APERTURE ADMITTANCE MATRIX BY FINITE ELEMENT METHOD FOR SCATTERING FROM A CAVITY-BACKED APERTURE

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The generalized network formulation has been proposed for aperture coupling problems for a long time[1]; however, to the author's knowledge, there is still no systematic and efficient way to numerically solve the aperture admittance matrix (AAM) of a cavity. This paper will apply the finite element method to compute the AAM. Numerical results for the aperture field excited by a plane wave normally incident upon a 2D cavity-backed aperture on a ground plane are also included. Although only 2D TE cases are considered here, the approach can be easily generalized for more complicated problems.

Consider a 2D cavity excited by a surface magnetic current (Fig. 1) $N_0(x)$ along the y-direction. The magnetic field in the cavity will posess only a y-component H(x, z), which satisfies the following variational equation

$$(1) \qquad \delta L = 0$$

$$L = \iint_{\Omega} [(\frac{\partial H}{\partial x})^2 + (\frac{\partial H}{\partial z})^2 - H] dx dz + 2j \int_{-A}^{A} H \cdot N_o \, dx$$

derived from a Helmholtz equation and natural boundary conditions for H. Note that all space coordinates have been multiplied by the wavenumber k, and N_0 has been normalized by dividing η, the intrinsic impedance of the free space. The relative permittivity and permeabilty are set to be1.

The problem domain, then, is subdivided into many elements. For a rectangular cavity, we may expand the H field by linear bases. For an arbitrary cavity, other basis functions like the higher order triangular element [2] could be adopted.

The basis functions for the magnetic current, then, are selected as rectangular pulses to fullfill the requirement that the electric field depends on the derivatives of H. The whole aperture is divided into N segments, and let sampling point x_q' be the center of the qth segment (Fig. 2).

With such a discretization, and by Ritz procedure [2], the approximate solution of (1) may be solved through

$$[K_{mn}][h_n] = [R_{mq}][V_q]$$

where h_n's are the magnetic field at finite element node n, and V_q's are the magnetic current at the qth sampling point. The indices m and n run through all finite element nodes, while q varies simply from 1 to N. This is indeed a very large sparse matrix equation, however, we need only the magnetic field on the aperture, and thus most unknowns are unnecessary to solve. A numerical technique called static condensation [2], which is common in finite element method, can use Gaussian elimination to reduce the matrix equation to one which involves only the needed unknowns. By inverting the reduced matrix equation, we get the magnetic field on the finite element nodes at the aperture

$$[h_p] = [\, Y_{pq}' \,][V_q]$$

where p = 1, 2, ..., N+1, and q = 1, 2, ..., N.

The AAM of the cavity, however, relates the magnetic field at sampling point x_r' to the qth magnetic current segment, with q and r running from 1 to N. A close look reveals that the magnetic field at the center of each segment can be interpolated as the average of the magnetic field at both ends. Thus the desired AAM is

$$[\, Y^C_{rq} \,] = -\frac{1}{2}\begin{bmatrix} 1\,1\,0\,0 \ldots 0\,0\,0 \\ 0\,1\,1\,0 \ldots 0\,0\,0 \\ \ldots\ldots \\ 0\,0\,0\,0 \ldots 0\,1\,1 \end{bmatrix} [\, Y_{pq}' \,]$$

The minus sign here, as in [1], is determined by power consideration.

The procedure stated above seems to be tedious; however, with the aid of a frontal solution program written by the author, the static condensation and matrix inversion are automatically accomplished. The algorithm of this program is modified and enhanced from the one proposed by Iron [3].

The computed AAM, then, is added to the AAM of an aperture on a ground plane, which may be evaluated simply by moment method. The total admittance matrix now can be used to solve the aperture field at the sample points.

In Fig. 3 we consider the aperture field for waves normally incident upon a rectangular cavity. In addition to those obtained by the finite element method, fields computed by mode-matching are also presented. 40 segments for the aperture field and 40 X 40 elements for the interior field have been applied. It is seen that both results match rather well. All computation was done on a VAX-11/780 minicomputer. Several minutes (turn-around time) are required for each finite elemet solution.

With little modification, the aperture field of a two-stage cavity excited by a normally incident plane wave may be evaluated easily. Fig. 4 represents a typical case.

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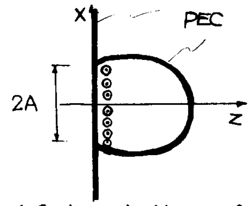


Fig. 1. Cavity excited by a surface magnetic current.

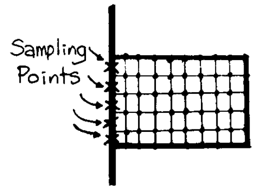
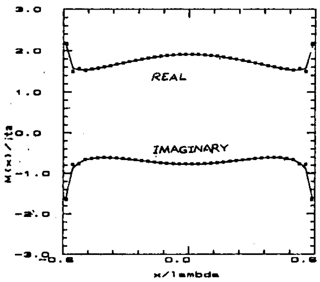


Fig. 2. Finite element mesh and sampling points on the aperture of a 2D rectangular cavity.



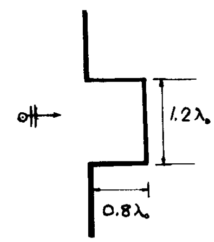


Fig. 3. Aperture fields by finite element method and by mode-matching. ——— : by mode-matching; ▪ : by finite element method.

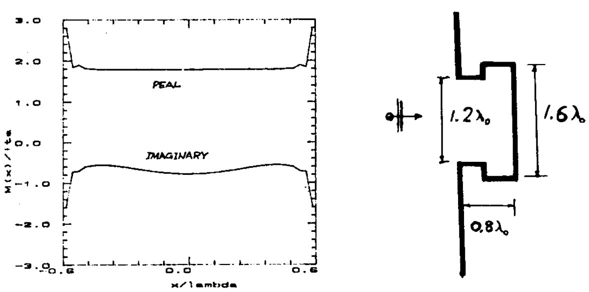


Fig. 4. Aperture fields for waves normally incident upon a two-stage cavity.