we can also use the DQCR for the application of color pattern recognition. We will show, when using DQCR, we can detect the objects that have the same shape, size, color, and brightness as the reference pattern. And at the same time, we can also find the objects partially match (such as shape, size, and color) the reference pattern, and calculate the difference of color or brightness. Many works can be done together by using the DQCR for color pattern recognition.

In Sec. 2, we will introduce our algorithm, and illustrate why we can use DQCR for the application of color pattern recognition. In Sec. 3, we will do some experiments to prove the theories in Sec. 2. And in Sec. 4, we make a conclusion.

2. ALGORITHM OF COLOR PATTERN RECOGNITION BY DQCR

When we use the DQCR defined as Eq. (3) for color pattern recognition, we can use $f(m, n)$ to express the reference pattern:

$$f(m,n) = f_R(m,n)i + f_G(m,n)j + f_B(m,n)k, \qquad (5)$$

where $f_R(m, n)$, $f_G(m, n)$, $f_B(m, n)$ represent the R, G, B parts of reference pattern. By similar way, we can use $h(m, n)$ in Eq. (3) to represent the input object. Then, we can use the output $b(m, n)$ to conclude whether the input matches the reference pattern. In the case that $h(m, n)$ is the space shift of the reference pattern:

$$h(m,n) = f(m-m_0, n-n_0), \qquad (6)$$

then, after some calculation, we can prove

$$Max(b_r(m,n)) = b_r(-m_0,-n_0), \qquad (7)$$

where $b_r(m, n)$ means the real part of $b(m, n)$, and

$$b_r(-m_0,-n_0) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} |f(m,n)|^2. \qquad (8)$$

So the real part of the maximum of $b_r(m, n)$ is independent of m_0 and n_0 (i.e., shift invariant). Besides, at the location $(-m_0, -n_0)$ the *i*-part, *j*-part, *k*-part of $b(m, n)$ are all equal to 0:

$$b_i(-m_0,-n_0) = b_j(-m_0,-n_0) = b_k(-m_0,-n_0) = 0. \qquad (9)$$

We can also prove that when $h(m, n)$ does not match the reference pattern, then the maximum of $b_r(m, n)$ will usually be far below the Eq. (8), and Eq. (9) will not be satisfied.

Thus, from the above discussion, if we use DQCR for color pattern recognition, we can follow the process as below:

(1) Calculate the energy of reference pattern $f(m, n)$:

$$E_f = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} |f(m,n)|^2, \qquad (10)$$

then normalize the reference $f(m, n)$ and the input $h(m, n)$ as:

$$f_a(m,n) = f(m,n)/\sqrt{E_f}, \quad h_a(m,n) = h(m,n)/\sqrt{E_f}. \qquad (11)$$

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(2) Then calculate the output of DQCR of $f_a(m, n)$ and $h_a(m, n)$:

$$g_a(m,n)=\sum_{\tau=0}^{M-1}\sum_{\eta=0}^{N-1} f_a(\tau,\eta)\cdot\overline{h_a(\tau-m,\eta-n)}, \quad (12)$$

and do the space reverse operation for $g(m, n)$:

$$g(m,n)=g_a(-m,-n). \quad (13)$$

(3) Then we search all the local peaks of $|g(m, n)|$, and record the locations of local peaks.

(4) Then we choose the thresholds c_1, c_2, d_1, where

$$c_1 < 1 < c_2, \qquad d_1 < 1, \quad (14)$$

and c_1, c_2, d_1 are all near to 1. Then at the locations of local peaks (denoted by (m_s, n_s)) found by the step 3, if both of the 2 requirements are satisfied:

- requirement 1: $c_1 < |g(m_s,n_s)| < c_2$, (15)
- requirement 2: $\rho \ge d_1$, (16)

where

$$\rho=\frac{|g_r(m_s,n_s)|}{|g_r(m_s,n_s)|+|g_i(m_s,n_s)|+|g_j(m_s,n_s)|+|g_k(m_s,n_s)|}, \quad (17)$$

and $g_i(m_s, n_s)$, $g_j(m_s, n_s)$, $g_k(m_s, n_s)$ mean the i-, j-, k-parts of $g(m_s, n_s)$, then we can conclude at the location (m_s, n_s), there is an object that has the **same shape**, **size**, **color**, and **brightness** as the reference pattern.

