signal bandwidth of 16·5 GHz was achieved by reducing the laser parasitics and enhancing the resonance frequency. This result indicates that the lasers with this structure are suitable for a light source of 10 Gbit/s optical communication systems.

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UNDETECTED ERROR PROBABILITIES OF CODES FOR SINGLE-ERROR CORRECTION AND ERROR DETECTION

Indexing terms: Information theory, Codes and coding, Error-correction codes, Error detection codes

It is shown that the probabilities of undetected errors for maximum-length codes are upper bounded by $(n+1).2^{-(n-k)}$, if the codes are used for correcting every single error and detecting other errors over a binary symmetric channel of which the transition probability is less than 1/2. Moreover, it is shown that binary linear codes with poor distance property do not satisfy the aforementioned bound.

Introduction: In pure ARQ systems, linear codes are used solely for detecting errors. Suppose that we apply codes to a binary symmetric channel (BSC) with transition probability ε. It has been proven[1] that for each ε with $0 \le \varepsilon \le 1/2$, there exists an (n, k) binary linear code whose probability of undetected errors (PUDE) is upper bounded by $2^{-(n-k)}$. Hamming codes and double-error-correcting BCH codes[2] have been proved to satisfy the upper bound. In Reference 3, it was shown that binary linear codes with poor distance property used for pure error detection will have their PUDE exceed the bound of $2^{-(n-k)}$ for certain values of ε.

Pure ARQ systems have the problems of low throughput when ε is high. Therefore, some hybrid ARQ systems are proposed to increase the throughput. In hybrid ARQ systems, we usually require linear codes for correcting some low weight error patterns and detecting many other error patterns. Hence, it is interesting to study the PUDE of linear codes which are used for both error correction and detection. In Reference 4, the class of $(n, k, d \ge 3)$ systematic binary linear codes which can be used for correcting every single error and detecting other error patterns has been studied. It was shown[4] that for each ε, $0 \le \varepsilon \le 1/2$, there exists a code whose PUDE is upper bounded by $(n+1).[2^{n-k} - n]^{-1}$. It is expected that a better upper bound would be $(n+1).2^{-(n-k)}$. However, the existence of codes whose PUDE satisfy the bound of $(n+1).2^{-(n-k)}$ for each class of $(n, k, d \ge 3)$ code has not yet been proved. We show that each maximum-length code[5] which is used for correcting every single error and detecting other error patterns will have its PUDE upper bounded by $(n+1).2^{-(n-k)}$. In addition, the result in Reference 3 is generalised. We show that a binary linear code with poor distance property, i.e. a code which falls below the asymptotic Gilbert–Varshamov bound, will have its PUDE exceed $(n+1).2^{-(n-k)}$ for a certain value of ε if it is used for correcting every single error and detecting other errors.

Bounds on maximum-length codes: Consider an (n, k, d) maximum-length code V over GF (2), where $n = 2^m - 1$, $k = m$, $d = 2^{m-1}$ and $m \ge 3$. For V, we have $A_0 = 1$, $A_d = 2^m - 1$ and $A_w = 0$ for $w \ne 0$ and d, where A_w is the number of codewords of weight w in V. Suppose V is used for correcting any single error and detecting other errors. Its PUDE is

$$\begin{aligned} P(\varepsilon) &= \sum_{w=2}^{n} [(w+1)A_{w+1} + A_w + (n-w+1)A_{w-1}] \\ &\quad \times \varepsilon^w (1-\varepsilon)^{n-w} \\ &= (1-\varepsilon)^n . \{d . A_d(\varepsilon/1-\varepsilon)^{d-1} + A_d(\varepsilon/1-\varepsilon)^d \\ &\quad + (n-d)A_d(\varepsilon/1-\varepsilon)^{d+1}\} \\ &= (2^m - 1)\varepsilon^{2^{m-1}-1}(1-\varepsilon)^{2^{m-1}-2} \\ &\quad \times \{2^{m-1} + (-2^m + 1)\varepsilon + (2^m - 2)\varepsilon^2\} \end{aligned} \tag{1}$$

For $\varepsilon = 1/2$, eqn. 1 becomes

$$\begin{aligned} P(\tfrac{1}{2}) &= (2^m - 1) . 2^{-(2^m - m - 1)} \\ &= n . 2^{-(n-k)} < (n+1)2^{-(n-k)} \end{aligned} \tag{2}$$

