

given targets of TP_{max} . It should be noted that our comparisons are thus different from most previous comparisons in which networks with the same replication and expansion factors were compared (e.g. comparing a K -RDN to a K -RDN [3]), and in which it was always implied that the crossbar switch is more complex.

For all networks under consideration, Fig. 2 shows C_{min} as a function of the target TP_{max} for $N = 128$. We notice that expansion costs the least C_{min} for a wide range of target TP_{max} . We also notice that for target values of TP_{max} greater than 0.56 (0.61), a CS requires lower C_{min} than both RDN and DDN (EDN). When the target TP_{max} is less than 0.52, replication requires lower C_{min} compared to dilation. Fig. 3 shows C_{min} against $\log_2 N$ for target $TP_{max} = 0.61$. We notice that for $N \leq 128$, a CS requires the lowest C_{min} , whereas for $N > 128$, an EDN requires the lowest C_{min} . Finally, the RDN and the DDN keep interchanging their positions while being close to each other.

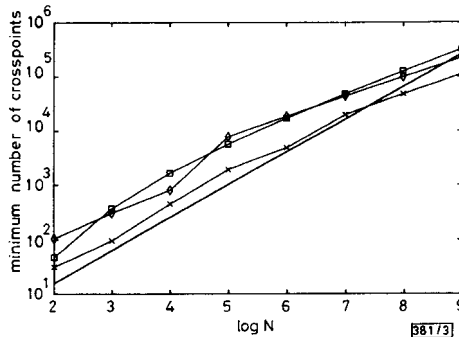


Fig. 3 C_{min} against n at target $TP_{max} = 0.61$

□ RDN
◇ DDN
× EDN
— CS

Summary: Based on extensive results obtained by the approach described in this Letter, the findings, applicable to unbuffered interconnection networks for which one packet only can be accepted at any output in each cycle, are summarised as follows:

- (i) Among delta-based networks, the EDN has the lowest number of crosspoints, almost always.
- (ii) For target values of the maximum throughput close to the maximum achievable throughput, the CS requires the lowest number of crosspoints for small to moderate network sizes.
- (iii) For a given network size, replication outperforms dilation up to a certain target maximum throughput, above which they seem to have comparable complexities.

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R. Y. Awdeh and H. T. Moutah (Department of Electrical Engineering, Queen's University at Kingston, Ontario K7L 3N6, Canada)

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Mapping iterative networks to parallel lookahead circuits

J.-C. Shyur and T.-M. Parng

Indexing term: Finite state machines

Finite state machines have sequential iterative network implementations. The authors show that by the proposed algorithm and Boolean matrix operations, nonlinear parts of recurrence sequences can be eliminated. Linear mapping is thus obtained, which by use of the prefix technique, results in parallel lookahead circuits.

Introduction: Finite state machines can be implemented in iterative networks [1]. However, with parallel input and output, there are signals propagating along the networks, resulting in the worst case $\Omega(N)$ time. However, if the network is associative, by using the prefix technique [2], a product circuit can be achieved with depth exactly $\lceil \log_2 N \rceil$ and size bounded by $4N$. For simplicity, such circuits are called parallel lookahead circuits [3,4].

In this Letter, we present a mapping from iterative networks to parallel lookahead circuits. We first show how linear recurrence sequences can be mapped to parallel lookahead circuits, and then give an algorithm to transform the nonlinear parts in recurrence sequences into linear parts. The resultant recurrence sequences are shown to be associative in the domain of the propagating signals, and parallel lookahead circuits can be realised.

Linear recurrence sequence: Let the j th input and propagating signals of an iterative array be vectors A^i and Y^i ; the recurrence relation of Y^i in the form of sum-of-products is

$$y_j^i = \sum \left(\prod a_k^i \prod y_l^{i-1} \right) + \sum \prod a_p^i \quad (1)$$

for $j = 1, 2, \dots, n$, where n is the dimension of Y^i , Π and Σ are the massive AND and OR operations.

