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Transient Analysis of Dispersive Transmission Lines with Nonlinear Loads Using Staircase Approximation

I-Ting Chiang and Shyh-Kang Jeng
Graduate Institute of Communication Engineering and
Department of Electrical Engineering
National Taiwan University
Taipei , Taiwan , ROC.
Email:skjeng@ew.ee.ntu.edu.tw

Abstract

This paper introduces a staircase approximation to solve for the transients of dispersive transmission lines with nonlinear loads. Such an approach is with easy formulation and general enough to solve a complete microwave circuit like mixers or oscillators. Furthermore, it can handle independently without the help of CAD tools if the terminated circuit network is complicated.

I Introduction

As the clock rate of digital circuits goes higher and higher, dispersion and other frequency-dependent property of transmission lines takes a more crucial role than before. Besides, nonlinear problems are of practical interest and importance, too. Therefore, an approach to handle these two issues simultaneously is necessary, which is what we want to do in this paper.

To treat the transient response of dispersive transmission lines, the frequency-domain method is adopted traditionally. It suffers, however, not only the difficulty with the presence of nonlinear loads but also the slow numerical integration when taking the inverse Fourier transform. The FDTD [1], another candidate, is capable of settling nonlinear loads, but has difficulties when frequency-dependent factors must be considered. In addition, the FDTD becomes involved in dealing with multisection transmission lines, which further limits its applications.

The staircase approximation we proposed here can solve for the transients of dispersive lines with nonlinear loads through a straightforward formulation. The next section will give a sketch of the staircase approximation. Section III provides some

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numerical examples, and the last section draws some conclusions.

Ⅱ Formulation

The dispersive telegraphist's equations in time domain are

$$-\frac{\partial}{\partial x}v = Ri + \frac{\partial}{\partial t}(L * i) \tag{1}$$

$$-\frac{\partial}{\partial x}i = Gv + \frac{\partial}{\partial t}(C * v) \tag{2}$$

where the star "*" represents the convolution operation to account for dispersion. Approximate the signals by

$$v(x,t) \approx \sum_{j=1}^{n} v_j(x)h_j(t) \tag{3}$$

$$i(x,t) \approx \sum_{j=1}^{n} i_j(x)h_j(t) \tag{4}$$

where $h_j(t)$ is the unit rectangular pulse with duration Δt. This is called the staircase approximation since the voltage and current signals at a given x is now approximated by a staircase-like function. By inserting (3) and (4) into (1) and (2) and integrating with respect to t and then taking inner product by Galerkin scheme, the original system of continuous time-dependent equations transforms to a system of discrete time-independent matrix equations

$$-\frac{d}{dx}[v] = ([R] + \frac{1}{\Delta t}[L])[i] \tag{5}$$

$$-\frac{d}{dx}[i] = ([G] + \frac{1}{\Delta t}[C])[v] \tag{6}$$

where $[v]$ and $[i]$ are column matrices and $[R]$, $[L]$, $[G]$, $[C]$ are square matrices which can be obtained accordingly.

Once these matrices are determined, equations can be manipulated in a way similar to those in the time-harmonic case. Problems with linear loads thus can be solved directly, and nonlinear loads can be solved iteratively [2].

Ⅲ Numerical Results

To prove the usefulness of the staircase approximation, consider a lossless ($R = G = 0$) transmission line shown in Fig. 1, and shunt with a capacitor and a nonlinear load at the terminal. A generator with an internal resistor is placed at the left end.

As the first example, assume the transmission line is with the parameters $\ell = 1.0(m)$, $L = 0.5(\mu H/m)$, and $C = 0.2(nF/m)$. In addition, a matched generator excites a unit step function. The nonlinear loads are described by $i = 0.01 \times v^2(A)$ for $v > 0$ and $i = 0$ for $v \le 0$, and shunt with capacitors $C_L = 50(pF)$. The solid curve represents the result by the FDTD method. Both the dotted and the dashed lines exhibit the response calculated by the staircase approximation. The parameters for it are $\Delta t = 0.15625(n\sec)$ with 128 bases (dotted curve), and $\Delta t = 0.125(n\sec)$ with 160 bases (dashed curve). Both match well.

Next, let's remove the nonlinear loads and introduce the Debye dispersion with parameters $\varepsilon_s = 9$, $\varepsilon_\infty = 0$ and $w_0 = 1/t_0 = 5\pi \times 10^8$. Apply a rectangular pulse excitation with pulse width $w = 1(n\sec)$, remove the nonlinear load and replace the length of transmission line by $\ell = 0.5(m)$, internal resistor by $R_g = 0(\Omega)$ and load capacitor by $C_L = 20(pF)$. The voltage and current response calculated at $x = 0.5(m)$ by the staircase approximation (solid line) and the frequency-domain transform method (dotted line) are illustrated in Fig. 3. The agreement of both curves validates the capability of our method in dealing with dispersive transmission lines.

