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Data Fusion of Three Attitude Sensors

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Abstract: A reliable attitude determination system, which integrate the measurements coming from the star tracker (STR), Global Positioning System (GPS) and gyros, is presented. The linearized state propagation and state update are two key algorithms in the data fusion. For state propagation, the primary information is provided by the high rate (16 Hz) sensors (gyros), whose measurement offsets are calibrated by the integrated filter. For state update, the primary information is provide by the low rate (1 Hz) sensors (STR and GPS). To minimize the update errors, the covariance matrices associated to different sensors are selected as the weightings. From the simulation results, our ideas are verified to be sound and effective.

Keywords: Data Fusion, Star Tracker, Global Positioning System

1 Introduction

To meet the requirements of the attitude determination for spacecrafts, many sensors such as STRs, horizon scanners, sun sensors, magnetometers, or gyros have been used. Among those sensors, STRs and gyros can be integrated with onboard systems to supply angular displacement. Gyros provide continuous and nearly instantaneous information of angular velocities. STRs provide precise attitude information and allow for estimation without prior information. However, the consideration of bandwidth limitation, requirement of initial acquisition and the problem of "loss in space", the STRs have large field of view [1] and supported by complicate software with expensive cost to perform tracking and identification. The advent of GPS, whose carrier phase measurements have the resolution in the level of centimeters, plays the role of a new attitude sensor. The data fusion of STR-GPS-gyro has the advantage that the requirement on the field of view of STR may be less stringent. Hence, the cost can be reduced. Moreover, this fusion can improve the system sensitivity and calibrate the drift errors of gyros simultaneously.

For fusing the system outputs of STRs, GPS and gyros, a pair of Kalman filters is needed as shown in Figure 1. The state propagation is based on the gyro measurements. The perturbation, caused by state errors due to system nonlinearity, is predicted by gyro measurements and updated by fusing STR and GPS measurements. The error covariance matrices of the two sensors, according to the fusing algorithm, are used to determine the weightings during the update procedure. The cross correlation can be applied in the data association since the two estimations originated from the same

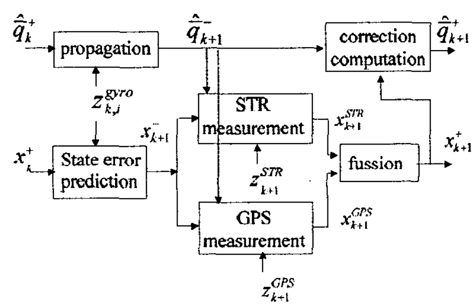

Figure 1 Fusion architecture for STR-GPS-gyro measurements

To verify our ideas, the software simulations are performed to assess the performance of the proposed method. Three GPS receivers, three gyros and one STR are considered as attitude sensors of a low-earth-satellite. By integrating the angular velocity measurements from gyros and the updated attitude from STR and GPS measurements, the proposed fusion scheme is sound and effective as shown by simulation results.

2 Models

2.1 Star Tracker Models

Star trackers measure star coordinates in the star tracker frame and provide attitude information by comparing these observed coordinates with known star directions from a star catalog. From observing at least two remote stars, the star tracker can provide full three-axis attitude determination. Let $\begin{bmatrix} x_m & y_m & z_m \end{bmatrix}^T$ denote the measured star direction vector in the star tracker frame, and $\begin{bmatrix} x_{inert} & y_{inert} & z_{inert} \end{bmatrix}^T$ denote the listed star direction vector from the star catalog in the inertial frame. Their relation can be expressed as

$$\begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix} = \overset{STR\Leftarrow inert}{A} \begin{bmatrix} x_{inert} \\ y_{inert} \\ z_{inert} \end{bmatrix} \tag{1}$$

where $\overset{STR\Leftarrow inert}{A}$ denotes the attitude matrix from inertial reference frame to the star tracker frame. In the filtering methods, the measurements are sometimes necessary to be compared with the predictions to form the so-called innovation process. Let $\hat{A}$ denote the estimated attitude matrix for $\overset{STR\Leftarrow inert}{A}$. The predicted star direction vector can be expressed as

$$\begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} = \hat{A} \begin{bmatrix} x_{inert} \\ y_{inert} \\ z_{inert} \end{bmatrix}. \tag{2}$$

The innovations shall be computed in later discussions after the kinematics of the attitude matrix being introduced.

2.2 GPS Models

With three antennas on board, the GPS receiver may be used to determine the attitude of the spacecraft. Due to the accuracy requirement, it is necessary to adopt the GPS carrier phase observables. However, the problem of integer ambiguity may occur. In the following discussion, it is assumed that the problem of integer ambiguities have been resolved. After compensating the ambiguity values, the double difference carrier phase observables become ambiguity-free. The ambiguity-free double difference, $\psi_i^j(t_k)$, is the inner product of the i-th baseline vector and the difference of the direction vectors to two GPS satellites (the $j+1$-th and the 1st) at time t_k. Let A be the attitude matrix which transforms the coordinates of vectors in the inertial reference frame to those in the spacecraft frame. Including the measurement noises, we have

$$\psi_i^j(t_k) = \mathbf{a}_i^T A \mathbf{s}^j + \omega_i^j, \tag{3}$$

where the vector $\mathbf{a}_i$ is the i-th baseline vector represented in the spacecraft frame; $\mathbf{s}^j$ denotes the difference between the direction vectors of satellite $j+1$ and satellite 1 , expressed in the inertial reference frame, and the error vector ω_i^j is assumed to be white with distribution $\mathbb{N}(0, 0.01\lambda)$.