A special case of eqn. 1 is that each Πy^{i-1} contains only one component from Y^{i-1} ; eqn. 1 can thus be written in linear form:

$$y_j^i = \sum_k e_{jk}(A^i) y_k^{i-1} + c_j(A^i) \quad (2)$$

where $e_{jk}(A^i)$ and $c_j(A^i)$ are Boolean expressions of variables from A^i . It then forms a linear mapping by augmenting Y^i by a '1':

$$\begin{aligned} (Y^i)^t &= ((Y^{i-1})^t \ 1)^t \\ &= E^t(A^i)(Y^{i-1})^t \end{aligned} \quad (3)$$

where $E(A^i)$ is the augmented matrix of $(e_{jk}(A^i))$ by $(c_j(A^i))$:

$$E(A^i) = \begin{pmatrix} E(A^i) & C(A^i) \\ 0 & 1 \end{pmatrix} \quad (4)$$

We note that Boolean matrix multiplication is implied in eqn. 3 which can be further expanded as

$$(Y^i)^t = E^t(A^i) E^{t-1}(A^{i-1}) \dots E^1(A^1) (Y^0)^t \quad (5)$$

where Y^0 represents the initial values propagated to the first cell of the iterative network. Without proof that Boolean matrix multiplication is associative, eqn. 5 becomes

$$(Y^i)^t = (E^t(A^i) E^{t-1}(A^{i-1}) \dots E^1(A^1)) (Y^0)^t \quad (6)$$

where the term $E^t(A^i) E^{t-1}(A^{i-1}) \dots E^1(A^1)$ can be parallel-computed by a product circuit, forming the following prefix problem.

Prefix problem formulation: It is revealed by eqn. 6 that the prefix technique can be applied to solve the prefix problem [2] defined as follows:

Let \circ be an associative operation on a domain D . The prefix problem is to compute, for given $x_1, \dots, x_n \in D$, each of the products $x_1 \circ x_1 \circ \dots \circ x_k$, $1 \leq k \leq n$.

Define the Boolean matrix multiplication as operation \circ and $E(A^i)$ as x_n , $i = 1, \dots, N$; we have a prefix problem that computes $E^t(A^i) E^{t-1}(A^{i-1}) \dots E^1(A^1)$, $1 \leq k \leq N$.

Transform algorithm: Finally we are left with the problem of eliminating the nonlinear parts in eqn. 1 such that the recurrence

sequences can be described by eqn. 2. The algorithm introduces new variables to Y^i to substitute for inversion or product terms, and decomposes them by recurrence relations of the substituted terms.

Algorithm 1:

(a) Restate eqn. 1 as

$$y_j^i = \sum \prod a_k^i \overbrace{\prod y_l^{i-1}}^{\text{nonlinear}} + c_j(A^i) \quad (7)$$

for $j = 1, \dots, n$

for $j = 1, \dots, n$

(b) While there is an inversion signal \bar{y}_l^{i-1} or a product term Πy_l^{i-1} composed of more than one component from Y^{i-1} ,

(i) Add a new component $y_{n+1}^{i-1} = \bar{y}_l^{i-1}$ or Πy_l^{i-1} .

(ii) Replace \bar{y}_l^{i-1} or Πy_l^{i-1} by y_{n+1}^{i-1} .

(iii) Decompose y_{n+1}^i by substituting recurrence relations of \bar{y}_l^i or every y_l^i , such that y_{n+1}^i can be expressed by a recurrence relation of components of Y^{i-1} :

$$y_{n+1}^i = \sum \prod a_k^i \prod y_l^{i-1} + c_{n+1}(A^i) \quad (8)$$

(c) Describe components of Y^i as a linear form like in eqn. 2.

Examples: We use two examples to illustrate the mapping: the parity checker and the comparator. Let p^i be the parity of a^1 to a^i , that is $p^i = 1$ if there are odd numbers of 1s in $\{a^1, \dots, a^i\}$. We have the recurrence relation with $p^0 = 0$:

$$p^i = \bar{a}^i p^{i-1} + a^i \bar{p}^{i-1} \quad (9)$$

where the inversion \bar{p}^i can be expanded by algorithm 1:

$$\bar{p}^i = \bar{a}^i p^{i-1} + a^i \bar{p}^{i-1} = a^i p^{i-1} + \bar{a}^i \bar{p}^{i-1} \quad (10)$$