If $P(\varepsilon)$ is a nondecreasing function of ε for $0 \le \varepsilon \le 1/2$, then $P(\varepsilon)$ is upper bounded by $(n+1).2^{-(n-k)}$ for $0 \le \varepsilon \le 1/2$. Taking the derivative of eqn. 1, we have

$$\begin{aligned} \frac{1}{2^m - 1} . \frac{dP(\varepsilon)}{d\varepsilon} &= \varepsilon . [\varepsilon(1-\varepsilon)]^{2^{m-1}-3} \\ &\quad \times \{2^{2m-2} - 2^{m-1} + (2^{m+1} - 2^{2m}) . \varepsilon \\ &\quad + (2^{2m} + 2^{2m-1} - 3 . 2^m) . \varepsilon^2 \\ &\quad + (-2^{2m} + 3 . 2^m - 2) . \varepsilon^3\} \\ &= \varepsilon . [\varepsilon(1-\varepsilon)]^{2^{m-1}-3} \\ &\quad \times \{X + (-2^{2m} + 3 . 2^m - 2) . \varepsilon^3\} \end{aligned} \tag{3}$$

where

$$\begin{aligned} X &= 2^{2m-2} - 2^{m-1} + (2^{m+1} - 2^{2m}) . \varepsilon \\ &\quad + (3 . 2^{2m-1} - 3 . 2^m) . \varepsilon^2 \end{aligned} \tag{4}$$

$X = 2^{m-1}Y$, where $Y = (2^{m-1} - 1) + (4 - 2^{m+1}) . \varepsilon + (3 . 2^m - 6) . \varepsilon^2$. Because $(4 - 2^{m+1})^2 - 4 . (2^{m-1} - 1) . (3 . 2^m - 6) < 0$ for $m > 1$ and $3 . 2^m - 6 > 0$ for $m > 1$, then $Y > 0$ for all ε. Hence, $X > 0$ for all ε. Clearly, $X \ge X . 2\varepsilon$ for $0 \le \varepsilon \le 1/2$. From eqn. 3, we have

$$\begin{aligned} \frac{1}{2^m - 1} . \frac{dP(\varepsilon)}{d\varepsilon} &\ge \varepsilon . [\varepsilon(1-\varepsilon)]^{2^{m-1}-3} \\ &\quad \times \{X . 2\varepsilon + (-2^{2m} + 3 . 2^m - 2) . \varepsilon^3\} \\ &= \varepsilon . [\varepsilon(1-\varepsilon)]^{2^{m-1}-3} . Z \end{aligned}$$

where

$$Z = X \,.\, 2\varepsilon + (-2^{2m} + 3\,.\,2^m - 2)\,.\,\varepsilon^3$$
$$= \varepsilon\,.\,\{(2^{2m-1} - 2^m) + (2^{m+2} - 2^{2m+1})\,.\,\varepsilon + (2^{2m+1} - 3\,.\,2^m - 2)\,.\,\varepsilon^2\}$$
$$= \varepsilon\,.\,Z^*$$

and

$$Z^* = (2^{2m-1} - 2^m) + (2^{m+2} - 2^{2m+1})\,.\,\varepsilon + (2^{2m+1} - 3\,.\,2^m - 2)\,.\,\varepsilon^2$$

Because $(2^{m+2} - 2^{2m+1})^2 - 4\,.\,(2^{2m-1} - 2^m)\,.\,(2^{2m+1} - 3\,.\,2^m - 2) < 0$ for $m > 1$ and $2^{2m+1} - 3\,.\,2^m - 2 > 0$ for $m > 1$, we see that $Z^* > 0$. Therefore, we have $Z = \varepsilon Z^* \geq 0$ and hence the derivative of $P(\varepsilon)$ is non-negative. Thus, for $0 \leq \varepsilon \leq 1/2$, $P(\varepsilon)$ is nondecreasing and is upper bounded by $(n+1)\,.\,2^{-(n-k)}$.

Codes with poor distance property: We now give a general form of the result of Reference 3 to the applications of codes for correcting every single error and detecting other errors. The Gilbert–Varshamov bound states that there exists an infinite sequence of (n, k, d) binary linear codes of minimum distance d with $d/n \geq \delta$ and rate $R = k/n$ satisfying

$$R \geq 1 - H_2(\delta) \qquad \forall n$$

where $0 \leq \delta < 1/2$ and $H_2(\ )$ is the binary entropy function. We say that an (n, k) code falls below the asymptotic Gilbert–Varshamov bound if

$$k/n < 1 - H_2(d/n)$$

Suppose that V is such a code. Clearly, we have

$$k/n < 1 - H_2(d/n) + \log_2(A_d)/n \tag{5}$$

where A_d is the number of codewords of weight d in V. Multiplying eqn. 5 by n, using $H_2(d/n) = -(d/n)\,.\,\log_2(d/n) - [1 - (d/n)]\,.\,\log_2[1 - (d/n)]$ and rearranging terms, we have

$$-(n-k) < \log_2 A_d + d\,.\,\log_2(d/n) + (n-d) \times \log_2[1 - (d/n)] \tag{6}$$

