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IMPROVED DESIGN OF MINIMUM-PHASE FIR DIGITAL FILTERS BY CEPSTRUM AND FAST HARTLEY TRANSFORM

Indexing terms: Filters, Transforms

An equiripple minimum-phase FIR filter is designed using the cepstrum and the fast Hartley transform (FHT). This fast procedure is performed entirely in the real domain and requires only two short-length FHT computations. This method avoids complicated phase wrapping and greatly reduces the aliasing error.

Introduction: Minimum phase FIR filters have certain practical advantages, e.g., minimum group delay, reduced order for given gain specification and lower coefficient sensitivity,[1] when compared with the linear phase FIR filters. They are highly attractive in some applications, such as CTD transversal filters and communication channel filters.

Herrman and Schuessler[2] have developed a method for transforming equiripple linear-phase designs into equiripple minimum-phase designs with half the order and with a magnitude response equal to the square root of the prototype. A numerial root-finding procedure is required for this method. Main and Nainer[3] proposed a design procedure to overcome this difficulty by using homomorphic deconvolution. However, a complicated phase wrapping algorithm is required in the computation. Pei and Lu[4] introduced a differential cepstrum design avoiding the need for phase wrapping. However, differential cepstrum has the aliasing problem[5] and requires three larger-length FFT's in order to implement it. Reddy[6] has recently used the cepstrum computed from the spectral log magnitude function to overcome the aliasing problem. The procedure requires only two FFT computations and avoids the processing of phase. In that letter, the fast Hartley transform (FHT) is introduced to design equiripple minimum phase FIR filters through cepstrum. This fast procedure only needs two real FHT computations, and avoids the complicated phase wrapping and polynomial root-finding algorithms. The aliasing error is also greatly reduced.

Minimum phase filter design: According to Herrman and Schuessler's method,[2] the relationship between linear phase $H(z)$, second-order zeros $H_1(z)$ and minimum phase $H_2(z)$ is as follows:

$$\begin{aligned} H_1(z) &= H(z) + \delta_2 z^{-(N-1)/2} \\ H_1(z) &= z^{-(N-1)/2} H_2(z) H_2(z^{-1}) \\ H_2(e^{j\omega}) &= \sqrt{[H_1(e^{j\omega})]} \end{aligned} \tag{1}$$

where N and δ_2 are the length and the stopband ripple of the linear phase filter $H(z)$.

Let $\hat{h}_1(n)$ and $\hat{h}_2(n)$ be the complex cepstrum of the filters $H_1(z)$ and $H_2(z)$, respectively. The relation between $\hat{h}_2(n)$ and $\hat{h}_1(n)$ is[3]

$$\hat{h}_2(n) = \tfrac{1}{2}[\hat{h}_1(n) + \hat{h}_1(-n)] = c_1(n) \qquad n > 0 \tag{2}$$

$c_1(n)$ corresponds to the cepstrum of $H_1(z)$ and is defined as

$$\ln |H_1(e^{j\omega})| = \sum_{n=-\infty}^{\infty} c_1(n) e^{-j\omega n} \tag{3}$$

Once $\hat{h}_2(n)$ is computed using eqns. 2 and 3, the minimum phase filter impulse response $h_2(n)$ can be obtained from its complex cepstrum $\hat{h}_2(n)$.[3]

$$h_2(n) = \hat{h}_2(n) h_2(0) + \sum_{k=0}^{n-1} \frac{k}{n} \hat{h}_2(n) h_2(n-k) \qquad 0 \le n \le (N-1)/2 \tag{4}$$

Then the equiripple minimum-phase filters can be designed using only the magnitude function of the linear phase filter, thereby avoiding the processing of the phase. The complex cepstrum $\hat{h}_2(n)$ decays at least as fast as $1/n$, and its corresponding differential cepstrum[4] $\hat{h}_{d2}(n+1) = n\hat{h}_2(n)$ does not decay with an envelope of $1/n$. This approach can greatly reduce the aliasing error by using the cepstrum instead of the differential cepstrum.

The fast Hartley transform[7–9] has recently been considered as an interesting alternative to the FFT, we can use the real FHT here to calculate the log magnitude response and the cepstrum of the filter $H_1(z)$ very efficiently.

The discrete Fourier and Hartley transform of $x(n)$ are defined as

$$\text{DFT}: X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N} \tag{5a}$$

$$\text{DHT}: H_x(k) = \sum_{n=0}^{N-1} x(n) \operatorname{cas}\left(\frac{2nk}{N}\pi\right) \tag{5b}$$

where $\operatorname{cas}(x) = \sin(x) + \cos(x)$. The inverse relation is

$$\text{IDFT}: x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi nk/N} \tag{6a}$$

$$\text{IDHT}: x(n) = \frac{1}{N} \sum_{k=0}^{N-1} H_x(k) \operatorname{cas}\left(\frac{2nk}{N}\pi\right) \tag{6b}$$

The relation of the DHT and DFT of a real sequence $x(n)$ is

$$\begin{aligned} \text{Even}\,[H_x(k)] &= [H_x(k) + H_x(-k)]/2 \\ &= \text{Re}\,[X(k)] \end{aligned} \tag{7a}$$

$$\begin{aligned} \text{Odd}\,[H_x(k)] &= [H_x(k) - H_x(-k)]/2 \\ &= -\text{Im}\,[X(k)] \end{aligned} \tag{7b}$$

