Proceedings of the 29th Conference
on Decision and Control
Honolulu, Hawaii • December 1990

Decentralized Model Reference Adaptive Control of Interconnected Dynamic Systems using Variable Structure Design.

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ABSTRACT

This paper deal with decentralized model reference adaptive control of large-scale interconnected dynamic systems. Using a variable structure design concept, we show that the steady-state tracking errors can be made as small as desired despite the strength of the interconnections among the subsystems. Furthermore, the steady-state tracking errors will converge to zero with exponential decay rate provided the interconnections are absent.

I. INTRODUCTION

Motivated by the success in designing model reference adaptive control schemes [1] & [3], some authors explored the field of decentralized adaptive control [5], [6] & [7]. Owing to direct applications of model reference adaptive controllers [1] & [3] to each unknown subsystems as if they were decoupled from each other, the standard *M*-matrix conditions have been established to limit the size of interconnection terms and, hence, retain the stability of the overall system. As has been pointed out in [6], the standard *M*-matrix condition seems to be a little far from being practical so that a modified model reference adaptive control scheme is then developed. There, the unknown interconnection strength is taken care of through the use of some structural constraints on those interconnection terms. However, due to the unsatisfactory performance of traditional model reference adaptive control schemes, the decentralized adaptive controllers stated above give limited transient responses and convergence property.

In this paper, using a variable structure design concept motivated by Fu [2], a new decentralized model reference adaptive control scheme is developed. We show that the steady state tracking error can be made as small as desired despite the existence of output interconnections among subsystems which may be nonlinear and time-varying with any possible strengths. It is also noted here that our problem formulation based on only I/O measurements which ordinarily meets the practical environments in industrial applications is different from those in [5], [6] & [7].

II. PROBLEM STATEMENT

Consider a class of large-scale interconnected dynamic systems described as follows

$$S_i:\ y_i = G_i(s)u_i + \sum_{\substack{j=1\\ i\neq j}}^{N} H_{ij}(s)\,F_{ij}(y_j,t), \qquad i\in N \tag{1}$$

where $u_i(t),\ y_i(t) \in \mathbb{R}$ are the input and the output respectively of the subsystem S_i, and $N = \{1,2,\ldots,N\}$. The function $F_{ij}(y_j,t) \in \mathbb{R}$ is the interconnection input to the subsystem S_i arising from the output of the subsystems S_j whereas $G_i(s)$ is the transfer function from u_i to y_i and $H_{ij}(s)$ is the transfer function from the interaction input $F_{ij}(y_j,t)$ to y_i.

We assume that for each i there exist nonnegative but unknown numbers α_{ij}, $j \neq i$, $j = 1, 2,\ldots, N$, such that

$$|\,F_{ij}(y_j,t)\,| \leq \alpha_{ij}\,|\,y_j\,| \tag{2}$$

Furthermore, we assume that the dynamics of the subsystem S_i are also unknown, that is, the transfer functions $G_i(s)$ and $H_{ij}(s)$, $j \neq i$, $j = 1, 2,\ldots, N$, are not specified. An implication shows that the interactions to each subsystem may not always appear at its control channel. Although transfer functions $H_{ij}(s)$, $j \neq i$, $j = 1, 2,\ldots, N$, may differ from $G_i(s)$, it is reasonable to assume that they have the same denominator.

The control objective is to force $y_i(t)$ to track the output of a given reference model for all $i \in N$. A model for S is formed by assigning to each subsystem S_i a local model

$$M_i:\ y_{mi} = M_i(s)\,r_i \tag{3}$$

which possesses the desired characteristics. With appropriate, elementary assumptions, it is clear that in the decoupled case the exact model matching through an input and output feedback techniques is obvious[1]. However, because of the use of switching control input and the interconnections among subsystem, the ideal setting of parameters may not be such that the isolated subsystems match the corresponding models. Instead, different, possibly higher gain settings may have to be chosen by the switching and adaptive mechanism in order to cancel the effects of the interconnection terms and to give faster and better tracking performance.

