Performance of DPSK Signals with Nonlinear Phase Noise for Systems with Small Number of Fiber Spans

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Abstract

When the dependence between linear and nonlinear phase noise is taken into account, the exact error probability of DPSK signals with nonlinear phase noise is derived analytically for a fiber system with finite number of fiber spans. For the same mean nonlinear phase shift, the SNR penalty is reduced with the number of fiber spans. The discrepancy between the exact error probability and independence approximation increases with the number of fiber spans.

I. Introduction

Nonlinear phase noise, often called Gordon-Mollenauer effect [1], or more precisely, self-phase modulation induced nonlinear phase noise, adds directly to the phase of a signal and degrades differential phase-shift keying (DPSK) signals [1]–[6] that has received renewed attention for either long-haul [7]–[10] or spectrally efficiency [11]–[13] transmission.

Nonlinear phase noise is found to be non-Gaussian distributed both experimentally [5] and theoretically [14], [15]. While uncorrelated to the linear phase noise, as non-Gaussian random variable, nonlinear phase noise is weakly depending on the linear phase noise. For systems with more than 32 fiber spans, the dependence between linear and nonlinear phase noise increases the error probability [4], [16]. Recently, DPSK signals have been used in systems with small number of fiber spans [10], [17], [18]. When DPSK signal is used in typical terrestrial systems with small (< 32) number of fiber spans, other than the approximation that linear and nonlinear phase noise is independent [6], an accurate model of the nonlinear phase noise must take into account the dependence between linear and nonlinear phase noise.

II. Joint Statistics of Linear and Nonlinear Phase Noise

For an N-span systems, for simplicity and without loss of generality, the overall quadratic nonlinear phase noise is [1], [15]

$$\begin{aligned}\Phi_{\mathrm{NL}} &= |\vec{E}_0 + \vec{n}_1|^2 + |\vec{E}_0 + \vec{n}_1 + \vec{n}_2|^2 \\ &\quad + \cdots + |\vec{E}_0 + \vec{n}_1 + \cdots + \vec{n}_N|^2, \quad (1)\end{aligned}$$

where $\vec{E}_0 = (A, 0)$ is a two-dimensional vector representing the transmitted electric field, $\vec{n}_k, k = 1, \ldots, N$, are independent identically distributed (i.i.d.) zero-mean circular Gaussian random complex number as the optical amplifier noise introduced into the system at the k^{th} fiber span. The noise variance is $E\{|\vec{n}_k|^2\} = 2\sigma_0^2$, $k = 1, \ldots, N$, where σ_0^2 is the noise variance per span per dimension. Without affected the SNR, both signal and noise in (1) can be scaled by the same ratio for different mean nonlinear phase shift of $<\Phi_{\mathrm{NL}}>= NA^2 + N(N+1)\sigma_0^2$.

In the linear regime, the signal received after N spans is

$$\vec{E}_N = \vec{E}_0 + \vec{n}_1 + \vec{n}_2 + \cdots + \vec{n}_N \quad (2)$$

with an instantaneous power of $P_N = |\vec{E}_N|^2$ and SNR of $\rho_s = A^2/(2N\sigma_0^2)$.

The joint characteristic function of the nonlinear phase noise and electric field is

$$\Psi_{\Phi,\vec{E}}(\nu, \vec{\omega}) = E\left\{\exp(j\nu\Phi_{\mathrm{NL}} + j\vec{\omega}\cdot\vec{E}_N\right\}, \quad (3)$$

where $\vec{\omega} = (\omega_1, \omega_2)$. Without going into detail, after some algebra, we obtain

$$\Psi_{\Phi,\vec{E}}(\nu, \vec{\omega}) = \Psi_{\mathrm{NL}}(\nu)\exp\left[j\omega_1 m_N(\nu) - \sigma_N^2(\nu)\frac{|\vec{\omega}|^2}{2}\right], \quad (4)$$

where

$$\Psi_{\Phi_{\mathrm{NL}}}(\nu) = \prod_{k=1}^{N} \frac{\exp\left[\frac{j\nu A^2(\vec{v}_k^T\vec{w})^2/\lambda_k}{1-2j\nu\sigma_0^2\lambda_k}\right]}{1-2j\nu\sigma_0^2\lambda_k}, \quad (5)$$

$$m_N(\nu) = A\sum_{k=1}^{N}\frac{(\vec{v}_k^T\vec{w})(\vec{v}_k^T\vec{w}_I)/\lambda_k}{1-2j\nu\sigma_0^2\lambda_k}, \quad (6)$$

$$\sigma_N^2(\nu) = \sigma_0^2\sum_{k=1}^{N}\frac{(\vec{v}_k^T\vec{w}_I)^2}{1-2j\nu\sigma_0^2\lambda_k}, \quad (7)$$

where $\vec{w} = (N, N-1, \ldots, 2, 1)^T$, $\vec{w}_I = (1, 1, \ldots, 1)^T$, and λ_k, $\vec{v}_k$, $k = 1, 2, \ldots, N$ are the eigenvalues and eigenvectors of the covariance matrix $\mathcal{C}$, respectively. The covariance matrix is $\mathcal{C} = \mathcal{M}^T\mathcal{M}$ with