This is just our proposed algorithm that uses the DQCR for color pattern recognition. We can also use

$$\frac{|g_r(m_s,n_s)|}{|g(m_s,n_s)|}\ge d_1 \quad (18)$$

instead of Eq. (16). Their effects are similar. Eq. (18) has better physical meaning, but in Eq. (16), the value of ρ decays faster with the color difference.

In fact, the requirement of Eq. (15) comes from Eq. (8), and the requirement of Eq. (16) comes from Eq. (9). Their roles are:

- Eq. (15) is used for determining whether the **shape**, **size**, and **brightness** of the objects match those of the reference pattern.
- Eq. (16) is used for determining whether the **color** of the objects matches that of the reference pattern.

Thus, except for the objects that fully match the reference pattern can be detected, we can also find the objects that partially match the reference pattern by just using one of the Eqs. (15) and (16). We illustrate it as below.

Suppose there is an object $h_0(m, n)$ that have the same shape, size, color, but different brightness with the reference $f(m, n)$, i.e.,

$$h_0(m,n)=\sigma\cdot f(m-m_s,n-m_s), \quad (19)$$

where σ is some real constant. If we use the algorithm described as above, then after some calculation, we can prove

$$g(m_s,n_s)=\sigma, \quad (20)$$

where $g(m, n)$ is defined as Eq. (13). In this case, since in Eq. (17), $g_i(m_s, n_s) = g_j(m_s, n_s) = g_k(m_s, n_s) = 0$, so,

$$\rho=1, \quad (21)$$

and Eq. (16) is still satisfied. But the requirement of Eq. (15) would not be satisfied. So the object $h_0(m, n)$ will not be recognized to match the reference pattern $f(m, n)$. But from above discussion, we obtain a useful result:

- If (m_s, n_s) is the local peak of $g(m, n)$ defined as Eq. (13), and the requirement of Eq. (16) is satisfied, but Eq. (15) is not satisfied, then we can conclude at the location (m_s, n_s), there is an object that has the *same shape, size, color*, but *different brightness* with the reference pattern.

Thus, Eq. (16) can find the objects that match the reference pattern in the case that the difference of brightness is ignored.

Similarly, Eq. (15) can find the objects that match the reference pattern when the difference of color is ignored. Suppose $h_0(m, n)$ has the same shape and size as $f(m, n)$, and both of them just has one color and uniform brightness, as the pattern and objects shown in Fig. 1. Then they can be expressed as:

$$f(m,n)=A_f\cdot(i\,R_f+j\,G_f+k\,B_f),$$

$$h(m+m_s,n+n_s)=A_h\cdot(i\,R_h+j\,G_h+k\,B_h) \quad (22)$$

when $(m, n) \in \Omega$,

$$f(m,n)=h(m+m_s,n+n_s)=0 \quad \text{when } (m,n)\notin\Omega, \quad (23)$$

where A_f, A_h are real numbers, (m_s, n_s) is the amount of space shift, Ω is the range of the reference pattern $f(m, n)$, and

$$\sqrt{R_f^2+G_f^2+B_f^2}=\sqrt{R_h^2+G_h^2+B_h^2}=1.$$

Then, after some calculation, we can prove in Eq. (13),

$$g(m_s,n_s)=A_h/A_f\cdot(iR_f+jG_f+kB_f)\overline{(iR_h+jG_h+kB_h)}, \quad (24)$$

so

$$|g(m_s,n_s)|=A_h/A_f. \quad (25)$$

Thus, the value of $|g(m_s, n_s)|$ is near to the ratio of the brightness of the object at (m_s, n_s) to the brightness of reference pattern. So when the object has the same shape with the reference, we can use Eq. (15) to determine whether their brightness are similar.