III. THE VARIABLE STRUCTURE DECENTRALIZED ADAPTIVE CONTROLLER

The minimal state space realization of (1) can be written as:

$$S_i:\dot{x}_i = A_i\,x_i + b_i\,u_i + \sum_{\substack{j=1\\ i\neq j}}^{N} d_{ij}\,F_{ij}(y_j,t)$$
$$y_i = c_i\,x_i \tag{4}$$

where $c_i(sI-A_i)^{-1}b_i = G_i(s)$. For each transfer function $G_i(s)$ in (1), we assume it is of the following form

$$G_i(s)=y_i(s)/u_i(s)=k_{pi}\hat{n}_{pi}(s)/\hat{d}_{pi}(s) \tag{5}$$

The reference model M_i is given by

$$M_i(s)=y_{mi}(s)/r_i(s)=k_{mi}\hat{n}_{mi}(s)/\hat{d}_{mi}(s) \tag{6}$$

In order to make this control design problem to be more tractable, the following conditions are also assumed:

A1) The plant is strictly proper with relative degree one, that is, $\hat{n}_{pi}(s)$ and $\hat{d}_{pi}(s)$ are both monic, coprime polynomials of degree (n_i-1) and n_i respectively, and $\hat{n}_{pi}(s)$ is Hurwitz.

A2) The sign of the high frequency gain, k_{pi}, is known and without loss of generality we assume that $k_{pi} > 0$.

A3) The reference model is strictly positive real (SPR), and for simplicity we assume $\hat{n}_{mi}(s)=1$ and $\hat{d}_{mi}(s)$ is a monic first order Hurwitz polynomial. The high frequency gain is chosen such that $k_{mi}>0$.

Our proposed variable structure adaptive decentralized control law is given by

$$u_i = \theta_i^T\,w_i + S_{pi} \tag{7}$$

where

$$w_i = (\,r_i,\ w_{pi}^T,\ y_i,\ w_{fi}^T\,)^T$$

is a vector of available signals,

$$\theta_i = (\,\hat{c}_{oi},\ \hat{c}_{pi}^T,\ \hat{d}_{oi},\ \hat{d}_{fi}^T\,)^T$$

is a vector of adaptation gains, S_{pi} is an auxiliary signal to be specified, and $e_{oi}=y_i-y_{mi}$ is the i–th output error. The signals w_{Pi} and w_{fi} in (3.5) are states of the precompensator and the postcompensator defined as follows :

$$\dot{w}_{Pi}=\Lambda_i w_{Pi}+b_{\lambda_i}u_i,\quad \dot{w}_{fi}=\Lambda_i w_{fi}+b_{\lambda_i}y_i \qquad (8)$$

Under the assumptions *A1)*–*A3)*, it is well known [1] that there exist unique $\theta_i^* = (c_{oi}^*, c_{Pi}^{*T}, d_{oi}^*, d_{fi}^{*T})^T$ such that the decoupled closed–loop subsystem's model match the corresponding reference model. Since, θ_i^* is known, we use their estimate $\theta_i(t)$ in the control law (7) and update them according to the following adaptive law:

$$\dot{\theta}_i(t)=-\Gamma_i(e_{oi}w_i+\sigma\theta_i) \qquad (9)$$

where Γ_i is a constant, symmetric, positive definite matrix of dimension $2n_i \times 2n_i$ and σ is a scalar constant chosen to perform a similar " σ–modification " originally proposed in [3]. Define the parameter errors $\phi_i(t) = \theta_i(t) - \theta_i^*$, so that the i–th isolated closed–loop subsystem is described by

$$\mathcal{S}_{ci} : \dot{x}_{ci} = A_{ci}x_{ci} + b_{ci}(\phi_i^T w_i - d_{oi}^* y_{mi} + S_{Pi}) + \sum_{\substack{j=1 \\ i\neq j}}^{N} d_{cij}\, F_{ij}(y_j, t)$$

