TWO-DIMENSIONAL POLYPHASE FILTER BANKS WITH ARBITRARY NUMBER OF SUBBAND CHANNELS

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ABSTRACT

This paper concerns two-dimensional (2-D) polyphase filter banks for 2-D subband signal processing systems. New 2-D polyphase filter banks with arbitrary number of subband channels are presented. The concept of nonsymmetric frequency band allocation for the construction of subband bandpass filters is extended to 2-D case. The subband bandpass filters required in both analysis and synthesis filter banks are then constructed from the frequency translations of a 2-D prototype lowpass filter. The constraints of perfect aliasing cancellation and signal reconstruction are derived for both cases of separable and nonseparable 2-D prototype lowpass filters. Simulation results which confirm the theoretical analyses are also provided.

I. INTRODUCTION

Subband signal processing systems employing polyphase filter bank have been successfully used in splitting a signal into N subbands and allowing the resynthesis of the signal from the subbands. Originally, their applications are mainly in the areas of short-time spectral analysis [1], subband coding of speech signals [2], etc. Recently, the application of subband systems has been extended to two dimensions, for example, the subband coding of images [3] and 2-D short-time spectral analysis [4]. However, the 2-D polyphase filter banks of [4] are the direct extension of the well-known results of 1-D case described in [1].

In this paper, based on our previous work on the development 1-D polyphase filter bank [5], we further present a new 2-D polyphase filter bank for 2-D subband signal processing systems with subband channels $N_1 x N_2$, where N_1 and N_2 are arbitrary integers and represent the numbers of subband channels in each dimension. The concept of nonsymmetric frequency band allocation for the construction of subband bandpass filters presented in [5] is extended to 2-D case. The subband bandpass filters for both analysis and synthesis filter banks are then constructed from four filters. Each of them is a frequency-shifted version of a 2-D prototype lowpass filter.

This paper is organized as follows. Section II describes the formulation of 2-D subband filter banks. In Section III, efficient structure consisting of polyphase network and fast Fourier transform for the 2-D subband filter banks is presented. Computer simulations confirming the theoretical analyses are given in Section IV.

II. FORMULATION OF 2-D SUBBAND FILTER BANKS

Consider the 2-D subband system with channels $N_1 x N_2$ shown in Figure 1. The (i, j)th subband bandpass filter $H_{i,j}(\omega_1,\omega_2)$ in the analysis filter bank is given as

$$H_{i,j}(\omega_1,\omega_2) = G_{i,j}(\omega_1,\omega_2) + G_{i,2N_2-1-j}(\omega_1,\omega_2) + G_{2N_1-1-i,j}(\omega_1,\omega_2) + G_{2N_1-1-i,2N_2-1-j}(\omega_1,\omega_2) \tag{1}$$

where $G_{\cdot}(\omega_1,\omega_2) \triangleq G(\omega_1 - i\pi/N_1, \omega_2 - j\pi/N_2)$, i=0, 1,...,$N_1$-1 and j=0,i,....., N_2-1. $G(\omega_1, \omega_2)$ is a 2-D prototype lowpass filter with frequency response shown in Figure 2. The (i,j)th subband bandpass filter in the synthesis filter bank is given as

$$\tilde{H}_{i,j}(\omega_1,\omega_2) = G_{i,j}(\omega_1,\omega_2) - G_{i,2N_2-1-j}(\omega_1,\omega_2) + G_{2N_1-1-i,j}(\omega_1,\omega_2) - G_{2N_1-1-i,2N_2-1-j}(\omega_1,\omega_2) \tag{2}$$

Hence, we can see that the $H_{i,j}$ and $\tilde{H}_{i,j}$ are the nonsymmetric frequency-shifted versions of $G(\omega_1,\omega_2)$.