Besides, it is also possible to use the value of $|g(m_s, n_s)|$ to conclude whether the object has the same shape and size as the reference pattern. Since if the object has different shape or size with the reference pattern, then the value of $|g(m_s, n_s)|$ is usually very small, so except for c_1, c_2 in Eq.(15), we can choose another threshold c_3 for $|g(m_s, n_s)|$, and $0 < c_3 < c_1$. If

$$|g(m_s,n_s)|<c_3, \quad (26)$$

then we can conclude the object at (m_s, n_s) has *different shape* with the reference, or there is no object at (m_s, n_s). If

$$|g(m_s,n_s)|>c_3, \quad \text{but} \quad g_r(m_s,n_s)\notin(c_1,c_2), \quad (27)$$

then we can conclude the object at (m_s, n_s) has *similar* (may not be all the same) *shape* and *size* as those of the reference pattern, but the *difference of brightness* is *large*. And when

$$c_1<|g(m_s,n_s)|<c_2, \quad (28)$$

we can conclude the object not only has *similar shape* and *size* as the reference, but the *difference of brightness* is also *small*.

Besides, when the object has the **same shape** and **size** with the reference pattern (i.e., Eq. (27) or (28) is satisfied), we can also calculate the **brightness ratio** (ratio of the brightness of input object to that of the reference) and the **.color difference** between objects and reference pattern. They are calculated from:

- *brightness ratio*: $|g(m_s,n_s)|$, (29)
- *Color difference*: $\left|\dfrac{g(m_s,n_s)}{|g(m_s,n_s)|}-1\right|$. (30)

When the input object and the reference both have one color and uniform brightness, then the physical meaning of Eq. (29) can be seen clearly from Eq. (25). And from Eqs. (24), (25)

$$\left|\frac{g(m_0,n_0)}{|g(m_0,n_0)|}-1\right| = \left|i(R_f - R_h)+j(G_f - G_h)+k(B_f - B_h)\right|. \quad (31)$$

It just corresponds to the intuitive sense of color difference. This is why we use Eq. (30) to calculate the amount of color difference. In the case that the reference and object have more than one color and non-uniform brightness, then we can view Eqs. (29), (30) as the averages of brightness ratio and color difference between the reference and input object.

In summary, there are many results and information can be obtained from DQCR. We are able to classify the objects into 5 classes by the above algorithm:

(A) If Eqs. (15), (16) are satisfied:

The object has the **same shape, size, color, and brightness** as the reference pattern.

(B) If Eq. (16) is satisfied, but Eq. (15) is not satisfied:

The object has the **same shape, size,** and **color**, but **different brightness** with the reference pattern.

(C) If Eq. (15) is satisfied, but Eq. (16) is not satisfied:

The object has the **same shape, size,** and **similar brightness** with the reference pattern, but the **colors** are **different.**

(D) If the value of $|g(m_s, n_s)|$ is above some threshold c_3, but Eqs. (15), (16) are not satisfied:

The object has **similar** (may not be all the same) **shape** and **size** as the reference pattern, but has **different color** and **brightness** with the reference pattern.

(E) If the value of $|g(m_s, n_s)|$ is below the threshold c_3:

The object has **different shape or size** with the reference, or there is no object at the location (m_s, n_s).

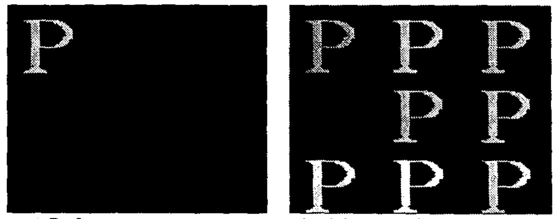
Besides, in cases (B), (D), the value of **brightness ratio** (or the average value) is near to $|g(m_s, n_s)|$, and in cases (C), (D), the amount (or the average amount) of **color difference** between the object and reference can be estimated from Eq. (30).

3. EXPERIMENTS

In this section, we use some experiments to prove the algorithm introduced in Sec. 2. Here we use the letter P with (R, G, B) = (64, 128, 192) as the reference pattern. We plot it in Fig. 1(a). We use an image containing 9 objects as the input (plotted in Fig. 1(b)). The (R, G, B) values of the 9 objects are:

RGB values	column 1	Column 2	column 3
row 1	(64, 128, 192) gray level: 128	(80, 160, 165) gray level: 135	(74, 138, 202) gray level: 138
row 2	(32, 64, 96) gray level: 64	(182, 96, 174) gray level: 150.7	(84, 148, 212) gray level: 148
row 3	(96, 192, 288) gray level: 192	(196, 200, 100) gray level: 165.3	(104, 168, 232) gray level: 168

