Decision Feedback Differential Detection Using Weighted Phase References

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Abstract : A conventional decision feedback differential detection technique can be regarded as a technique using L previously received signal samples and $L-1$ previously decided data phases to generate a phase reference for the phase detection operation of a currently received $MPSK$ signal sample. With this observation, we propose a differential detection technique which uses a new way to generate the phase reference. In the proposed technique, the phase reference called weighted phase reference is the weighted sum of primitive phase references derived from previously received signal samples. The weighted phase reference can be generated using a simple recursive form. The proposed detection technique can be easily implemented and has satisfactory error performance.

I Introduction

For an $MPSK$ transmission system, differential detection (DD) is preferred over coherent detection (CD) if phase coherence is hard to obtain. The conventional (one-symbol) differential detection technique uses the previously received signal sample as the phase reference to detect the currently received signal sample. Such a phase reference is likely to have been corrupted by noise. Hence, the error performance of the conventional differential detection technique is somewhat worse than that of the coherent differential detection technique. To improve the error performance of differential detection, several multiple-symbol differential detection techniques have been proposed [1-7]. Among them, decision feedback differential phase detection (DF-DPD) is the simplest one. The DF-DPD technique is a simplified version of a technique called decision feedback differential detection (DF-DD). By setting all the amplitudes of received samples to be constant, DF-DD reduces to DF-DPD. The DF-DD (including DF-DPD) technique employs L previously received signal samples and $L-1$ data phases which were previously decided to detect the currently received signal sample. For applications over the additive white Gaussian noise (AWGN) channel, larger L will result in better error performances. However, the complexity of DF-DD (including DF-DPD) with large L is much higher than conventional DD.

In this paper, we show that the conventional DF-DD (including DF-DPD) can be detected using phase references like the conventional DD. The difference is that the phase reference of the conventional DD is generated by the previously received signal sample while the phase reference of the DF-DD (including DF-DPD) is generated by L previously received signal samples and $L-1$ previously decided data phases. We note that there is still much room for decreasing the complexity of the conventional DF-DD (including DF-DPD). Hence, we propose a new DF-DPD called DF-DPD using weighted phase references (WPR) for which the phase reference can be easily generated by a simple recursive form. The weighted phase reference for detecting the currently received signal sample is the weighted sum of primitive phase references derived from previously received signal samples. In the weighted sum, the primitive phase reference derived from a more recently received signal sample provides a more significant contribution. The degree of the significance can be adjusted by varying the weight factor. In some applications, the phase of a received signal sample is not available while the phase difference between a received signal and a delayed version of it is available. The recursive form for generating weighted phase references can be converted into an equivalent recursive form for generating weighted phase differences. The DF-DPD using WPR has the advantage of both low complexity of detection and good error performance.

In this paper, we also propose a technique called DF-DD using WPR. The DF-DPD using WPR is a simplified version of DF-DD using WPR by setting the amplitudes of received signal samples to be constant.

II Decision Feedback Differential Detection

In this section, we show that DF-DD [2-4] can be regarded as a technique which uses L previously received signal samples and $L-1$ previously decided data phases to generate a phase reference for the phase detection operation of a currently received $MPSK$ signal sample. Suppose that differentially encoded $MPSK$ signals are transmitted over an AWGN channel. Let $r(t)$ be the received signal for time t. Through some process, we have the received signal

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sample for time k, which is

$$r_k = \sqrt{2P} e^{j\phi_k} e^{j\theta_k} + N_k, \quad (1)$$

where P is the signal power, ϕ_k is the modulation phase, θ_k is an arbitrary phase introduced by the channel, and N_k is a sample of zero-mean complex Gaussian noise. The modulation phase $\phi_k = 2\pi m/M$ for some $m \in \{0,1,\cdots,M-1\}$. The arbitrary phase θ_k is assumed to be uniformly distributed in the interval $(-\pi, \pi)$. Define $x \ mod \ 2\pi = x + 2K\pi$, where K is an integer such that $\pi < x + 2K\pi \leq \pi$. The information data for time k is carried by the data phase $\Delta\phi_k = (\phi_k - \phi_{k-1}) \ mod \ 2\pi$. We can express r_k by

$$r_k = \mid r_k \mid e^{j\psi_k}, \quad (2)$$

where $\mid r_k \mid$ and ψ_k are the amplitude and phase of the received signal r_k respectively. We have

$$\psi_k = (\phi_k + \eta_k + \theta_k) \ mod 2\pi, \quad (3)$$

where η_k is the phase noise due to AWGN.