For the (i,j)th subband channel, the input signal $X(\omega_1,\omega_2)$ is filtered by $H_{i,j}(\omega_1,\omega_2)$ to give the (i, j)th subband signal $X_{i,j}(\omega_1,\omega_2)$. The decimated subband signal $U_{i,j}(\omega_1,\omega_2)$ can be expressed as

$$U_{i,j}(\omega_1,\omega_2) = \frac{1}{N_1 x N_2}\sum_{p=0}^{N_1-1}\sum_{q=0}^{N_2-1} X_{i,j}((\omega_1-2\pi p)/N_1, (\omega_2-2\pi q)/N_2) \tag{3}$$

For the (i,j)th subband channel of the synthesis filter bank, the channel signal is first interpolated by $N_1 x N_2$ to produce

$$V_{i,j}(\omega_1,\omega_2) = \frac{1}{N_1 x N_2}\sum_{p=0}^{N_1-1}\sum_{q=0}^{N_2-1} X_{i,j}(\omega_1-2\pi p/N_1, \omega_2-2\pi q/N_2) \tag{4}$$

After filtering $V_{i,j}(\omega_1,\omega_2)$ by $\tilde{H}_{i,j}(\omega_1,\omega_2)$ to produce $\hat{X}_{i,j}(\omega_1,\omega_2)$, we then add $\hat{X}_{i,j}(\omega_1,\omega_2)$ to give the reconstructed signal

$$\hat{X}(\omega_1,\omega_2) = \sum_{(i,j)}\sum \hat{X}_{i,j}(\omega_1,\omega_2) = \frac{1}{N_1 x N_2}\sum_{p=0}^{N_1-1}\sum_{q=0}^{N_2-1} X(\omega_1-2\pi p/N_1, \omega_2-2\pi q/N_2)\, F_{p,q}(\omega_1,\omega_2) \tag{5}$$

where

$$F_{p,q}(\omega_1,\omega_2) = \sum_{i=0}^{N_1-1}\sum_{j=0}^{N_2-1} (-1)^{i+j} H_{i,j}(\omega_1-2\pi p/N_1, \omega_2-2\pi q/N_2)\,\tilde{H}_{i,j}(\omega_1,\omega_2) \tag{6}$$

for p=0, 1, ..., N_1-1 and q=0, 1, . . . , N_2-1.

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From (5), we can see that the $F_{p,q}(\omega_1, \omega_2)$ must be zero for $(p,q) \neq (0,0)$ and $F_{0,0}(\omega_1, \omega_2)$ must be of the form

$$F_{0,0}(\omega_1, \omega_2) = N_1 N_2 \exp(-j(\omega_1 Q_1 + \omega_2 Q_2)), \qquad (7)$$

where Q_1 and Q_2 are integers, in order to obtain perfect reconstruction of $X(\omega_1, \omega_2)$ from $\hat{X}(\omega_1, \omega_2)$. From (7), we see that the following constraints must be satisfied

$$|G(\omega_1, \omega_2)| = 0 \quad \text{for} \quad |\omega_1| > \pi/N_1 \text{ and } |\omega_2| > \pi/N_2$$

$$M_1 - 1 = Q_1 = N_1(2L_1+1) \text{ and } M_2 - 1 = Q_2 = N_2(2L_2+1)$$

$$\sum_{\ell_1=0}^{2N_1-1} \sum_{\ell_2=0}^{2N_2-1} |G(\omega_1 - \ell_1\pi/N_1, \omega_2 - \ell_2\pi/N_2)|^2 = N_1 N_2 . \qquad (8)$$

if the 2-D prototype lowpass filter has its linear phase frequency response given as

$$G(\omega_1, \omega_2) = |G(\omega_1, \omega_2)| \exp\{-j(\omega_1(M_1-1)/2 + \omega_2(M_2-1)/2)\} . \qquad (9)$$

The reconstructed signal of (5) then becomes

$$\hat{X}(\omega_1, \omega_2) = X(\omega_1, \omega_2) \exp\{-j(\omega_1(M_1-1) + \omega_2(M_2-1))\} . \qquad (10)$$

which is just a delayed version of the input signal.