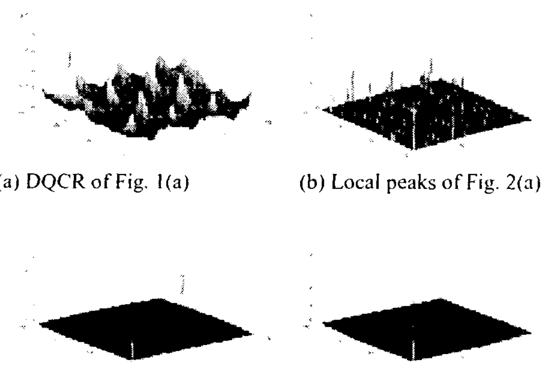
The gray level values are calculated by (R+G+B)/3. The object in (R1, C1) (i.e., row 1 and column 1) is all the same as the reference. The other 2 objects in column 1 also have the same colors as the reference, but the brightness are different. The objects in column 2 have different colors with the reference, and the objects in column 3 increase the RGB values of the reference pattern.



(a) Reference pattern, (b) 9 input objects.

Fig. 1 Reference and input objects of the experiment.

Then, we do the color pattern recognition for Fig. 1(b) to find the objects that are **all the same** as the reference pattern. We follow the process introduced in Sec. 2, and plot the result of step 2 in Fig. 2(a), plot the result of step 3 in Fig. 2(b), and plot the result of step 4 in Fig. 2(e), 2(f), where ρ is defined as Eq. (17). In Eqs. (15), (16), if we choose the thresholds as c_1=0.8, c_2=1.2, d_1=0.8, then there are 3 objects (at (R1, C1), (R1, C3), (R2, C3)) can be detected, as Fig. 2(c). If we choose the thresholds closer to 1, as the case of Fig. 2(d) (c_1=0.9, c_2=1.1, d_1=0.9), then only one object (at (R1, C1)) is detected, and its shape, size, brightness, and color are all the same as those of the reference pattern.



(a) DQCR of Fig. 1(a) (b) Local peaks of Fig. 2(a)

(c) Peaks, $1.2>|g(m_s, n_s)|>0.8$, and $\rho > 0.8$. (d) Peaks, $1.1>|g(m_s, n_s)|>0.9$, and $\rho > 0.99$.

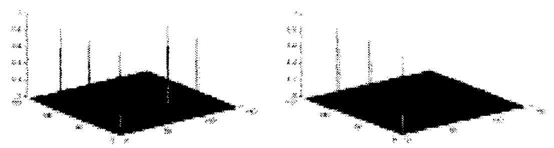
Fig. 2 Searching the objects that fully match the reference pattern by the algorithm in Sec. 2.

Then we just use the constraint of Eq. (15) to detect the objects that has **similar brightness** with the reference pattern (color may be different), and the results are shown in Fig. 3. From the left table, we find the objects detected in Fig. 3 indeed have the gray levels near to the reference. So Eq. (15) is useful for testing whether the objects has similar brightness as the reference.

(a) Peaks, $1.2>|g(m_s, n_s)|>0.8$. (b) Peaks, $1.1>|g(m_s, n_s)|>0.9$.

Fig. 3 Searching the objects with the same shape, size, and similar brightness with the reference pattern.

Then, we do the experiment that just uses Eq. (16) (without using Eq. (15)) to detect the objects with the **same shape, size, and color** with those of the reference pattern (but the brightness may be different). We show the results in Fig. 4, where ρ is defined as Eq. (17).



(a) Peaks, $\rho > 0.9$. (b) Peaks, $\rho > 0.99$.

Fig. 4 Experiment for finding the objects with the same shape, size, and color with the reference pattern.

And the objects with the same color, or almost the same color with the reference are detected successfully.

Then, we use Eq. (26) to detect the objects that have the **same shape and size** as the reference pattern (this experiment is not shown in figure). We choose the threshold c_3 as 0.6. Then, the objects at (R1, C1), (R3, C1), (R1, C2), (R2, C2), (R3, C2), (R1, C3), (R2, C3) and (R3, C3) can all be detected successfully. But the object at (R2, C1) can't be detected successfully. This is because the brightness of the object at (R2, C1) is too low. The difference of brightness will affect the ability of detecting the objects with the same size and shape as the reference by Eq. (26).

