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SIGNAL EXTRAPOLATION FROM HARTLEY TRANSFORM MAGNITUDES

Indexing terms: Signal processing, Transforms, Mathematical techniques

A novel approach to the problem of signal extrapolation from its Hartley transform magnitudes is presented. Using a newly defined function, it is proved that using only one known sample and the associated Hartley transform magnitudes a finite extended signal can be completely reconstructed. An algorithm for signal reconstruction from short-time Hartley transform (STHT) magnitudes with minimal window overlap can consequently be derived.

Introduction: The problem of signal reconstruction from partial information in the Fourier domain has been widely considered.[1–3] Most works are only concerned with the reconstruction of signal from magnitude information or phase information. A signal can be reconstructed from its partial Fourier domain information with the aid of some *a priori* information about the signal. In this case, the task changes to signal extrapolation from partial Fourier domain information.[4] The development of signal extrapolation from partial information needs an investigation of how many samples of a signal are required to uniquely estimate other unknown samples of the signal. The results obtained play a kernel role in deriving conditions for which the partial information of short time Fourier transform (STFT) is a unique signal representation.

Bracewell[5] has introduced the Hartley transform, which uses the real variable $\mathrm{cas}(2\pi fn)$ as the transform kernel and is intuitively simpler and faster than the Fourier transform. Signal reconstruction from partial information in the Hartley domain has been discussed in Reference 6. Signal extrapolation from partial Hartley domain information is the theme of this letter. A novel algorithm for extrapolating signals from Hartley transform magnitude information with one known sample is derived. This result can be applied to signal reconstruction from short time Hartley transform (STHT) magnitudes.[7]

Signal extrapolation from Hartley transform magnitudes: The theorem of signal extrapolation from Fourier transform magnitudes is first quoted from Reference 4.

Theorem 1: For $N > 0$, let $x(n)$ be a sequence that is zero outside the interval $[0, N]$ and assume that $x(0) \neq 0$. Then the Fourier transform magnitude $|F(\omega)|$ and the Q samples of $x(n)$ in the interval $[0, Q)$ will uniquely specify the entire sequence $x(n)$ if and only if $Q \geq \lceil M/2 \rceil$ (where $M = N + 1$ and $\lceil \alpha \rceil$ is the smallest integer greater or equal to α).

Intuitively, from the relations between Hartley transform and Fourier transform, theorem 1 should also be valid for Hartley transform based signal extrapolation. However, from theorem 2 stated below, we prove that $Q = 1$ is satisfactory for the Hartley transform based approach.

Theorem 2: Under the same conditions and assumptions in theorem 1, the Hartley transform magnitudes $|H(\omega)|$ and the Q samples of $x(n)$ in the interval $[0, Q)$ will uniquely specify the entire sequence $x(n)$, if and only if $Q \geq 1$.

Proof: Throughout this proof, the samples of $x(n)$ for $0 \leq n < Q$ will be referred to as the initial Q samples of $x(n)$. We first show that the unknown samples of $x(n)$ are uniquely specified when $Q = 1$. Clearly, if uniqueness holds for $Q = 1$, uniqueness will also hold for $Q > 1$. From the Hartley transform magnitude $|H(\omega)|$, the following function can be constructed:

$$D(n) = \mathrm{IHT}\,\{H^2(\omega) - H^2(-\omega)\} \tag{1}$$

$$= x(n)*x(n) - x(-n)*x(-n) \tag{2}$$

$$= \begin{cases} 0 & n = 0 \\ \sum_{m=0}^{n} x(m)x(n-m) & 1 \leq n \leq N \\ \sum_{m=n-N}^{N} x(m)x(n-m) & N+1 \leq n \leq 2N \end{cases} \tag{3}$$

where * denotes the operation of digital convolution and $x(-n)$ and $H(-\omega)$ are obtained from the time inversing of $x(n)$ and $H(\omega)$, respectively. It is obvious that $D(n)$ is an odd function of n and from eqn. 3, N equations with N unknowns, $x(1)$, $x(2), \ldots, x(N)$ can be obtained. In matrix form, these equations can be written as

$$\begin{bmatrix} 2x(0) & & & & \\ x(1) & 2x(0) & & & \\ x(2) & x(1) & 2x(0) & & \\ \vdots & & & \ddots & \\ x(N-1) & x(N-2) & & \cdots & 2x(0) \end{bmatrix} \begin{bmatrix} x(1) \\ x(2) \\ x(3) \\ \vdots \\ x(N) \end{bmatrix} = \begin{bmatrix} D(1) \\ D(2) \\ D(3) \\ \vdots \\ D(N) \end{bmatrix} \tag{4}$$