Now if the $G(\omega_1, \omega_2)$ is assumed to be separable, then each of $H_{i,j}(\omega_1, \omega_2)$ and $\tilde{H}_{i,j}(\omega_1, \omega_2)$ is a product of two 1-D bandpass filters, i.e., $H_{i,j}(\omega_1, \omega_2) = H_i(\omega_1)H_j(\omega_2)$ and $\tilde{H}_{i,j}(\omega_1, \omega_2) = \tilde{H}_i(\omega_1)\tilde{H}_j(\omega_2)$. Thus, $F_{p,q}(\omega_1, \omega_2)$ is also separable and given as

$$F_{p,q}(\omega_1, \omega_2) = F_p(\omega_1) F_q(\omega_2) \qquad (11)$$

where

$$F_p(\omega_1) \triangleq \sum_{i=1}^{N_1-1} (-1)^i H_i(\omega_1 - 2\pi p/N_1) \tilde{H}_i(\omega_1) \qquad (12)$$

and

$$F_q(\omega_2) \triangleq \sum_{j=1}^{N_2-1} (-1)^j H_j(\omega_2 - 2\pi q/N_2) \tilde{H}_j(\omega_2) \qquad (13)$$

Therefore, the resulting 2-D subband system is reduced to a cascade of two 1-D subband systems of [5]. Hence, each of the associated 2-D polyphase networks of the analysis and synthesis filter banks can be formed as a cascade of two 1-D polyphase networks with respect to ω_1 and ω_2, respectively. On the other hand, if $G(\omega_1, \omega_2)$ is nonseparable, then substituting (1) and (2) into (6) yields

$$F_{p,q}(\omega_1, \omega_2) = \sum_{i=0}^{N_1-1} \sum_{j=0}^{N_2-1} (-1)^{i+j} [G_{i,j}(\omega_1 - 2\pi p/N_1, \omega_2 - 2\pi q/N_2) + G_{i,2N_2-1-j}(\omega_1 - 2\pi p/N_1, \omega_2 - 2\pi q/N_2) + G_{2N_1-1-i,j}(\omega_1 - 2\pi p/N_1, \omega_2 - 2\pi q/N_2) + G_{2N_1-1-i,2N_2-1-j}(\omega_1 - 2\pi p/N_1, \omega_2 - 2\pi q/N_2)] \cdot [G_{i,j}(\omega_1, \omega_2) - G_{i,2N_2-1-j}(\omega_1, \omega_2) - G_{2N_1-1-i,j}(\omega_1, \omega_2) + G_{2N_1-1-i,2N_2-1-j}(\omega_1, \omega_2)] \qquad (14)$$

Concerning the possible aliased terms in (14), we should discuss the following two situations. The first one is $p \neq 0$ and $q=0$, or $p=0$ and $q \neq 0$. Since the $G(\omega_1, \omega_2)$ is of finite bandwith as shown in Figure 2, the product terms of nonoverlapping filter responses in (14) are zero. Examine the first product term of (14) with $p \neq 0$, $q=0$, the aliasing exists when $i=N_1-p-1$ or $i=N_1-p$. If $i=N_1-p-1$, then the first term of (14) becomes $-(-1)^{(N_1-p-1)+j} G(\omega_1-(N_1+p-1)/N_1, \omega_2 - j\pi/N_2) \cdot G(\omega_1-(N_1+p)\pi/N_1, \omega_2 - j\pi/N_2)$. If $i=N_1-p$, it becomes $-(-1)^{(N_1-p+j)} G(\omega_1 - (N_1+p)\pi/N_1, \omega_2 - j\pi/N_2) \cdot G(\omega_1 - (N_1+p-1)\pi/N_1, \omega_2 - j\pi/N_2)$. Thus, we can see that they cancel each other automatically. Next, examine the second term of (14) with $p \neq 0$ and $q=0$. The aliasing due to this term exists when $i=N_1-p$ or N_1-p-1, $j=0$ or N_2-1. For $i=N_1-p$ and $j=0$, this term becomes

$(-1)^{(N_1-p)} G(\omega_1 - (N_1+p)\pi/N_1, \omega_2) G(\omega_1 - (N_1+p-1)\pi/N_1, \omega_2 - (2N_2-1)\pi/N_2)$.