Last, but not least, assume the same parameters used in the second example but drop the capacitor and apply the nonlinear loads $i = 0.002 \times v^2(A)$ for $v > 0$ and $i = 0$ for $v \le 0$. The results are exhibited in Fig. 4, in which the solid line is with the parameters $\Delta t = 0.1$ $(n\sec)$ and 512 bases while the dashed line adopts $\Delta t = 0.2(n\sec)$ and 256 bases. The results show that the convergence of our method is pretty good.

Ⅳ Conclusion

We have proposed the staircase approximation and shown its usefulness in dealing with transients of dispersive transmission lines with non-linear loads. Numerical results verified with the FDTD and the conventional frequency-domain method have been exhibited. This method can be easily formulated and applied to problems with frequency-dependent loads and multiconductor transmission lines, which are important for more realistic applications.

Ⅴ Reference

[1] A. Taflove, " *Computational Electro-dynamics: The Finite-Difference Time-Domain Method,* " Artech House, Boston, 1995.

[2] I.T. Chiang and S.K. Jeng, " Haar wavelet scale domain method for solving the transient response of dispersive transmission lines with nonlinear loads, " submitted to IEICE Transaction on Communications.

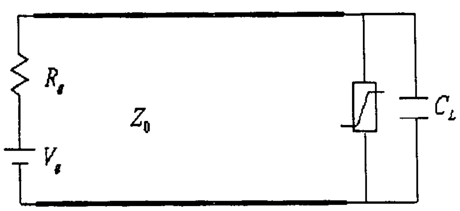

Figure 1

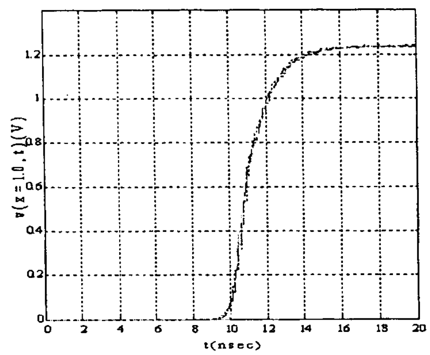

Figure 2

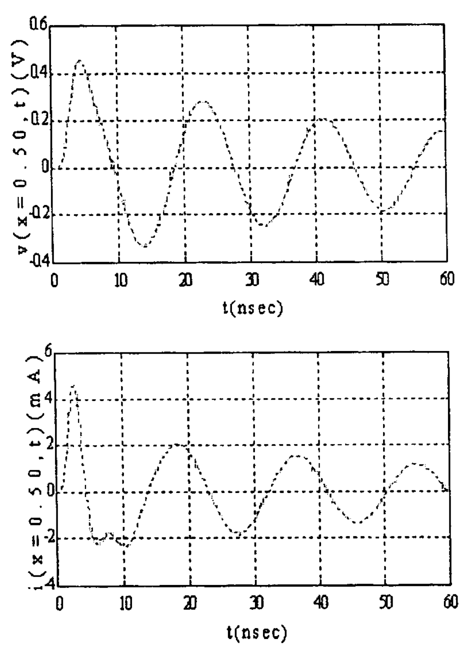

Figure 3

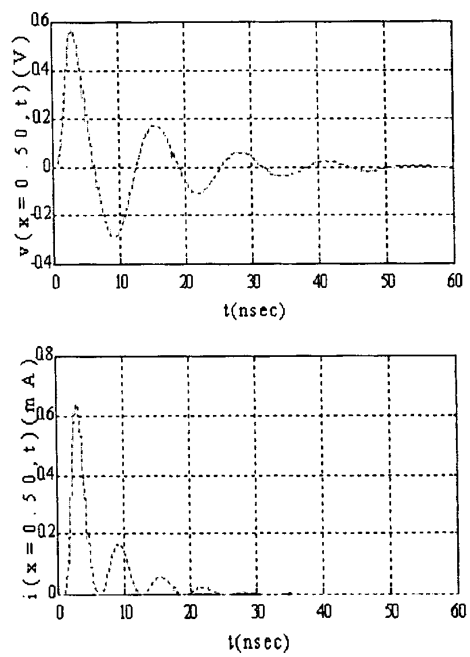

Figure 4