

## DESIGN OF ADAPTIVE ALL-PASS BASED NOTCH FILTER FOR NARROWBAND ANTI-JAMMING GPS SYSTEM

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### ABSTRACT

**The application of an adaptive all-pass based notch filter (ANFA) for the narrowband interference suppression in global positioning system (GPS) is proposed. The GPS that provides accurate positioning and timing information has become a commonly used navigation instrument for many applications. The adaptive notch filter is used to accurately estimate the narrowband jamming in real-time. It is based on all-pass filter with the Gaussian-Newton algorithm that can significantly improve the convergence rate by incorporating the so-called Hessian matrix. Simulation results show that the ANFA achieves a better performance than conventional linear predictors in terms of signal to noise ratio (SNR) improvement and mean square prediction error (MSPE) on the narrowband interference rejection in the GPS receiver.**

The discussion in this paper is focus on the anti-jamming technology named adaptive all-pass notch filter (ANFA) presented in [3]. The ANFA is a kind of filter used to suppress sine wave or narrow-band component in the input signal adaptively. Ideal fixed notch filter has a unity gain at all frequencies except at the notched point having a zero gain. When the input signal is unknown or time varying, it is necessary to adjust the notched point adaptively. The ANFA is based on all-pass filter and the Gaussian-Newton algorithm is applied to execute the anti-jamming work. This paper deal with the examination of the performance of the ANFA in the GPS system accomplished stationary or time-varying jamming both in steady state and in transient state.

The remainder of this paper is organized as follows. In section 2, the GPS received signal model and jamming signal are described. Section 3 introduces the structure and the adaptive algorithm of the ANFA. The simulation results are demonstrated in Section 4. In the last section, some conclusions are stated.

### 1. INTRODUCTION

A GPS receiver can provide accurate positioning and timing information by processing its received navigation data. GPS signals are 20-30 dB below background thermal noise. Because of its signal characteristics, the GPS inherently have anti-jamming ability. However, as jamming is higher than the system's limitation, the GPS performance can be degraded severely. Therefore, it needs additional strategy to improve the anti-jamming ability.

In recent years, the study of narrowband interference rejection has attracted great attention. There have been many different methods to achieve narrowband interference suppression in DS-SS. Vijayan and Poor proposed a new enhanced nonlinear adaptive (ENA) algorithm [1] that can outperform the existing linear and nonlinear adaptive filters. The pipelined recurrent neural network proposed by Haykin and Li [2] is one of the alternative methods used to narrowband jamming rejection.

### 2. SIGNAL MODEL

A simplified block diagram of the anti-jamming GPS model is shown in Figure 1, which is similar to [4]. The transmitted spread spectrum signal is given by

$$Z(t) = [B(t) \oplus CA(t)] \cos(2\pi f_{L1}t + \theta) \quad (1)$$

where  $B(t)$  represents the transmitted baseband navigation data that is in binary form ( $\pm 1$ ) with duration  $T$  ( $T=20$ ms).  $CA(t)$  is pseudo random number (PRN) code sequence representing GPS C/A code with chip duration  $T_c$  ( $R_c = 1/T_c = 1.023$ MHz).  $\theta$  is phase delay and  $L1$  represents primary GPS frequency (1575.42MHz). The integer  $PG = T/T_c = 20460 = 43$ (dB) is the processing gain of the GPS system.

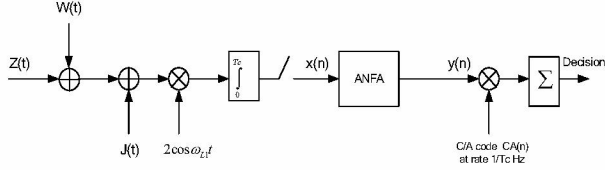


Fig.1 GPS anti-jamming system

Two kinds of jamming signals are investigated [4]:

(1) Multi-tone CWI

$$J_{multi}(t) = \sum_{i=1}^m A_i \cos[\omega_{\Delta i} n T_c + \theta_i] \quad (2)$$

where  $m$ ,  $A_i$ ,  $\omega_{\Delta i}$ , and  $\theta_i$  represent the number of narrowband jamming, amplitude, frequency offset, and random phase of the  $i$ -th interference respectively.

(2) Swept (linear FM) CWI

$$J_{swept}(k) = A \cos[-(l-1)\omega_{\Delta} n T_c + 0.5 * \xi * (n T_c)^2 + \theta]$$

$$(l-1)K \leq n < lK, l = 1, 2, 3, \dots$$

$$\xi = \Omega / K \quad (3)$$

where  $\omega_{\Delta}$  represents frequency offset.  $A$  and  $\theta$  are the amplitude and random phase of the swept CWI.  $\Omega$  is the sweep bandwidth,  $K$  is the sweep period, and  $\xi$  is the sweep rate.