Consider a sequence of received signal samples of length $L+1$. We assume that θ_k is constant for all the samples of this sequence. In a DF-DD algorithm [2-4], the decision rule is to decide the data phase $\Delta\bar{\phi}_n$ for time n as the one out of all the M possible $\Delta\phi_n$ which maximizes

$$X_n = Re\{\sum_{i=1}^{L} r_n r^*_{n-i} exp(-j(\Delta\phi_n + \sum_{l=1}^{i-1} \Delta\bar{\phi}_{n-l}))\}, \quad (4)$$

where $\Delta\bar{\phi}_{n-l}$ is the data phase which have already been chosen for time $n-l$. This is equivalent to maximizes

$$\sum_{l=1}^{L} \mid r_{n-l} \mid cos(\psi_n - \psi_{n-l} - \Delta\phi_n - \sum_{i=1}^{l-1} \Delta\bar{\phi}_{n-i})$$
$$= \sum_{l=1}^{L} \mid r_{n-l} \mid cos(\psi_n - \Psi^{(l)}_{n-1} - \Delta\phi_n), \quad (5)$$

where r^*_{n-i} is the complex conjugate of r_{n-i} and

$$\Psi^{(l)}_{n-1} = \psi_{n-l} + \sum_{i=1}^{l-1} \Delta\bar{\phi}_{n-i}, \quad (6)$$

which is a primitive phase reference for time $n-1$ and is derived from ψ_{n-l}. Define

$$\mu^{(l)}_n = (\psi_n - \Delta\phi_n - \Psi^{(l)}_{n-1}) \ mod \ 2\pi$$
$$= (\psi_n - \Delta\phi_n - \Psi^{(l)}_{n-1}) + 2K^{(l)}_{n-1}\pi, \quad (7)$$

where $K^{(l)}_{n-1}$ is an integer which is used such that $-\pi < \mu^{(l)}_n \leq \pi$. The parameter $\mu^{(l)}_n$ is an estimated phase error of ψ_n obtained by comparing ψ_n with $\Psi^{(l)}_{n-1} + \Delta\phi_n$.

In case that the phase ψ_n is not available while the phase difference $\Delta\psi_n = \psi_n$ - ψ_{n-1} is available, the parameter $\mu^{(l)}_n$ can be generated from the recursive form given by

$$\mu^{(l)}_n = \begin{cases} (\mu^{(l-1)}_n + [\Delta\psi_{n-l+1} - \Delta\bar{\phi}_{n-l+1}]) \ mod \ 2\pi, & \text{for } l > 1, \\ \Delta\psi_n - \Delta\bar{\phi}_n, & \text{for } l = 1. \end{cases} \quad (8)$$

Since the shape of the curve of $cosx$ is similar to the shape of the curve of $1 - x^2$ for $-\pi < x \leq \pi$, we use an approximation to simplify the decision rule of DF-DD. The simplified decision rule is to find the one among M possible $\Delta\phi_n$ which minimizes

$$Y_n = \sum_{l=1}^{L} \mid r_{n-l} \mid (\mu^{(l)}_n)^2. \quad (9)$$

The minimization of Y_n can be achieved if and only if

$$\sum_{l=1}^{L} \mid r_{n-l} \mid (\psi_n - \Psi^{(l)}_{n-1} - \Delta\phi_n + 2K^{(l)}_{n-1}\pi)^2$$
$$< \sum_{l=1}^{L} \mid r_{n-l} \mid (\psi_n - \Psi^{(l)}_{n-1} - \Delta\phi_n - \frac{2\pi}{M} + 2K^{(l)}_{n-1}\pi)^2 \quad (10)$$

and

$$\sum_{l=1}^{L} \mid r_{n-l} \mid (\psi_n - \Psi^{(l)}_{n-1} - \Delta\phi_n + 2K^{(l)}_{n-1}\pi)^2$$
$$< \sum_{l=1}^{L} \mid r_{n-l} \mid (\psi_n - \Psi^{(l)}_{n-1} - \Delta\phi_n + \frac{2\pi}{M} + 2K^{(l)}_{n-1}\pi)^2 \quad (11)$$