The power spectrum of $x(n)$ can be efficiently calculated directly form the DHT $H_x(k)$ as[7]

$$|X(k)|^2 = [H_x^2(k) + H_x^2(-k)]/2 \tag{8}$$

Also by eqn. 3, the cepstrum $c_1(n)$ can be computed by the IDFT of In $|H_1(k)|$

$$c_1(n) = \frac{1}{N} \sum_{k=0}^{N-1} \text{In } |H_1(k)| e^{j2\pi nk/N} \tag{9}$$

Since the output of the IDFT (the cepstrum) $c_1(n)$ is guaranteed to be real, then the IDFT equals the IDHT, we can obtain $c_1(n)$ from the IDHT of In $|H_1(k)|$ as

$$c_1(n) = \frac{1}{N} \sum_{k=0}^{N-1} \text{In } |H_1(k)| \text{ cas} \left(\frac{2nk}{N} \pi\right) \tag{10}$$

In summary, the steps necessary to obtain $h_2(n)$ from $h_1(n)$ are

(*a*) Prepare $h_1(n)$ for length N odd (eqn. 1).

(*b*) Take forward FHT of $h_1(n)$ and compute the power spectrum of $h_1(n)$ (eqn. 8).

(*c*) Compute the log magnitude response In $|H_1(k)|$.

(*d*) Calculate $c_1(n)$ by taking the IDHT of In $|H_1(k)|$ (eqn. 10).

(*e*) Obtain a new sequence $t(n)$ from the recursive formula in eqn. 4.

$$t = 0 \qquad n < 0$$

$$t = h_1(n) \qquad n = 0$$

$$t = c_1(n)t(0) + \sum_{k=0}^{n-1} \frac{k}{n} c_1(n)t(n-k) \qquad 0 \le n \le \frac{N-1}{2} \tag{11}$$

(*f*) Get the scaling factor R for unity in the passband.

$$R = \frac{\sqrt{\left|\sum_{n=0}^{N-1} h_1(n)\right|}}{\sum_{n=0}^{(N-1)/2} t(n)} \tag{12}$$

(*g*) Scale the impulse response $t(n)$

$$h_2(n) = Rt(n) \qquad 0 \le n \le \frac{N-1}{2}$$

The above steps are performed entirely in the real domain and require only two FHT computations. The proposed algorithm has been tested for designing several mininum phase FIR filters. Two short length 256-point real FHTs are sufficient to give satisfactory results equivalent to Reddy's two FFT computations.[6] The differential cepstrum method needs three long 1024-point complex FFTs.[4]

Conclusions: The cepstrum and fast Hartley transform have been used for designing equiripple minimum-phase FIR filters. This fast procedure requires only two short real FHT computations and avoids the complicated phase wrapping and polynomial root-finding algorithms.

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HIGH EFFIENCY SMALL-SIZE PLANAR HYPERBOLICAL LENSES

Indexing terms: Integrated optics, Integrated lenses, Focussing

Planar hyperbolical lenses with an average efficiency of 85% for coupling a 40 μm wide beam into a 4 μm wide waveguide are realised. The total length required for the beam compression is 600 μm, with a lateral index contrast of 0·01. Good agreement is found between the BPM predicted and the measured lens efficiencies.

Introduction: Planar lenses are important components for coupling wide beams into narrow waveguides. Miki *et al.*[1] reported geodesical lenses with a focal length of 17 mm and 75% efficiency. Valette *et al.*[2] obtained 60–70% efficiency with Fresnel lenses having a focal length of 8·5 mm. In this letter hyperbolical lenses will be presented combining a 300 μm focal length with efficiencies up to 92% which is the highest for planar lenses so far reported.

Basic principle: Fig. 1 shows the geometry and the ray propagation through a hyperbolical lens. The lens design is based on Fermat's principle which states that in the focal point the optical length $n_1 l_1 + n_2 l_2$ is constant for all ray paths, where l_1 and l_2 are the path lengths in the media with effective refractive indices n_1 and n_2, respectively. The lens shape for aberration-free focusing of a plane wave is thus found to be[3]

$$x^2 = 2fz\left[1 - \frac{n_1}{n_2}\right] + z^2\left[\left(\frac{n_1}{n_2}\right)^2 - 1\right] \tag{1}$$

Lens design and analysis: To determine the optimal lens parameters for coupling a Gaussian beam of width $2w_0$ into a waveguide of width, w_g, we proceeded as follows: The equiva-

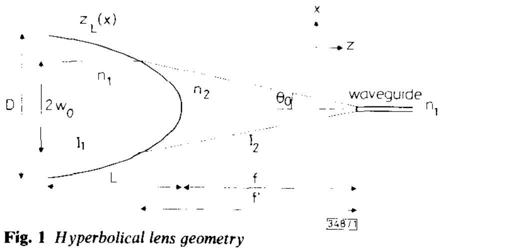

Fig. 1 *Hyperbolical lens geometry*