The matrix of the left hand side of eqn. 4 is a lower triangular matrix with all nonzero diagonal elements $2x(0)$. It follows that a unique solution exists for each $x(n)$, $n = 1, 2, \ldots, N$, as long as $D(n)$ are known. The general form of $x(n)$ can be represented as

$$x(n) = \left[D(n) - \sum_{m=1}^{n-1} x(m)x(n-m) \right] \Big/ 2x(0) \tag{5}$$

For $Q = 0$, i.e., no known samples, the following lemma states that the Hartley transform based extrapolation will fail:

Lemma 1: For $N > 0$, let $x(n)$ and $y(n)$ be two finite extended sequences that are zero outside the interval $[0, N]$. If the magnitudes of the Hartley transforms of $x(n)$ and $y(n)$ are equal, then $y(n) = \pm x(n)$.

Lemma 1 coincides with the results obtained from the consideration of the 'Hartley phase problem' as stated in Reference 6 and its proof is trivial. Lemma 1 and previous discussions complete the proof of theorem 2.

Discussion and conclusion: The function $D(n)$ is the only information derived from the Hartley transform magnitudes $|H(\omega)|$. Since $x(n)$ is $N + 1$ points long, $D(n)$ is $4N + 1$ points long and is an odd function of n. The entire sequence $D(n)$ can be obtained without aliasing by using a $4N + 1$ point inverse DHT of $\{H^2(\omega) - H^2(-\omega)\}$. Using a $4N$ point inverse DHT, the sample $D(2N)$ is aliased with the sample $D(-2N)$. Since $D(2N)$ is not used in the extrapolation procedure, it follows that $D(n)$ can be obtained through a $4N$ point inverse DHT of the $4N$ uniformly spaced samples of $\{H^2(\omega) - H^2(-\omega)\}$. Thus, the fast DHT algorithm developed in Reference 5 can be directly utilised if N is a power of two. Furthermore, since $\{H^2(\omega) - H^2(-\omega)\}$ is an odd function, it can be easily seen that $4N$ uniformly spaced samples of $\{H^2(\omega) - H^2(-\omega)\}$ over the frequency interval $[0, 2\pi]$ are equivalent to $2N + 1$ samples in the interval $[0, \pi]$. It can also be shown that $D(n)$ can be obtained even if the $2N + 1$ samples of $\{H^2(\omega) - H^2(-\omega)\}$ in $[0, \pi]$ are not uniformly sampled.

Based on theorem 2, a novel signal extrapolating algorithm using Hartley transform magnitudes and only one known sample can now be derived. Fig. 1 shows the block diagram of the proposed algorithm. The proposed algorithm has been implemented by using the C programming language to verify its correctness.

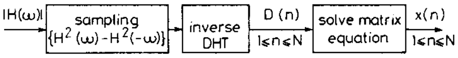

Fig. 1 *Extrapolation procedure*

The proposed algorithm has also been successfully applied to the signal representation from short time Hartley transform magnitudes. A novel algorithm, based on theorem 2, for signal reconstruction from STHT magnitudes with minimal window overlap has been developed.[8]

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ANALYSIS OF ARBITRARILY SHAPED COAX-FED MICROSTRIP ANTENNAS WITH THICK SUBSTRATES

Indexing terms: Antennas, Microstrip

Arbitrarily shaped, coax-fed microstrip patch antennas with thick substrates are studied using a mixed-potential integral equation approach. This incorporates a triangle-element model of the patch and a rigorous treatment of the probe-to-patch junction. Computed input impedance data are shown to agree well with measured results.

Introduction: Considerable progress has been made in the numerical modelling of coax-fed microstrip patch antennas, both in the spectral and spatial domains.[1,2] Spectral domain methods, which rely on Fourier-transformable entire-domain expansion functions, are limited to antennas of a few simple shapes. Space domain techniques using basis functions defined on rectangular or triangular subdomains are applicable to a much wider class of microstrip geometries. The triangle-element model employing the basis functions introduced by Rao *et al.*[3] appears to be particularly attractive in this respect. Pichon *et al.*[4] have used this approach in conjunction with the mixed-potential integral equation (MPIE) of Mosig and Gardiol[5] in the method of moments (MOM) analysis of a coax-fed triangular patch microstrip antenna. To model the probe-to-patch junction, they introduced a simple attachment mode, which enforces the current continuity condition only in the average and does not attempt to model the singular behaviour of the patch current near the feed point. The coaxial probe current was assumed constant, which is a good approximation only for electrically thin substrates. In a recent study, Hall and Mosig[2] eliminated this thin-substrate restriction and used the magnetic current frill to model the coax aperture. However, their analysis employed a rectangular cell model of the microstrip patch, which is not suited for arbitrarily shaped antennas.