For $i=N_1-p-1$ and $j=0$, it becomes $(-1)^{(N_1-p-1)} G(\omega_1 - (N_1+p-1)\pi/N_1, \omega_2) \cdot G(\omega_1 - (N_1+p)\pi/N_1, \omega_2 - (2N_2-1)\pi/N_2)$. Thus, we see that they cancel each other if the following additional constraint on the frequency response $G(\omega_1, \omega_2)$ is satisfied

$$G(\omega_1 - \ell_1\pi/N_1, \omega_2 - \ell_2\pi/N_2) G(\omega_1 - (\ell_1+1)\pi/N_1, \omega_2 - (\ell_2+1)\pi/N_2) = G(\omega_1 - \ell_1\pi/N_1, \omega_2 - (\ell_2+1)\pi/N_2) G(\omega_1 - (\ell_1+1)\pi/N_1, \omega_2 - \ell_2\pi/N_2) \qquad (15)$$

for $\ell_1 = 0,1,\ldots\ldots, 2N_1-1$ and $\ell_2 = 0,1,\ldots, 2N_2-1$. Checking the other product terms of (14) with $p \neq 0$, $q=0$, we find that (15) is the only additional constraint for aliasing cancellation. Similar results can be obtained for the case of $p=0$ and $q \neq 0$. The second situation is $p \neq 0$ and $q \neq 0$. For the first product term of (14), again, we can see that the possible aliasing exists when $i=N_1-p-1$ or N_1-p, $j=N_2-q-1$ or N_2-q. If $i=N_1-p-1$ and $j=N_2-q-1$, the product term becomes $(-1)^{(N_1-p-1)+(N_2-q-1)} G(\omega_1 - (N_1+p-1)\pi/N_1, \omega_2 - (N_2+q-1)\pi/N_2) \cdot G(\omega_1 - (N_1+p)\pi/N_1, \omega_2 - (N_2+q)\pi/N_2)$. For $i=N_1-p$ and $j=N_2-q-1$, then it becomes $(-1)^{(N_1-p)+(N_2-q-1)} G(\omega_1 - (N_1+p)\pi/N_1, \omega_2 - (N_2+q-1)\pi/N_2) \cdot G(\omega_1 - (N_1+p-1)\pi/N_1, \omega_2 - (N_2+q)\pi/N_2)$. Thus, we see that they cancel each other if (15) is satisfied for $\ell_1 = N_1+p-1$ and $\ell_2 = N_2+q-1$. Similar results can be obtained for the other cases.

In order to achieve perfect signal reconstruction, (7) must be satisfied. From (14), $F_{0,0}(\omega_1, \omega_2)$ is given as

$$F_{0,0}(\omega_1, \omega_2) = \sum_{i=0}^{N_1-1} \sum_{j=0}^{N_2-1} (-1)^{i+j} [G^2_{i,j}(\omega_1, \omega_2) - G^2_{i,2N_2-1-j}(\omega_1, \omega_2) - G^2_{2N_1-1-i,j}(\omega_1, \omega_2) + G^2_{2N_1-1-i,2N_2-1-j}(\omega_1, \omega_2) + 2G_{i,j}(\omega_1, \omega_2) G_{2N_1-1-i,2N_2-1-j}(\omega_1, \omega_2) - 2G_{i,2N_2-1-j}(\omega_1, \omega_2) G_{2N_1-1-i,j}(\omega_1, \omega_2)] . \qquad (16)$$

Examining (16), we note that the last two terms cancel each other if the constraint of (15) is satisfied. Furthermore, from the assumption of (9), (16) can be rewritten as

$$F_{0,0}(\omega_1, \omega_2) = \sum_{\ell_1=0}^{2N_1-1} \sum_{\ell_2=0}^{2N_2-1} (-1)^{\ell_1+\ell_2} G^2_{\ell_1,\ell_2}(\omega_1, \omega_2) = \sum_{\ell_1=0}^{2N_1-1} \sum_{\ell_2=0}^{2N_2-1} (-1)^{\ell_1+\ell_2} \exp\{j\pi[\ell_1(M_1-1)/N_1 + \ell_2(M_2-1)]/N_2\} |G_{\ell_1,\ell_2}(\omega_1, \omega_2)|^2 \exp\{-j[\omega_1(M_1-1) + \omega_2(M_2-1)]\} \qquad (17)$$

Hence, if conditions of (8) are met, then the final reconstructed signal will be the same as that of (10).