$$y_i = c_{ci}\, x_{ci} \qquad (10)$$

where $x_{ci} =(x_i^T , w_{Pi}^T ,w_{fi}^T)^T$ and

$$A_{ci} = \begin{bmatrix} A_i+b_i d_{oi}^* c_i & b_i c_{Pi}^{*T} & b_i d_{fi}^{*T} \\ b_{\lambda_i} d_{oi}^* c_i^T & \Lambda_i+b_{\lambda_i} c_{Pi}^{*T} & b_{\lambda_i} d_{fi}^{*T} \\ b_{\lambda_i} c_i^T & 0 & \Lambda_i \end{bmatrix}$$

$$b_{ci} = [\, b_i^T , b_{\lambda_i}^T , 0 \,]^T,\quad c_{ci} = [\, c_i , 0 , 0 \,]$$

$$d_{cij} = [\, d_{ij}^T, 0, 0 \,]^T .$$

Then, it is easy to derive the error model of the i–th subsystem as: [4]

$$\mathcal{S}_{ei}: \dot{e}_i = A_{ci}\, e_i + b_{ci} (\phi_i^T w_i - d_{oi}^* y_{mi} + S_{Pi}) + \sum_{\substack{j=1 \\ i\neq j}}^{N} D_{cij}\, F_{ij} (y_j, t)$$

$$e_{oi} = c_{ci}\, e_i \qquad (11)$$

If we choose S_{Pi}, a switching compensation signal defined as :

$$S_{Pi}=-\mathrm{sgn}(e_{oi})[M_{i1}\|w_i\|+M_{i2}|y_{mi}|] \qquad (12)$$

where positive numbers $M_{i1} \geq \| \theta_i^* \|$ and $M_{i2} \geq | d_{oi}^* |$. We can conclude that the state error e as well as controller parameter θ of the system $\mathcal{S}_e$ are globally ultimately bounded. The above results can be refer to [4] for a more detailed insight.

V. SIMULATION

Consider the following possibly nonlinear time varying interconnected dynamic systems:

$$y_1 = G_1(s)\, u_1 + H_{12}(s)p(t)y_2$$
$$y_2 = H_{21}(s)p(t)y_1 + G_2(s)u_2$$

where transfer functions

$$G_1(s) = \frac{s+1}{s^2+2s-1},\quad H_{12} = \frac{2s+4}{s^2+2s-1}$$

$$G_2(s) = \frac{s-3}{s^2-3},\quad H_{21}(s) = \frac{s+1}{s^2-3}$$

Function p(t) is the proportional gain of the interconnections. The results of simulations are shown in two cases

Case 1) $p(t) = 1$ for all $t \geq 0$.

Case 2) $p(t) = 10\sin(100t)$ for all $t \geq 0$, and it results in fast time–varying interconnections.

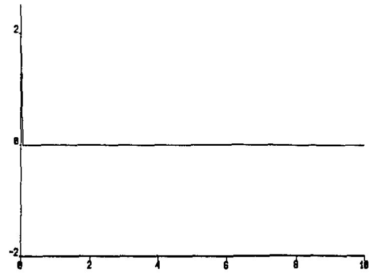

Fig. 1 The output error of subsystem 1 using current scheme–case 1

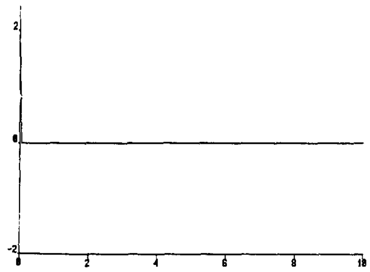

Fig. 2 The output error of subsystem 1 using current scheme–case 2

In Fig. 1–2, it is obvious to observe that in the above two cases the variable structure decentralized adaptive scheme force the output error to drop into zero in finite time.

VI. CONCLUSION

In this paper, a variable structure decentralized model reference adaptive control scheme for a class of large–scale interconnected dynamic systems has been developed. Inaccessibility to states of each subsystem is assumed here. The controller is constructed based on the concept of variable structure design which, in turn, to provided much better transient performance than those obtained so far. Computer simulations are performed in two cases, namely, coupled case and strongly fast time–varying coupled case.

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