We propose an approach based on a recently developed MPIE.[6] It incorporates the triangle-element patch model of Rao *et al.*,[3] the coax probe model of Hall and Mosig,[2] and the rigorous junction treatment introduced in a different context by Hwu *et al.*[7] The Hwu[7] junction model accurately predicts the diverging behaviour of the patch current near the feed point and is applicable even for edge or corner fed microstrip antennas.

Formulation: The MPIE for the surface current distribution $\boldsymbol{J}$ on the patch S_P and the coax probe S_C has the form

$$\hat{\boldsymbol{n}} \times \left[j\omega \int_S \bar{\bar{G}}^A(\boldsymbol{r}|\boldsymbol{r}') \cdot \boldsymbol{J}(\boldsymbol{r}')\, dS' + \nabla \int_S G^\Phi(\boldsymbol{r}|\boldsymbol{r}') q(\boldsymbol{r}')\, dS' \right] = \hat{\boldsymbol{n}} \times \boldsymbol{E}^i(\boldsymbol{r}) \qquad \boldsymbol{r} \in S \quad (1)$$

where $S = S_P \cup S_C$, $\hat{\boldsymbol{n}}$ is a unit vector normal to S, $\boldsymbol{E}^i$ is the incident field caused by the magnetic current frill radiating in the grounded substrate environment and q is the charge density related to $\boldsymbol{J}$ by the continuity equation. The dyadic kernel $\bar{\bar{G}}^A$ can be expressed as

$$\bar{\bar{G}}^A = (\hat{x}\hat{x} + \hat{y}\hat{y})G^A_{xx} + \hat{x}\hat{z}G^A_{xz} + \hat{y}\hat{z}G^A_{yz} + \hat{z}\hat{x}G^A_{zx} + \hat{z}\hat{y}G^A_{zy} + \hat{z}\hat{z}G^A_{zz} \quad (2)$$

where it is assumed that the dielectric/air interface is normal to the unit vector $\hat{z}$. The elements of this dyadic, as well as G^Φ, have been derived by the authors.[6] This formulation requires a single scalar potential kernel G^Φ for both the horizontal and vertical components of $\boldsymbol{J}$. In contrast to Hall and Mosig's[2] approach no additional point charges at the probe-to-patch junction are required. This advantage is partially offset by the appearance of two additional entries in eqn. 2.

The MPIE in eqn. 1 is solved using the well-established MOM procedure[3] utilising a triangle-element approximation of the arbitrarily shaped microstrip patch and the associated vector basis functions to represent $\boldsymbol{J}$. As in Hall and Mosig,[2] the surface current on the probe and the coax aperture field are assumed to be azimuthally symmetric. The latter is taken to be that of a TEM coaxial transmission line mode with known voltage V_i. The axial probe current is approximated in terms of piecewise linear, subsectional expansion functions. A special attachment mode, originally introduced by Hwu *et al.*,[7] is used to model the current behaviour near the probe-to-patch junction. The resulting integral equation is then reduced to an algebraic system by a testing procedure.[3,7] Once this system is solved for the current expansion coefficients, the antenna input impedance is found as V_i/I_i, where I_i is the current at the base of the coax probe.

Results: In Figs. 1 and 2, we compare computed and measured input impedance data for triangular and rectangular patch antennas, respectively, on a substrate with $\varepsilon_r = 2{\cdot}484$ and $\tan\delta = 6 \times 10^{-4}$, driven by a coaxial cable with the inner and outer radii of 0·635 and 2·095 mm, respectively. The former antenna was analysed by Pichon *et al.*,[4] and the latter by Hall and Mosig,[8] using a rectangular mesh model. In the numerical analysis, the triangular and rectangular antennas were modelled by 144 and 160 triangular elements, respectively. For the triangular antenna, which had a moderately thick substrate, only one basis function (in addition to the attachment mode) was placed on the coax probe. The rectangular antenna, which had a thicker substrate, required two