At sample  $n$ , the received signal is

$$x(n) = Z(n) + J(n) + W(n) \quad (4)$$

where  $W(n)$  is additive white Gaussian noise (AWGN) with variance  $\sigma^2$ . The received signal is band-pass filtered, amplified, and down converted to baseband frequency. The components of  $x(n)$  are assumed to be mutually independent.  $W(n)$  can be modeled as band-limited and white, and the jamming source has a bandwidth much narrower than  $1/T_c$ .

### 3. ADAPTIVE ALL-PASS BASED NOTCH FILTER

Assume the order of the filter is two. The all-pass based notch filter (NFA) is

$$T(z) = \frac{1}{2}(1 + A(z)) = \frac{1}{2} \left( 1 + \frac{a_2 - a_1 z^{-1} + z^{-2}}{1 - a_1 z^{-1} + a_2 z^{-2}} \right) \quad (5)$$

where  $A(z)$  is a 2<sup>nd</sup> order all-pass filter. The coefficients are

$$a_1 = \frac{2 \cos(\omega_0)}{1 + \tan\left(\frac{BW}{2}\right)}, \quad (6)$$

$$a_2 = \frac{1 - \tan\left(\frac{BW}{2}\right)}{1 + \tan\left(\frac{BW}{2}\right)} = \rho^2 \quad (7)$$

where  $\omega_0$  is the notch frequency,  $\rho$  is the pole radius, and  $BW$  is the 3dB bandwidth. To make sure the stability,  $\rho$  should be less than unity. From (5), (6), and (7), the relationship of the pole's angle and the zero's angle can be found as

$$\cos(w') = \frac{1}{2} \left( \frac{1}{\rho} + \rho \right) \cos(\omega_0). \quad (8)$$

Similarly, a  $2p^{\text{th}}$ -order NFA is

$$T(z) = \frac{1}{2}(1 + A(z)) = \frac{1}{2} \left( 1 + \frac{\prod_{i=1}^p (\rho^2 - 2 \cos(w'_i) \rho z^{-1} + z^{-2})}{\prod_{i=1}^p (1 - 2 \cos(w'_i) \rho z^{-1} + \rho^2 z^{-2})} \right) \quad (9)$$

where  $p$  is the number of the total notch frequencies,  $\rho$  is the pole radius, and  $w'_i$  is the  $i^{\text{th}}$  pole's angle. An alternative transform function of  $T(z)$  can be rewrote as

$$T(z) = \frac{1 + r_1 z^{-1} + \dots + r_p z^{-p} + \dots + r_1 z^{-2p+1} + z^{-2p}}{1 + \rho \beta_1 z^{-1} + \dots + \rho^p \beta_p z^{-p} + \dots + \rho^{2p-1} \beta_1 z^{-2p+1} + \rho^{2p} z^{-2p}},$$

$$\beta_i = \frac{1 + \rho^{2p}}{\rho^i + \rho^{2p-i}} r_i, \quad i = 1 \dots p. \quad (10)$$

The adaptive algorithm used here is so-called Gauss-Newton algorithm, and there are some modifications to increase its convergence speed [3]. Besides, it needs stability check during each iteration. Summarize the adaptive algorithm as follows:

Main Loop:

$$\bar{y}(n) = x(n) + x(n-2p) - \rho^{2p}(n) \bar{y}(n-2p) - \varphi^T(n) \hat{\theta}(n) \quad (11)$$

$$\varphi_i(n) = \begin{cases} -x(n-i) - x(n-2p+i) \\ + \left[ \rho^j(n) \bar{y}(n-i) + \rho^{2p-i}(n) \bar{y}(n-2p+i) \right] \frac{1 + \rho^{2p}}{\rho^i + \rho^{2p-i}} \\ , 1 \leq i \leq p-1 \\ -x(n-p) + \rho^p(n) \bar{y}(n-p) \frac{1 + \rho^{2p}}{2\rho^p}, i = p \end{cases} \quad (12)$$

$$x_F(n) = x(n) - \rho^{2p}(n) x_F(n-2p) - \rho^p(n) x_F(n-p) \hat{r}_p(n) \frac{1 + \rho^{2p}}{2\rho^p}$$

$$- \sum_{i=1}^{p-1} \left[ \rho^j(n) x_F(n-i) + \rho^{2p-i}(n) x_F(n-2p+i) \right] \hat{r}_i(n) \frac{1 + \rho^{2p}}{\rho^i + \rho^{2p-i}} \quad (13)$$

$$\bar{y}_F(n) = \bar{y}(n) - \rho^{2p}(n)\bar{y}_F(n-2p) - \rho^p(n)\bar{y}_F(n-p)\hat{r}_p(n)\frac{1+\rho^{2p}}{2\rho^p} - \sum_{i=1}^{p-1} [\rho^i(n)\bar{y}_F(n-i) + \rho^{2p-i}(n)\bar{y}_F(n-2p+i)]\hat{r}_i(n)\frac{1+\rho^{2p}}{\rho^i + \rho^{2p-i}} \quad (14)$$