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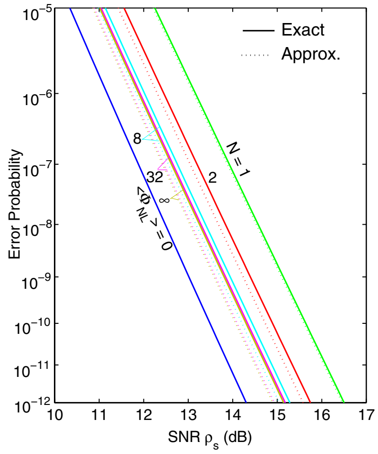

Fig. 1. The error probability of DPSK signal as a function of SNR for N =1, 2, 4, 8, 32, and infinite number of fiber spans and mean nonlinear phase shift of $<\Phi_{\rm NL}>= 0.5$ rad.

$$\mathcal{M} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix}. \tag{8}$$

III. Exact Error Probability

Similar to the approaches of [3], [4], [16], the exact error probability is

$$p_e = \frac{1}{2} - \frac{1}{2}\sum_{k=0}^{\infty} \frac{(-1)^k \left|\lambda_k e^{-\lambda_k}\right|}{2k+1} \left|\Psi_{\Phi_{\rm NL}}(2k+1)\right|^2 \times \left| I_k\left(\frac{\lambda_k}{2}\right) + I_{k+1}\left(\frac{\lambda_k}{2}\right)\right|^2 \tag{9}$$

where $I_k(\cdot)$ is the $k^{\rm th}$-order modified Bessel function of the first kind and the "angular frequency" depending SNR is

$$\lambda_k = \frac{m_N^2(2k+1)}{2\sigma_N^2(2k+1)}. \tag{10}$$

The error probability of (9) is the same as that in [4], [16] but with different parameter of λ_k from (10) with (6) and (7).

Figure 1 shows the exact error probability as a function of SNR for $<\Phi_{\rm NL}>= 0.5$ rad. Figure 2 shows the SNR penalty for an error probability of 10^{-9} as a function of mean nonlinear phase shift $<\Phi_{\rm NL}>$. Both Figs. 1 and 2 are calculated using (9) and the independence approximation of [6]. The independence approximation of [6] underestimates and the SNR penalty of a DPSK signal with quadratic phase noise of (1). The exact and approximated error probability for $N = \infty$ are the distributed model from [16] and [19], respectively. From Figs. 1 and 2, for the same mean nonlinear phase shift of

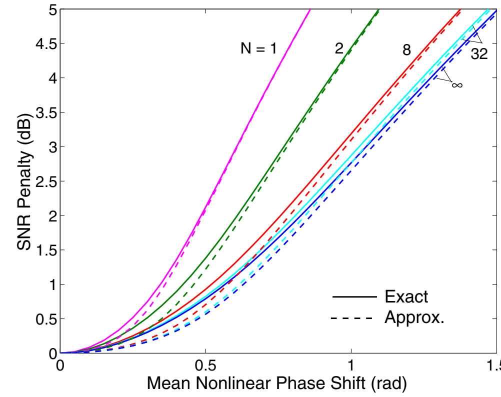

Fig. 2. The SNR penalty vs. mean nonlinear phase shift $<\Phi_{\rm NL}>$.

$<\Phi_{\rm NL}>$, the SNR penalty is larger for smaller number of fiber spans. The independence approximation of [15] is closer to the exact error probability for small number of fiber spans. In all cases, the independence assumption of [6], [19] underestimates the error probability of the system, contradicting to the conservative principle of system design. The dependence between linear and nonlinear phase noise increases the SNR penalty up to 0.23 dB. The distributed model of [16], [19] can be used when the number of fiber spans is larger than 32.

IV. Conclusions

For a system with small number of fiber spans, the exact error probability of a DPSK signal with nonlinear phase noise is derived analytically the first time when the dependence between linear and nonlinear phase noise is taking into account. For the mean nonlinear phase shift, the error probability increases for small number of fiber spans.

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