Because the frequency band allocation of subband bandpass filters are nonsymmetric, the channel signals will be complex. This can be solved by multiplying the input and the recovered signals with 2-D exponential factors as shown in Figure 3. However, these multiplications are incorporated into an efficient structure for realizing the above 2-D subband system. It will be derived in the next section.

III. THE 2-D POLYPHASE FILTER BANKS

In this section, we consider the efficient structure for the realization of the proposed 2-D subband system with nonseparable 2-D prototype lowpass filter. From Figure 1, we note that

$$x_{i,j}(n_1,n_2) = x(n_1,n_2)**h_{i,j}(n_1,n_2) \quad (18)$$

where $h_{i,j}(n_1, n_2)$ denotes the impulse response of $H_{i,j}(\omega_1, \omega_2)$ and $**$ the 2-D discrete convolution. Let $g(n_1, n_2)$ be the impulse response of $G(\omega_1, \omega_2)$, then $g_{i,j}(n_1, n_2)$ can be written as

$$g_{\ell_1,\ell_2}(n_1,n_2) = g(n_1,n_2)\exp(j(n_1\ell_1\pi/N_1 + n_2\ell_2\pi/N_2)) \quad (19)$$

The convolution $x(n_1, n_2)** g_{\ell_1,\ell_2}(n_1, n_2)$ of (18) is given as

$$x(n_1,n_2)**g_{\ell_1,\ell_2}(n_1,n_2) = \sum_{k_1=0}^{2N_1-1}\sum_{k_2=0}^{2N_2-1}\exp(j(k_1(\ell_1+½)\pi/N_1 + k_2(\ell_2+½)\pi/N_2)) \cdot \sum_{r_1=0}^{L_1}\sum_{r_2=0}^{L_2} g(k_1+2N_1r_1,\ k_2+2N_2r_2)\,(-1)^{r_1+r_2}\cdot x(n_1-k_1-2N_1r_1,\ n_2-k_2-2N_2r_2) \quad (20)$$

We see that the last two summations in (20) are in 2-D convolution form. Next, define the (k_1, k_2)th polyphase component of $g(n_1, n_2)$ as

$$g'_{k_1,k_2}(n_1,n_2) = \begin{cases} g(n_1,n_2)(-1)^{r_1+r_2}, & n_1 = k_1+2N_1r_1,\ n_2 = k_2+2N_2r_2 \\ 0, & \text{otherwise.}\end{cases} \quad (21)$$

The the last two summations of (20) can be expressed as

$$\sum_{r_1=0}^{L_1}\sum_{r_2=0}^{L_2} g(k_1+2N_1r_1,\ k_2+2N_2r_2)(-1)^{r_1+r_2}x(n_1-(k_1+2N_1r_1),\ n_2-(k_2+2N_2r_2)) = x(n_1,n_2)**g'_{k_1,k_2}(n_1,n_2) \triangleq gx_{k_1,k_2}(n_1,n_2) \quad (22)$$

Substituting (22) and (20) into (18) yields

$$x_{i,j}(n_1,n_2) = \sum_{k_1=0}^{2N_1-1}\sum_{k_2=0}^{2N_2-1} gx_{k_1,k_2}(n_1,n_2)\,(W_{2N_1}^{k_1(i+½)}\cdot W_{2N_2}^{k_2(j+1/2)} + W_{2N_1}^{k_1(2N_1-i-½)}\cdot W_{2N_2}^{k_2(2N_2-j-½)} + W_{2N_1}^{k_1(i+½)}\cdot W_{2N_2}^{k_2(2N_2-j-½)} + W_{2N_1}^{k_1(2N_1-i-½)}\cdot W_{2N_2}^{k_2(j+½)}) \quad (23)$$

where $W_{2N} \triangleq \exp(j2\pi/2N)$. From (3), (23) and the property of complex conjugate, we obtain

$$u_{i,j}(n_1,n_2) = \mathrm{Re}\Big(\sum_{k_1=0}^{2N_1-1}\sum_{k_2=0}^{2N_2-1} 2gx_{k_1,k_2}(n_1N_1,n_2N_2)\,(W_{2N_1}^{-k_1(i+½)}\cdot W_{2N_2}^{-k_2(j+½)} + W_{2N_1}^{-k_1(i+½)}\cdot W_{2N_2}^{-k_2(2N_2-j-½)})\Big) \quad (24)$$