It follows from (10) and (11) that, the decision rule of DF-DD can be transformed into finding the the one among M possible $\Delta\phi_n$ such that

$$-\frac{\pi}{M} < \psi_n - \psi_{n-1,DF} - \Delta\phi_n < \frac{\pi}{M}. \quad (12)$$

where

$$\psi_{n-1,DF} = \frac{\sum_{l=1}^{L} \mid r_{n-l} \mid (\Psi^{(l)}_{n-1} + 2K^{(l)}_{n-1}\pi)}{\sum_{l=1}^{L} \mid r_{n-l} \mid}. \quad (13)$$

When $L = 1$, this decision rule of DF-DD is identical to the decision rule of the conventional (one symbol) differential detection. A DF-DD technique can be regarded as a modification of the conventional (one-symbol) differential detection by using $\psi_{n-1,DF}$ as the phase reference to replace ψ_{n-1}.

III Detection Using Weighted Phase Reference

Decision feedback differential phase detection (DF-DPD) [7] is a simplified version of DF-DD, for which the decision rule neglects the amplitude factor $| r_{n-l} |$ from Y_n given in (9). That means the decision rule of DF-DPD is to decide $\Delta\bar{\phi}_n$ as the one among M possible $\Delta\phi_n$ such that

$$Z_n = \sum_{l=1}^{L} (\mu_n^{(l)})^2 \tag{14}$$

is minimized. With a procedure similar to that used for DF-DD, we can transform the decision rule of DF-DPD into finding $\Delta\phi_n$ such that

$$-\frac{\pi}{M} < \psi_n - \psi_{n-1,DFP} - \Delta\phi_n < \frac{\pi}{M}, \tag{15}$$

where $\psi_{n-1,DFP}$ is the phase reference for time $n-1$ obtained by averaging $\Psi_{n-1}^{(1)}, \cdots, \Psi_{n-1}^{(L)}$, i.e.,

$$\psi_{n-1,DFP} = \frac{\sum_{l=1}^{L} \Psi_{n-1}^{(l)} + 2K_{n-1}^{(l)}\pi}{L}. \tag{16}$$

Assume that feedback symbols $\Delta\bar{\phi}_{n-1}, \cdots, \Delta\bar{\phi}_{n-L+1}$ are correct. Inequality (15) implies that $\Delta\phi_n$ is correct if

$$-\frac{\pi}{M} < \eta_n - \eta_{n-1,DFP} < \frac{\pi}{M}, \tag{17}$$

where η_n is the phase noise of ψ_n,

$$\eta_{n-1,DFP} = \frac{\sum_{l=1}^{L} \eta_{n-1}^{(l)}}{L}, \tag{18}$$

is the phase noise of $\psi_{n-1,DFP}+\Delta\phi_n$ and $\eta_{n-1}^{(l)}$ is the phase noise of $\Psi_{n-1}^{(l)} + 2K_{n-1}^{(l)}\pi + \Delta\phi_n$.

The phase reference for time n, $\psi_{n,DFP}$, can be recursively derived from the phase reference for time $n-1$, $\psi_{n-1,DFP}$. The relation is given by

$$\begin{aligned}\psi_{n,DFP} &= \frac{1}{L}[(L-1)(\psi_{n-1,DFP} + \Delta\bar{\phi}_n) + (\psi_{n-1,DFP} - \\ &\quad \Psi_{n-1}^{(L)} + \psi_n + 2\sum_{l=1}^{L}(K_n^{(l)} - K_{n-1}^{(l)})\pi]. \end{aligned} \tag{19}$$

This recursive relation is a bit complicated. Since both $\psi_{n-1,DFP}$ and $\Psi_{n-1}^{(L)}$ are estimated phase references for time $n-1$, we expect that the difference between them is small after the modulo 2π operation. Hence, we propose a new way of estimating the phase reference for time n, which has a simplified recursive relation that is obtained by removing the $\psi_{n-1,DFP} - \Psi_{n-1}^{(L)}$ term from (19). The new phase reference for time n is denoted φ_n. The recursive relation is given by

$$\varphi_n = \begin{cases} \frac{W(\varphi_{n-1}+\Delta\bar{\phi}_n)+\psi_n+2K_n\pi}{W+1}, & \text{for } n \geq 1, \\ \psi_0, & \text{for } n = 0, \end{cases} \tag{20}$$

where K_n is an interger which is used such that

$$-\pi < (\varphi_{n-1} + \Delta\bar{\phi}_n) - \psi_n - 2K_n\pi \leq \pi. \tag{21}$$

