

## ANALYSIS OF COPLANAR WAVEGUIDE-FED CIRCULAR PATCH ANTENNA

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A full-wave analysis of a circular patch antenna fed by an open-ended coplanar waveguide is presented. A set of integral equations are derived by using the Hankel transform and the two-dimensional Fourier transform. Method of moments is applied to solve these integral equations for the current distribution. The surface electric current on the circular patch is expanded by entire domain basis functions, and the surface magnetic currents on the open-ended slot and the coplanar waveguide are expanded by subsectional linear basis functions. Computed results for the reflection coefficient and the input impedance are obtained which compare favorably with measurements.

### 1 Introduction

Circular microstrip patches are used in microwave integrated circuits as planar resonator for antenna, oscillator, and filter applications [1]. Many resonant modes have been studied for circular patches. The fundamental  $TM_{11}$  mode and the higher order  $TM_{12}$  mode radiate a boresight-directed beam, while other higher modes such as  $TM_{21}$ ,  $TM_{31}$  and  $TM_{41}$  modes radiate conical beams with circular polarization [2]. Such antennas can be used in wireless systems such as GPS, GSM, and satellite phones. Recently, a circular patch with the linearly polarized  $TM_{02}$  excitation was proposed as a candidate for wireless LAN applications where a beam radiating away from boresight is required [3].

Different feed structures can be used to feed circular patch antennas. For example, coaxial probe [4], microstrip line, and proximity coupling [5]. So far, only one experimental result has been published on coplanar waveguide (CPW) feed [6]. CPW are gaining popularity as feeding lines in the design of rectangular patch antennas [7], especially in the millimetre-wave frequencies. The reasons are that the CPWs have low radiation leakages, and allow shunt and series connections on the same side of the substrate, thus reducing the number of vias. Since the bandwidth of the microstrip patch antennas is known to be very narrow, it is critical to develop accurate algorithms to calculate the resonant frequencies, reflection coefficient, and input impedances.

In this paper, we combine Hankel transform in the polar coordinate and two-dimensional Fourier transform in the Cartesian coordinate to formulate a set of coupled integral equations for a circular patch fed by a linear CPW. The entire domain basis functions derived from the cavity model are used on the circular patch. They are chosen differently from those in [5] and [8]. Only the fundamental mode is assumed to propagate on the CPW away from the open end. To account for the higher order modes in the vicinity of the discontinuity, subsectional linear basis functions are used for the magnetic-current density near the open end [9]. The magnetic-current density on the open-ended slot is also expanded by the same linear basis functions. Accelerating technique is also used for the computation of integrals. Experiment is also conducted to verify our approach.

### 2 Formulation

Fig. 1 shows the configuration of a circular patch antenna fed by an open-ended coplanar waveguide. The substrate and the ground plane is assumed to extend to infinity in the  $x$  and  $y$  directions, and the conductors are assumed to be perfect with negligible thicknesses. The coplanar

waveguide is symmetric about the  $y$ -axis, and the edge of the open end is placed along the  $x$ -axis. The patch is also placed symmetric about the  $y$ -axis.

By invoking the equivalence principle and applying proper boundary conditions, a set of integral equations are obtained in terms of the electric surface current on the patch and the magnetic surface current on the CPWs and the open end as

$$\begin{aligned} H_x^{sb}(\bar{J}^p) + H_x^{sb}(\bar{M}^{sb}) &= H_x^{sa}(\bar{M}^{sa}) \\ H_y^{sb}(\bar{J}^p) + H_y^{sb}(\bar{M}^{sb}) &= H_y^{sa}(\bar{M}^{sa}) \\ E_x^p(\bar{J}^p) + E_x^p(\bar{M}^{sb}) &= 0 \\ E_y^p(\bar{J}^p) + E_y^p(\bar{M}^{sb}) &= 0 \end{aligned} \quad (1)$$

where the superscripts  $a$  and  $b$  denote the fields or the currents just below and above the slot aperture, respectively,  $\bar{J}^p$  and  $\bar{M}^s$  are the electric and magnetic surface currents on the patch and in the slot aperture, respectively,