Taking the antilog (base 2) of both sides of eqn. 6 results in

$$2^{-(n-k)} < A_d\,.\,(d/n)^d\,.\,[1 - (d/n)]^{n-d} \tag{7}$$

Eqn. 7 was derived in Reference 3, where eqn. 7 was then used to show that the PUDE of V will exceed $2^{-(n-k)}$ for $\varepsilon = d/n$ if V is used for pure error detection. However, here we assume V is used for correcting every single error and detecting other errors. We have

$$P(\varepsilon) = \sum_{w=2}^{n} [(w+1)A_{w+1} + A_w + (n-w+1)A_{w-1}] \times \varepsilon^w(1-\varepsilon)^{n-w}$$
$$> d\,.\,A_d\,.\,\varepsilon^{d-1}(1-\varepsilon)^{n-d+1} + A_d\,.\,\varepsilon^d\,.\,(1-\varepsilon)^{n-d} + (n-d)\,.\,A_d\,.\,\varepsilon^{d+1}\,.\,(1-\varepsilon)^{n-d-1}$$
$$= A_d\,.\,\varepsilon^d\,.\,(1-\varepsilon)^{n-d} \times [d\,.\,(1-\varepsilon)/\varepsilon + 1 + (n-d)\,.\,\varepsilon/(1-\varepsilon)] \tag{8}$$

Substituting ε by d/n, eqn. 8 becomes

$$P(d/n) > A_d\,.\,(d/n)^d\,.\,[1 - (d/n)]^{n-d}\,.\,[(n-d) + 1 + d]$$
$$= A_d\,.\,(d/n)^d\,.\,[1 - (d/n)]^{n-d}\,.\,[n+1] \tag{9}$$

It follows from eqns. 7 and 9 that

$$P(d/n) > (n+1)\,.\,2^{-(n-k)}$$

Thus, we have shown that the PUDE of V will exceed $(n+1)\,.\,2^{-(n-k)}$ for $\varepsilon = d/n$ if V is used for correcting every single error and detecting other errors.

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GaAs VERTICAL *pin* DIODE USING MeV IMPLANTATION

Indexing terms: Diodes, Semiconductor devices and materials

Vertical *pin* diodes were fabricated using MeV Si/S coimplantation and keV Be/P coimplantation into undoped semi-insulating GaAs to obtain buried n^+ and surface p^+ regions, respectively. An exploratory device with a $500 \times 500\,\mu m^2$ junction area and a $3\,\mu m$ thick intrinsic region had a breakdown voltage of 70 V, reverse leakage current density of $40\,\mu A/cm^2$ at 20 V, an off-state capacitance of $3{\cdot}9\,nF/cm^2$ and a DC forward resistance of $2{\cdot}4\,\Omega$ at 100 mA.

For high-power microwave switching, *pin* diodes are preferred over MESFETs due to their higher breakdown voltage, lower on-state resistance, and lower off-state capacitance. Presently vertical GaAs *pin* diodes are fabricated from material grown by epitaxial techniques.[1] However, *pin* diodes made with epitaxial techniques cannot be easily integrated into a planar monolithic circuit. It would be useful to be able to make *pin* diodes (*pins*) with direct ion implantation into semi-insulating GaAs. Implanted *pins* would be planar structures which could be selectively fabricated on a single chip with other types of analogue and digital device. Conventional ion implantation cannot be used to make vertical *pin* structures, but in recent years a high energy implantation technology has been developed for GaAs.[2] It is now possible to implant Si and/or S into GaAs with energies of up to 20 MeV, in a manner compatible with integrated circuit manufacturing techniques. Si or S implants at 20 MeV energy have a range of $\sim 6\,\mu m$ in GaAs and hence buried n^+ layers suitable for *pin* diode fabrication can be produced.

In this study we fabricated GaAs *pin* diodes by using MeV energy Si/S coimplantation and keV energy Be/P coimplantation into SI GaAs to obtain n^+ and p^+ layers, respectively. The Si/S coimplantation is used to obtain a high peak n-type carrier concentration.[3] The Be/P coimplantation is used to minimise Be indiffusion during annealing.[4] The cross-sectional view of the structure investigated in this exploratory study is shown in the inset of Fig. 1. The junction area is $500 \times 500\,\mu m^2$. The large dimensions were chosen for convenience and do not represent practical device values. *pin* diodes (D1 and D2) of different i-layer thicknesses were fabricated by