We may consider φ_n as the weighted sum of two estimated phase references, one of which is $(\varphi_{n-1} + \Delta\bar{\phi}_n)$ and the other is $\psi_n + 2K_n\pi$. Hence, we call φ_n the weighted phase reference (WPR) for time n.

Then, we have a new differential detection technique called DF-DPD using weighted phase reference for which the decision rule is to find $\Delta\phi_{n+1}$ such that

$$-\frac{\pi}{M} < \Omega_{n+1} \leq \frac{\pi}{M}, \tag{22}$$

where

$$\Omega_{n+1} = (\psi_{n+1} - \varphi_n - \Delta\phi_{n+1}) \mod 2\pi \tag{23}$$

Equivalently, the decision rule is to find $\Delta\phi_{n+1}$ that minimizes $| \Omega_{n+1} |$.

We can express φ_n as the weighted sum of all the primitive phase references, i.e.,

$$\varphi_n = \sum_{l=1}^{n+1} a_n^{(l)}(\Psi_n^{(l)} + 2J_n^{(l)}\pi), \quad \text{for } n \geq 0, \tag{24}$$

where $J_n^{(l)}$ is an integer, $J_0^{(1)} = 0$, $a_0^{(1)} = 1$ and

$$a_n^{(l+1)} = \begin{cases} \frac{1}{W+1} \times (\frac{W}{W+1})^l & \text{for } n > l \\ (\frac{W}{W+1})^l & \text{for } n = l. \end{cases} \tag{25}$$

The proof of (24) and (25) are omitted for brevity.

In a conventional DF-DPD technique, L previously received signal samples are used to detect the current data phase, where all these L received signal samples are treated to be equally important and other signal samples are neglected. From (24) and (25), we see that in the DF-DPD using WPR proposed here, all the previously received signal samples are taken into account while the more recently received signal sample provides more significant contribution in detecting the currently received signal sample.

Simulation results for the AWGN channel are given in Fig. 1 and Fig. 2 which show that similar error performances can be achieved for $L = 2W + 1$. For the application of the DF-DPD using WPR over a time-varying channel, if the varying speed is fast then a small W should be used; if the varying speed is slow then a large W may be used.

In case that the phase ψ_n is not available while the phase difference $\Delta\psi_n = \psi_n - \psi_{n-1}$ is available, we rewrite Ω_{n+1} by

$$\Omega_{n+1} = (\Delta\varphi_{n+1} - \Delta\phi_{n+1}) \mod 2\pi, \tag{26}$$

where $\Delta\varphi_{n+1} = \psi_{n+1} - \varphi_n$. The reference of phase difference can be generated by

$$\Delta\varphi_{n+1} = \begin{cases} \Delta\psi_{n+1} + \frac{W(\Delta\varphi_n - \Delta\bar{\phi}_n + 2J_n\pi)}{W+1}, & \text{for } n \geq 1, \\ \Delta\psi_1, & \text{for } n = 0, \end{cases} \tag{27}$$

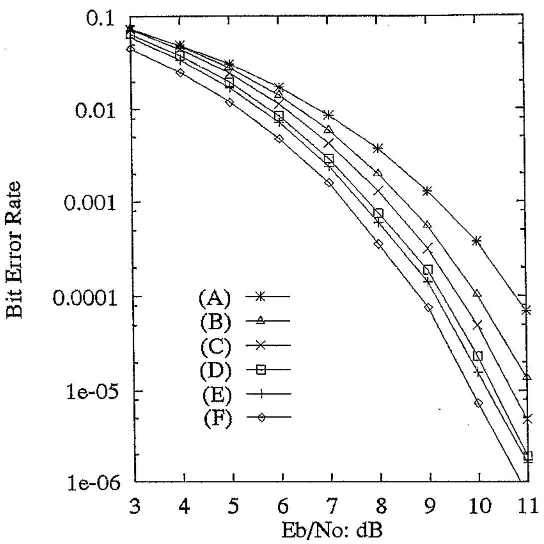

Figure 1: The error performance of DF-DPD for QPSK
(A) conventional differential decoding.
(B) DF-DPD with L=2.
(C) DF-DPD with L=3.
(D) DF-DPD with L=5.
(E) DF-DPD with L=7.
(F) coherent decoding.