In Sec. 2, we have stated when the shape, size of object matches the reference pattern, we can use $|g(m_s, n_s)|$ to estimate the **brightness ratio**. By this method, the brightness ratios of the 9 objects in Fig. 1(b) to the reference pattern we obtain are:

brightness ratios	column 1	column 2	column 3
row 1	1.0000	1.0163	1.0673
row 2	0.5000	1.1253	1.1352
row 3	1.5000	1.2417	1.2762

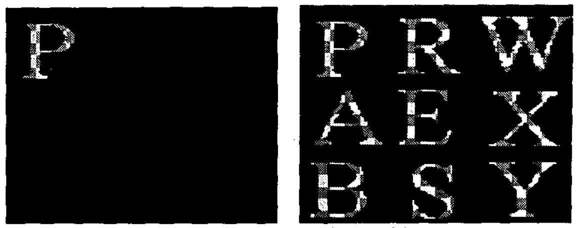
When the object is as bright as the reference, then the value of $|g(m_s, n_s)|$ is also near to 1. Although using $|g(m_s, n_s)|$ to estimate the brightness ratio is not accurate enough, but in usual if the object has higher brightness, the value of $|g(m_s, n_s)|$ is also large.

Then, we do the experiment that uses Eq. (30) to detect the **color difference** between the input objects and the reference. Substituting the locations of the objects and the values of $g(m, n)$ at these locations into Eq. (30), we obtain:

color difference	column 1	column 2	column 3
row 1	0.0000	0.1850	0.0256
row 2	0.0000	0.4719	0.0481
row 3	0.0000	0.6239	0.0860

We find the objects in column 1 have no color difference, the objects in column 3 have small color difference, and the objects in column 2 have larger color difference with the reference pattern. This is the same as the results of direct observation.

At last, we do the experiment that the input objects have **different shapes** with the reference pattern, as shown in Fig. 5



(a) Reference pattern, (b) 9 input objects.

Fig. 5 Experiments of color pattern recognition by DQCR when the objects have different shapes with the reference.

Then, we also follow the process, introduced in Sec. 2, and the values of brightness ratio (i.e., $|g(m_s, n_s)|$) we obtain are as below:

brightness ratios	column 1	column 2	column 3
row 1	1.0000	0.2846	0.2901
row 2	0.3466	0.2341	0.3822
row 3	0.4859	0.4758	0.3147

From the above results, we find, except for the object at (R1, C1), all the other objects have much lower value of $|g(m_s, n_s)|$. It prove our statement in Sec. 2, i.e., when the object has different shape with that of reference pattern, then the value of $|g(m_s, n_s)|$ will be far below 1. And if we choose the threshold c_3 in Eq. (26) properly, all the objects have different shape with the reference can be sifted out successfully.

4. CONCLUSION

In this paper, we have discussed how to use the discrete quaternion correlation (DQCR) for the application of color pattern recognition. When using the conventional correlation, the difference of color can't be distinguished. If we use DQCR for color pattern recognition, then we can detect the objects that have the same shape, size, color and brightness with the reference pattern. Besides, when using DQCR, many works (such as the description in the end of Sec. 2) can be done at the same time.

Quaternion and DQCR are useful tools for color image processing. Many trouble works about color image processing can be solved easily by them.

5. REFERENCES

[1] W. R. Hamilton, "*Elements of Quaternions*", Longmans, Green and Co., London, 1866.

[2] T. A. Ell, *Proceedings of the 32nd Conferences on Decision and Control*, p. 1830-1841, Dec. 1993.

[3] S. J. Sangwine and T. A. Ell, 'Hypercomplex auto- and cross-correlation of color images', p. 319-323, *ICIP*, 1999.

[4] T. A. Ell and S. J. Sangwine, *EUSIPCO2000*, p. 151-154.

[5] S. C. Pei, J. J. Ding, and J. Chang "Efficient implementation of quaternion Fourier transform, convolution, and correlation by 2-D FFT", submitted to *IEEE Trans. Signal Processing*.