To solve (1) by applying the Galerkin's moment method, first choose basis functions to expand the unknown currents. Traveling wave type of basis functions are chosen to represent the incident and the reflected current of the fundamental CPW mode, which are symmetric about the  $y$ -axis as follows

$$\begin{aligned} \bar{M}^{inc}(x, y) &= e^{-j\beta y} [\hat{x}F_x(x) + \hat{y}F_y(x)] \\ \bar{M}^{ref}(x, y) &= Re^{j\beta y} [-\hat{x}F_x(x) + \hat{y}F_y(x)], \text{ for } y \leq 0, \frac{G_s}{2} \leq |x| \leq W_s \end{aligned} \quad (2)$$

where  $R$  is the reflection coefficient referenced to the end of the line,  $\beta$  is the propagation constant of the fundamental mode of the uniform CPW.

$$\begin{aligned} F_x(x) &= \sum_{p=0}^{N_x-1} U_{xp} [f_{xp}(x) + f_{xp}(-x)] \\ F_y(x) &= \sum_{p=0}^{N_y-1} U_{yp} [f_{yp}(x) - f_{yp}(-x)] \end{aligned} \quad (3)$$

The travelling wave basis function used in [9] are modified in the transverse direction by using linear basis expansion.

Subsectional linear basis functions are used the magnetic-current density on the slots of the coplanar waveguide near the open end and on the open-end slot as

$$\bar{M}^L = \hat{x} \sum_{p=0}^{px-1} w_{xp} M_{xp}(x, y) + \hat{y} \sum_{p=0}^{py-1} w_{yp} M_{yp}(x, y) \quad (4)$$

where  $M_{xp}$  and  $M_{yp}$  are the linear basis defined in [7].

For the circular patch, entire domain basis functions derived from a cavity model are used to accelerate the convergence of integration. The explicit forms are

$$\bar{K}(\bar{r}_s) = \hat{\rho} \sum_{p=0}^{Q\rho-1} v_{\rho p} g_{\rho p}(\bar{r}_s) + \hat{\phi} \sum_{p=0}^{Q\phi-1} v_{\phi p} g_{\phi p}(\bar{r}_s) \quad (5)$$

where

$$\begin{aligned} g_{\rho p}(\rho, \phi) &= \frac{1}{\rho} J_m(\chi'_{mn} \rho / a) [\sin(m\phi) + \cos(m\phi)] \\ g_{\phi p}(\rho, \phi) &= J'_m(\chi'_{mn} \rho / a) [\sin(m\phi) + \cos(m\phi)] \end{aligned}$$

Substituting (2), (4) and (5) into (1), then choose the same set of basis functions as weighting functions, the resulting equations are converted to a matrix equation with unknown coefficients  $R$ ,

$w_{xp}$ ,  $w_{yp}$ ,  $v_{xp}$ , and  $v_{yp}$  to be solved. The input impedance and the return loss can be derived from the reflection coefficient  $R$ .

### 3 Results and Discussions

Fig. 2 shows the numerical results of an open-ended CPW-fed circular patch antenna. The dimension of the CPW is adjusted to have a  $50\ \Omega$  characteristic impedance. The center of the patch is located at the origin of the coordinate system. The open end is parallel to the  $x$ -axis, and is offset from the  $x$ -axis by  $O_s/2$ . For each frequency, it takes about 25 minutes to obtain the results using a Pentium III-550 PC. Measurement is also conducted using an HP8510B network analyser with the TRL calibration technique. Magnitude and phase of the measured and the calculated reflection coefficient  $R$  referenced to the end of the CPW are shown in Figs. 2 (a) and 2(b), respectively, and the normalized input impedances are shown in Fig. 2(c). Good agreement between numerical and measurement results are observed. Notice that the resonant frequency predicted by using IE3D is lower than those by using our approach and the measurement. Possible reasons may lie in the polygonal approximation for the circular patch in the IE3D. The measured resonant frequency and the bandwidth with  $VSWR < 2$  are 6.44 GHz and 3.726%, respectively. The resonant frequency and bandwidth using our approach are 6.47 GHz and 5.8 %, respectively. The discrepancies may be attributed to (1) the tolerance in the dimensions of the test piece, (2) the finite size of the substrate, and (3) the negligence of conductor thickness. Also, in modelling the slot magnetic surface current, the transverse variation was assumed to be uniform disregarding the singular-edge behavior. Comparison with other feeding structure like coaxial, microstrip line have also been conducted.