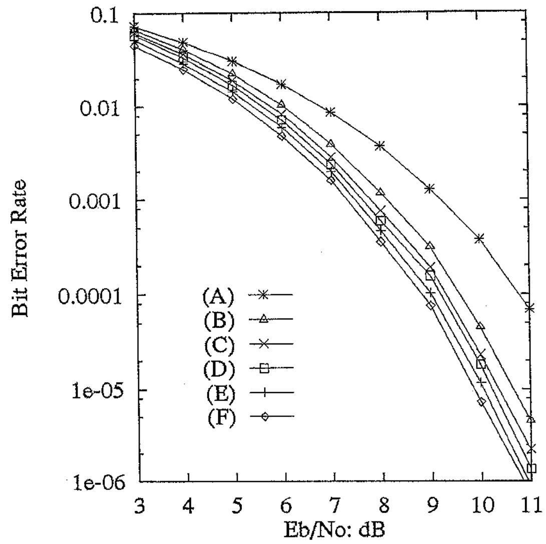

Figure 2: The error performance of DF-DPD using weighted phase references for QPSK
(A) conventional differential decoding.
(B) DF-DPD using WPR with W=1.
(C) DF-DPD using WPR with W=2.
(D) DF-DPD using WPR with W=3.
(E) DF-DPD using WPR with W=6.
(F) coherent decoding.

where J_n is an integer which is used such that

$$-\pi < (\Delta\varphi_n - \Delta\bar{\phi}_n) + 2J_n\pi \leq \pi. \tag{28}$$

The error performance of a DF-DPD using WPR with the recursive generation of $\Delta\varphi_{n+1} = \psi_{n+1} - \varphi_n$ is the same as that of a DF-DPD using WPR with the recursive generation of φ_n.

IV Modified Weighted Phase Reference

In section III, the proposed weighted phase reference is obtained by ignoring the amplitudes of received signal samples. We may slightly modify the recursive form given in (20) by including the amplitudes of recieved samples.

The modified weighted phase reference for time $n+1$ which takes the amplitude factor into account is

$$\varphi_n = \begin{cases} \frac{W\sqrt{2P}(\varphi_{n-1}+\Delta\bar{\phi}_n)+|r_n|(\psi_n+2K_n\pi)}{W\sqrt{2P}+|r_n|}, & \text{for } n \geq 1, \\ \psi_0, & \text{for } n = 0. \end{cases} \tag{29}$$

It can be shown that

$$\varphi_n = \sum_{l=1}^{n+1} a_n^{(l)}(\Psi_n^{(l)} + 2J_n^{(l)}\pi), \quad \text{for } n \geq 0, \tag{30}$$

where $J_n^{(l)}$ is an integer, $J_0^{(1)} = 0$, $a_0^{(1)} = 1$ and

$$a_n^{(l+1)} = \begin{cases} \frac{|r_{n-l}|}{W\sqrt{2P}+|r_{n-l}|} \times \Pi_{i=1}^{l} \frac{W\sqrt{2P}}{W\sqrt{2P}+|r_{n-l+i}|} & \text{for } n > l \\ \Pi_{i=1}^{l} \frac{W\sqrt{2P}}{W\sqrt{2P}+|r_{n-l+i}|} & \text{for } n = l. \end{cases} \tag{31}$$

Since each $|\, r_n \,|$ is close to $\sqrt{2P}$ for large signal to nosie natio, we can simplify the recursive relation of φ_n to be

$$\varphi_n \simeq \frac{W(\varphi_{n-1} + \Delta\bar{\phi}_n) + (|\, r_n \,| / \sqrt{2P})(\psi_n + 2K_n\pi)}{W+1}. \tag{32}$$

With the modified weighted phase reference φ_n and the decision rule of finding $\Delta\phi_{n+1}$ by minimizing $|\, \Omega_{n+1} \,|$, where Ω_{n+1} is given in (23), we have the DF-DD using weighted phase references technique. Fig. 3 shows the simulation results of DF-DD and Fig. 4 shows the simulation results of DF-DD using weighted phase references, where AWGN channel is assumed.