Finally the linear recurrence sequence can be obtained as follows:

$$\begin{pmatrix} p^i \\ \bar{p}^i \end{pmatrix} = \begin{pmatrix} \bar{a}^i & a^i & 0 \\ a^i & \bar{a}^i & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p^{i-1} \\ \bar{p}^{i-1} \\ 1 \end{pmatrix} \quad (11)$$

For comparators, we use g^i and l^i to denote that the number $x^N \dots x^i$ is greater/less than the number $y^N \dots y^i$, with initially $g^N = l^N = 0$. The recurrence relation is

$$\begin{pmatrix} g^i \\ l^i \end{pmatrix} = \begin{pmatrix} g^{i+1} + \bar{g}^{i+1} \bar{l}^{i+1} x^i \bar{y}^i \\ l^{i+1} + \bar{g}^{i+1} \bar{l}^{i+1} \bar{x}^i y^i \end{pmatrix} \quad (12)$$

where the nonlinear term $\bar{g}^i \bar{l}^i$ can be substituted by e^i and expanded:

$$e^i = \bar{g}^i \bar{l}^i = e^{i+1} (\bar{x}^i \bar{y}^i + x^i y^i) \quad (13)$$

resulting in the linear recurrence sequence

$$\begin{pmatrix} g^i \\ l^i \\ e^i \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & x^i \bar{y}^i & 0 \\ 0 & 1 & \bar{x}^i y^i & 0 \\ 0 & 0 & \bar{x}^i \bar{y}^i + x^i y^i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} g^{i+1} \\ l^{i+1} \\ e^{i+1} \\ 1 \end{pmatrix} \quad (14)$$

Summary: We have presented an approach to mapping iterative networks to parallel lookahead circuits. By using the proposed algorithm, nonlinear parts in the recurrence sequences can be eliminated and substituted by linear parts. The resultant linear recurrence sequences are then shown to be expressible by augmented Boolean matrices and matrix multiplications. As the multiplications are associative operations, the linear recurrence sequences can be implemented by parallel lookahead circuits by using the prefix technique. We conclude that all iterative networks can be transformed to parallel lookahead circuits.

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Diffraction coefficients at edges in artificially soft and hard surfaces

S. Maci, R. Tiberio and A. Toccafondi

Indexing terms: Electromagnetic wave diffraction, Geometrical theory of diffraction

In this Letter, both uniform GTD and incremental diffraction coefficients at edge discontinuities in artificially soft and hard surfaces are obtained from the exact solution of the relevant canonical wedge configuration. Numerical results are presented for a dipole on the axis of an artificially soft disk.

Introduction: Accurate control of the radiating characteristics of high-performance antenna systems is important in electromagnetic engineering applications. To this end, undesired scattering mechanisms may be reduced by conveniently shaping or corrugating the scattering surfaces. Recently, a systematic method for characterising corrugated surfaces for different applications has been presented [1]. There, it is suggested that suitable combinations of corrugations and dielectric loadings may usefully be employed to devise surfaces that exhibit artificially soft and hard boundary conditions (BCs). It has been shown that this technology may provide effective tools for controlling the sidelobe level and crosspolar components of radiation patterns. Within this framework, an accurate description of diffraction mechanisms at edge discontinuities is relevant for an exhaustive examination of the polarisation effects.

In this Letter, uniform, dyadic GTD diffraction coefficients are obtained, that provide basic tools for treating the scattering at actual edge configurations of practical interest. Furthermore, incremental diffraction coefficients are presented which provide a suitable correction near to and at caustics. These are obtained according to the incremental theory of diffraction (ITD) [2, 3]. Both GTD and ITD diffraction coefficients are derived from the exact solution of a canonical wedge with artificially soft and hard BCs on one face and a perfectly electric conducting (PEC) BC on the other face. This procedure yields a representation of the field which is uniformly valid at any observation aspect, including caustics of the corresponding ray-field description.

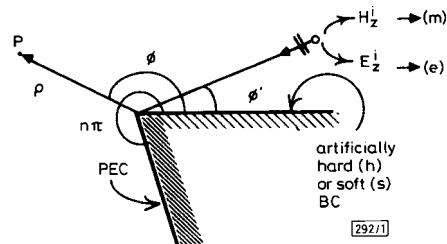


Fig. 1 Geometry at canonical wedge configuration