### 4 Conclusions

An integral equation approach is presented in the spectral domain combining Hankel transform in the polar coordinate and two-dimensional Fourier transform in the Cartesian coordinate. We use cavity mode basis functions to expand the currents on the patch and linear basis functions to expand the slot magnetic currents near the discontinuity to improve the solution accuracy. The numerical results compare favourably with the measurement data.

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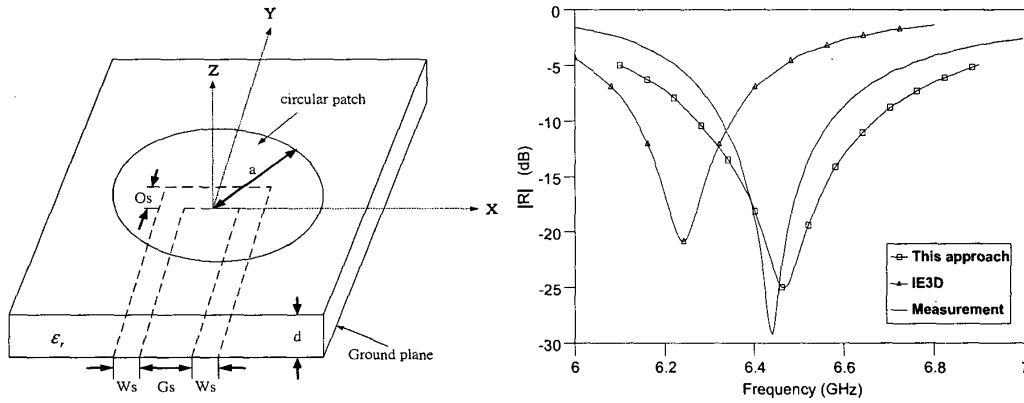


Fig. 1. Configuration of an open-ended CPW-fed circular patch antenna.

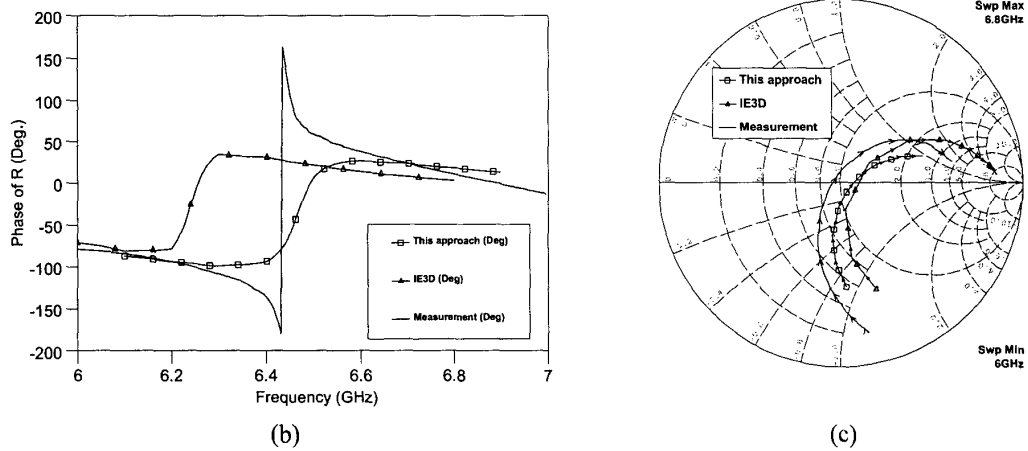


Fig 2. Reflection coefficient of an open-ended CPW-fed patch antenna,  $G_s=3$  mm,  $W_s=0.3$  mm,  $O_s=2$  mm,  $a=8$  mm,  $\epsilon_r=2.6$ , and  $h=1.5748$  mm. (a) magnitude of  $R$ , (b) phase of  $R$ , and (c) normalized input impedances.