The recursive form for generating $\Delta\varphi_{n+1}$ which is equivalent to the recursive form for generating φ_n given in (29) is given by

$$\Delta\varphi_{n+1} = \begin{cases} \Delta\psi_{n+1} + \frac{W\sqrt{2P}(\Delta\varphi_n - \Delta\bar{\phi}_n + 2J_n\pi)}{W\sqrt{2P}+|r_n|}, & \text{for } n \geq 1, \\ \Delta\psi_1, & \text{for } n = 0, \end{cases} \tag{33}$$

where J_n is an integer which is used such that

$$-\pi < (\Delta\varphi_n - \Delta\bar{\phi}_n) + 2J_n\pi \leq \pi. \tag{34}$$

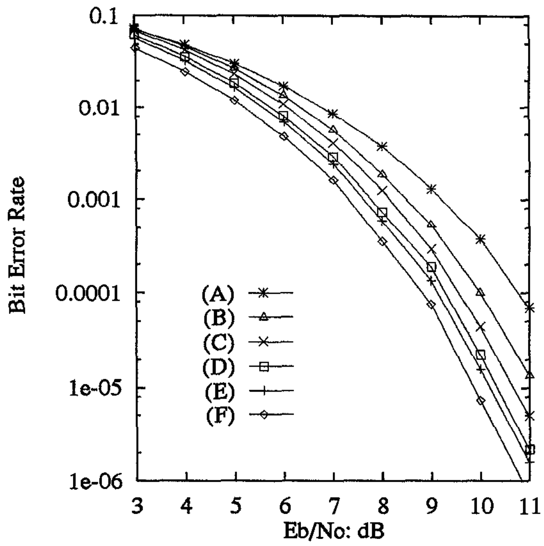

Figure 3: The error performance of DF-DD for QPSK
(A) conventional differential decoding.
(B) DF-DD with L=2.
(C) DF-DD with L=3.
(D) DF-DD with L=5.
(E) DF-DD with L=7.
(F) coherent decoding.

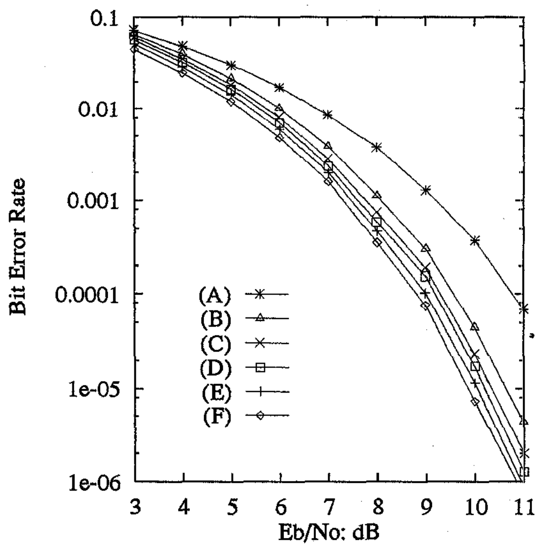

Figure 4: The error performance of DF-DD using weighted phase references for QPSK
(A) conventional differential decoding.
(B) DF-DD using WPR with W=1.
(C) DF-DD using WPR with W=2.
(D) DF-DD using WPR with W=3.
(E) DF-DD using WPR with W=6.
(F) coherent decoding.

V Conclusions

The conventional (one-symbol) differential detection for differentially encoded $MPSK$ signals has the merit of low complexity while the error performance is somewhat sacrificed as compared to the coherent detection. There were several techniques of differential detection such as DF-DD (including the simplified version, DF-DPD) which improves the error performance but appreciably increase the complexity. The new technique proposed in this paper called decision feedback differential detection using weighted phase references (including the simplified version, DF-DPD using weighted phase references) employs a simple recursive form to generate phase references for detection. The new technique can also be implemented by using a simple recursive form to generate references of phase differences for detection. In addition to the low complexity needed for detection, the proposed technique also has good error performance.

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