$$\psi_i(n) = \begin{cases} -x_F(n-i) - x_F(n-2p+i) \\ + [\rho^i(n)\bar{y}_F(n-i) + \rho^{2p-i}(n)\bar{y}_F(n-2p+i)]\frac{1+\rho^{2p}}{\rho^i + \rho^{2p-i}} \\ , 1 \leq i \leq p-1 \\ -x_F(n-p) + \rho^p(n)\bar{y}_F(n-p)\frac{1+\rho^{2p}}{2\rho^p}, i = p \end{cases} \quad (15)$$

$$P(n+1) = \frac{P(n) - \frac{P(n)\psi(n)\psi^T(n)P(n)}{\lambda(n) / \alpha(n) + \psi^T(n)P(n)\psi(n)}}{\lambda(n)} \quad (16)$$

$$\tau(n) = \psi^T(n)P(n)\psi(n) \quad (17)$$

$$\alpha_{opt}(n) = \frac{2}{1 + \sqrt{2 \cdot \tau(n) - 1}} \quad (18)$$

$$y(n) = x(n) + x(n-2p) - \rho^{2p}(n)\bar{y}(n-2p) - \varphi^T(n)\hat{\theta}(n-1) \quad (19)$$

$$\hat{\theta}(n+1) = \hat{\theta}(n) + \alpha(n)P(n+1)\psi(n)y(n) \quad (20)$$

$$\lambda(n+1) = \lambda_0\lambda(n) + (1 - \lambda_0) \quad (21)$$

$$\rho(n+1) = \rho_0\rho(n) + (1 - \rho_0)\rho(\infty) \quad (22)$$

$$\varphi(n+1) = [\varphi_1(n+1) \quad \varphi_2(n+1) \dots \varphi_p(n+1)]^T \quad (23)$$

$$\psi(n+1) = [\psi_1(n+1) \quad \psi_2(n+1) \dots \psi_p(n+1)]^T \quad (24)$$

where  $p$  is the total number of notch frequencies,  $x(n)$  is the filter input, and  $y(n)$  is the filter broadband output. The filter coefficient vector is defined as  $\theta(n) = [r_1(n) \dots r_p(n)]^T$ . The gradient of the output is

$$\psi(n) = \left[ -\frac{\partial y(n)}{\partial r_1(n)} \dots -\frac{\partial y(n)}{\partial r_p(n)} \right] = \frac{\varphi(n)}{N(\rho, z^{-1}, n)}$$

where  $N(\rho, z^{-1}, n)$  is the denominator of the transfer function in (11).  $P(n)$  is the inverse of the covariance matrix (Hessian matrix). Filtered input and output are defined as  $x_F(n) = \frac{x(n)}{N(\rho, z^{-1}, n)}$ ,  $y_F(n) = \frac{y(n)}{N(\rho, z^{-1}, n)}$ .

$\lambda(n)$  is the forgetting factor. At the beginning,  $\hat{\theta}(0) = 0$ ,  $\psi(0) = 0$ ,  $x(-i) = 0$  where  $i = 1, \dots, 2p$ , and  $P(0) = \sigma \cdot I_p$  where  $\sigma \cong 100 / E[x(n)^2]$ .

#### 4. SIMULATION RESULTS

The indexes used to verify the performance in transient state and steady state are mean squared prediction error (MSPE) and SNR improvement respectively [4], which are:

$$Y_{MSPE} = \log \left\{ \frac{1}{100} \left[ \sum_{i=(n-1) \cdot 100 + 1}^{n \cdot 100} y(i)^2 \right] \right\} \quad (25)$$

where  $y(i)$  is the adaptive filter output.

$$SNR_{output} = 10 \log \left[ \frac{E|S(n)|^2}{E|x(n) - S(n)|^2} \right] (dB) \quad (26)$$

$$SNR_{output} = 10 \log \left[ \frac{E|S(n)|^2}{E|y(n) - S(n)|^2} \right] (dB) \quad (27)$$

$$SNR_{imp} = 10 \log \left[ \frac{E|x(n) - S(n)|^2}{E|y(n) - S(n)|^2} \right] (dB) \quad (28)$$

where  $y(n)$  is the adaptive filter output,  $x(n)$  is the filter input, and  $S(n)$  represents the baseband GPS spreading signal, i.e.  $B(n) \oplus CA(n)$ .