where Re(.) denotes the real part of (.). Let

$$gxc_{k_1,k_2}(n_1,n_2) = gx_{k_1,k_2}(n_1,n_2)/\cos(k_1\pi/2N_1)\cos(k_2\pi/2N_2) \text{ for } k_1 \neq N_1,\ k_2 \neq N_2.$$
$$gxc_{k_1,N_2}(n_1,n_2) = gx_{k_1,N_2}(n_1,n_2)/\cos(k_1\pi/2N_1), \text{ for } k_1 \neq N_1.$$
$$gxc_{N_1,k_2}(n_1,n_2) = gx_{N_1,k_2}(n_1,n_2)/\cos(k_2\pi/2N_2), \text{ for } k_2 \neq N_2.$$
$$gxc_{N_1,N_2}(n_1,n_2) = 0. \quad (25)$$

Then substituting (25) into (24) and performing some manipulation gives

$$u_{i,j}(n_1,n_2) = (½).\ \mathrm{Re}\Big(\sum_{\substack{k_1=0,\,k_2=0\\ k_1\neq N_1,\,k_2\neq N_2}}^{2N_1-1,\,2N_2-1} gxc_{k_1,k_2}(n_1N_1,n_2N_2)\cdot (W_{2N_1}^{-k_1 i} + W_{2N_1}^{-k_1(i+1)})\cdot(W_{2N_2}^{-k_2 j} + W_{2N_2}^{-k_2(2N_2-j-1)} + W_{2N_2}^{-k_2(2N_2-j)} + W_{2N_2}^{-k_2(j+1)})\Big) \quad (26)$$

From (26), it can be seen that the input signal $x(n_1, n_2)$ is first convolved with the (k_1, k_2)th polyphase component $g'_{k_1,k_2}(n_1, n_2)$. The result is then divided by the cosine factors to give $gxc_{k_1,k_2}(n_1,n_2)$ as shown in (25). Finally, the output from the polyphase network is decimated by $N_1 \times N_2$ and then used as the input of a 2-D FFT operation. $u_{i,j}(n_1,n_2)$ is obtained by summing all the eight outputs of the 2-D FFT and taking the real part of this summation.

Next, we describe the derivation of the efficient structure for the synthesis filter bank. From Figure 1, we have

$$\hat{x}_{i,j}(n_1,n_2) = v_{i,j}(n_1,n_2)**\tilde{h}_{i,j}(n_1,n_2) \quad (27)$$

Hence,

$$\hat{x}(n_1,n_2) = \sum_{i=0}^{2N_1-1}\sum_{j=0}^{2N_2-1} v'_{\ell_1,\ell_2}(n_1,n_2)**g_{\ell_1,\ell_2}(n_1,n_2) \quad (28)$$

where $v'_{i,j}(n_1,n_2) = v'_{2N_1-1-i,\,2N_2-1-j}(n_1,n_2) = (-1)^{i+j}v_{i,j}(n_1,n_2)$ and $v'_{1,2N_2-1-j}(n_1,n_2) = v'_{2N_1-1-i,j}(n_1,n_2) = -(-1)^{i+j}v_{i,j}(n_1,n_2)$ for $i=0,1,\ldots,N_1-1$ and $j=0,1,\ldots,N_2-1$. Next, substituting (19) into (28) and performing some manipulation yields

$$\hat{x}(n_1,n_2) = \sum_{k_1=0}^{2N_1-1}\sum_{k_2=0}^{2N_2-1} g'_{k_1,k_2}(n_1,n_2)**P_{k_1,k_2}(n_1,n_2) \quad (29)$$

where

$$P_{k_1,k_2}(n_1,n_2) = 4N_1N_2(-1)^{(L_1+L_2+1)}W_{2N_1}^{k_1/2}\cdot W_{2N_2}^{k_2/2}\cdot\Big((1/4N_1N_2)\sum_{\ell_1=0}^{2N_1-1}\sum_{\ell_2=0}^{2N_2-1} v'_{\ell_1,\ell_2}(n_1,n_2)\,W_{2N_1}^{k_1\ell_1}\,W_{2N_2}^{k_2\ell_2}\Big). \quad (30)$$