In the simulation, the linear tapped delay line predictor with normalized least mean square (NLMS) and recursive least squares (RLS) adaptive algorithm are compared with the ANFA. The tap numbers of the predictors are 10. The step size of the NLMS is 0.01. The initialed diagonal elements of error covariance and the forgetting factor of the RLS are 0.01 and 1.0 respectively. For the ANFA, the forgetting factor  $\lambda$  is fixed at 0.8. The pole radius  $\rho$  is held constant at 0.84 in the multi-tone CWI case, and 0.89 in the swept CWI test. The order of the ANFA is set at  $2p$  where  $p$  is the number of the narrowband interferences. In the other settings, the down-conversion IF is 1.25Mz, sampling rate is 5MHz, the variance of the AWGN is held constant at  $\sigma^2 = 0.01$ . The total jamming power is 73 dB higher than the baseband GPS spreading signal  $S(n)$ . In the MSPE simulation, the first 1600 sampling points are adopted. In the steady state performance simulation, there are 9500 sampling points that is received, and use the last 1000 points to compute the SNR improvement.

The Multi-tone test results are presented in Figure 2 and Figure3. The multi-tone CWI offset frequencies are 0.5 MHz, 1.3 MHz, and 1.8 MHz. The figures show the ANFA converge faster than the NLMS and the RLS. It also offers the most SNR improvement than the other two algorithms in steady state. On averages, the ANFA can offers 3.20 dB and 8.94 dB more in terms of SNR improvements than the RLS and the NLMS respectively.

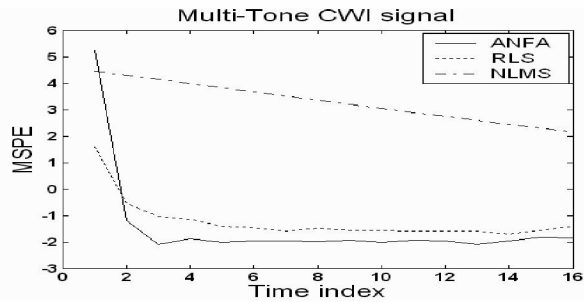


Fig. 2 MSPE for multi-tone jamming

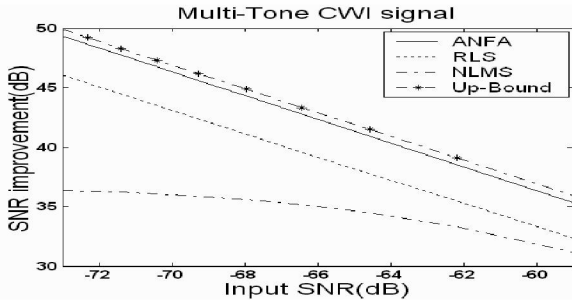


Fig. 3 SNR improvement for multi-tone jamming

In Figure 4 and 5, the simulation results of the swept jamming case are presented. The relative sweep rate is set to 526 MHz/sec. The starting frequency is 0.75MHz and the ending frequency is 1.75 MHz. The ANFA converge faster than the NLMS and the RLS, and in the steady state, it has the best SNR improvement than the other two algorithms. It can offer average 19.19 dB and 29.03 dB more SNR improvement than the NLMS and RLS respectively.

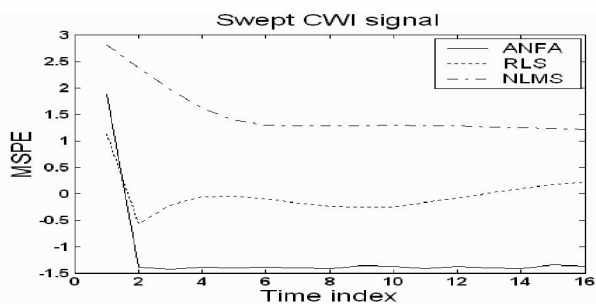


Fig. 4 MSPE for swept jamming

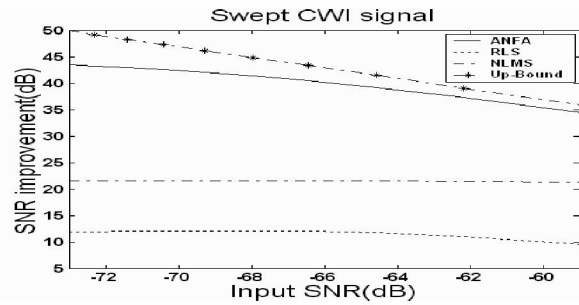


Fig. 5 SNR improvement for swept jamming

## 5. CONCLUSION

This paper presents an application of the adaptive all-pass based notch filter for the GPS interference cancellation with the performance comparison via MSPE and SNR improvement. In the simulations, there are two kind of high power narrowband jamming, multi-tone CWI, swept CWI, as the jamming sources, and the results indicate that the ANFA can achieve better SNR improvement and faster estimate the narrowband interferences than either the NLMS or RLS.

## 6. ACKNOWLEDGEMENT

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## 7. REFERENCES

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