We note that the double summation in (30) represents the 2-D IFFT of $v_{\ell_1,\ell_2}(n_1,n_2)$. Furthermore, from (30), we can also show that $P_{2N_1-k_1,2N_2-k_2}(n_1,n_2) = P_{2N_1-k_1,k_2}(n_1,n_2) = P_{k_1,k_2}(n_1,n_2)$. Therefore, it can be seen that in the synthesis filter bank, the (i,j)th input channel signal $v_{i,j}(n_1,n_2)$ is first multiplied by $(-1)^{i+j}$ (or $-(-1)^{i+j}$) to produce the (i,j)th and the $(2N_1-1-i,\ 2N_2-1-j)$th (or the $(i,\ 2N_2-1-j)$th) and the $(2N_1-1-i,j)$th) inputs of the2-D IFFT. Then the (k_1,k_2)th output of the 2-D IFFT is multiplied by $4N_1N_2(-1)^{(L_1+L_2+1)}\exp(j(k_1\pi/2N_1 + k_2\pi/2N_2))$. After interpolated by $(N_1 \times N_2)$, it is convolved with the (k_1, k_2)th polyphase component. Finally, the outputs from the polyphase network are summed together to produce the reconstructed signal.

IV. EXPERIMENTAL RESULTS

Several computer simulations were performed to illustrate the theoretical development of the proposed 2-D polyphase filter bank. Figure 4 shows the original 2-D image as the input 2-D signal. The reconstructed 2-D image using the proposed polyphase filter banks with $N_1 = N_2 = 2$ when $G(\omega_1, \omega_2)$ is a separable zero-phase FIR filter of size 69x69 is shown in Figure 5. Comparison to the original shows it to be virtually identical. Next, we performed the simulation using nonseparable $G(\omega_1, \omega_2)$ with magnitude response satisfying the additional constraint of (15). Figure 6 shows the obtained reconstruction. Again, the simulation result confirms our theoretical development. Some more detailed analyses concerning the characteristics of the proposed 2-D polyphase filter banks will be presented in a forthcoming paper.

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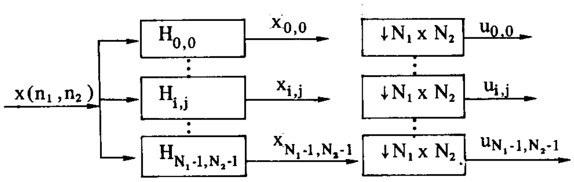
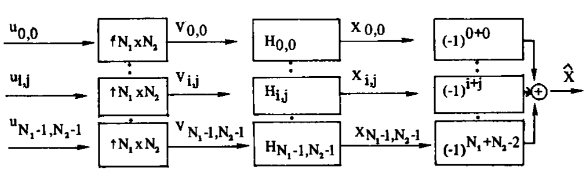


(a) The Analysis Filter Bank



(b) The Synthesis Filter Bank

Figure 1. The Filter Banks of A 2-D Subband System

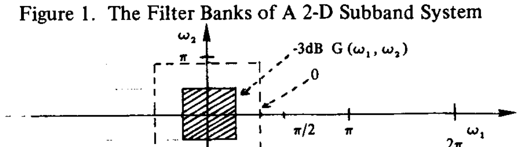
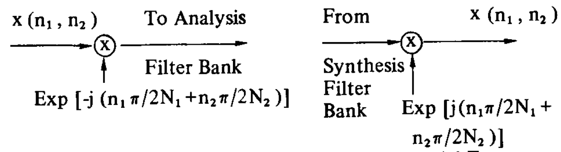


Figure 2. The 2-D Prototype Lowpass Filter



(a) Exponential Factor For Analysis Filter Bank

(b) Exponential Factor For Synthesis Filter Bank

Figure 3

Figure 4 The Original Image

Figure 5 The Reconstructed Image when $G(\omega_1, \omega_2)$ is separable

Figure 6 The Reconstructed Image when $G(\omega_1, \omega_2)$ is nonseparable