Through the estimation process, the estimated attitude may be computed as $\hat{A}$, and the predicted difference vector can be found as

$$\mathbf{s}^p = \hat{A}\mathbf{s}^j, \tag{4}$$

where $\mathbf{s}^p$ denotes the difference between predicted directional vector satellite $j+1$ and satellite 1 expressed in the spacecraft frame. The innovations can be then obtained, which will be discussed later.

2.3 Gyro models

The gyro output vector $\mathbf{u}$ is related to the angular velocity ω through the following equation [3]

$$\mathbf{u} = \omega + \mathbf{b} + \eta_1, \tag{5}$$

where $\mathbf{b}(= \mathbf{b}_0 + \mathbf{b}_w)$ denote the gyro drift bias vector, with constant bias $\mathbf{b}_0$ and a random walk $\mathbf{b}_w$, and η_1 represents zero mean white Gaussian noise with strength $\mathbf{Q}_1$. The drift bias is assumed to satisfy

$$\frac{d}{dt}\mathbf{b} = \eta_2, \tag{6}$$

where η_2 is another zero mean white Gaussian noise with strength $\mathbf{Q}_2$. The two noise processes are assumed to be uncorrelated

$$E[\eta_1 \eta_2^T] = 0. \tag{7}$$

The components of the drift vector shall be included in the filter model as state variables. Through the estimation process, we may then estimate the drift vector and denote it by $\hat{\mathbf{b}}$. The estimated angular velocity $\hat{\omega}$ may be then obtained by taking the expectation of (5), as

$$\hat{\omega} = \mathbf{u} - \hat{\mathbf{b}}, \tag{8}$$

3 Linearized Kalman Filter

3.1 Attitude Kinematics

Let $\mathbf{A}$ be an attitude matrix, represented by the quaternion $\overline{\mathbf{q}}$ defined as

$$\overline{\mathbf{q}} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \left[\begin{bmatrix} \mathbf{L}\sin(\theta/2) \\ \cos(\theta/2) \end{bmatrix}\right] = \begin{bmatrix} \mathbf{q} \\ q_4 \end{bmatrix}. \tag{9}$$

where $\mathbf{L}$ is a unit vector represents the axis of rotation; θ is the angle of rotation about the axis $\mathbf{L}$. It is obvious that the norm of $\mathbf{q}$ and $\overline{\mathbf{q}}$ are $\left|\sin(\frac{\theta}{2})\right|$ and 1, respectively.

According to the relation

$$\mathbf{A}(\overline{\mathbf{q}}) = (|q_4|^2 - |\mathbf{q}|^2)\mathbf{I} + 2\mathbf{q}\mathbf{q}^T + 2q_4[|\mathbf{q}|], \tag{10}$$

where $\mathbf{I}$ is the 3×3 identity matrix and

$$[|\mathbf{q}|] = \begin{bmatrix} 0 & q_3 & -q_2 \\ -q_3 & 0 & q_1 \\ q_2 & -q_1 & 0 \end{bmatrix}, \tag{11}$$

$$\overline{\mathbf{q}}^T\overline{\mathbf{q}} = 1. \tag{12}$$

The kinematic relation between the attitude quaternion and the angular velocity is

$$\frac{d}{dt}\overline{\mathbf{q}}(t) = \frac{1}{2}\mathbf{\Omega}(\omega(t))\overline{\mathbf{q}}(t), \tag{13}$$

where

$$\mathbf{\Omega}(\omega) = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix}, \tag{14}$$

and $\omega = [\ \omega_1 \quad \omega_2 \quad \omega_3\ ]$ denotes the angular velocity of the body.

To obtain the discretized model, we consider the time interval t_k to t_{k+1}. In such a short period, $\hat{\omega} = \mathbf{u}(t) - \hat{\mathbf{b}}(t)$ can be taken as a constant. Therefore, the expectation of equation (13) can be integrated directly from t_k to t_{k+1}:

$$\hat{\overline{\mathbf{q}}}_{k+1} = \mathbf{\Theta}_k\hat{\overline{\mathbf{q}}}_k, \tag{15}$$

where

$$\mathbf{\Theta}_k = exp(\frac{1}{2}\int_{t_k}^{t_{k+1}} \mathbf{\Omega}(\mathbf{u} - \hat{\mathbf{b}})d\tau). \tag{16}$$

3.2 Dynamic Equations

In stead of the $\overline{\mathbf{q}}(t)$ itself, the aim of our filter is to track its errors. Let define

$$[\Delta\overline{\mathbf{q}}] = \left[\overline{\mathbf{q}} \otimes \hat{\overline{\mathbf{q}}}^{-1}\right], \tag{17}$$

where $\Delta\overline{\mathbf{q}}$ is the quaternion error between the true attitude and the estimated attitude.

The advantage of the quaternion error representation is that the fourth component will be close to unity since the incremental quaternion corresponding to a small angle rotation. Thus the attitude information of interest is contained in the three tuple vector $\Delta\mathbf{q}$, where

$$\Delta\overline{\mathbf{q}} = \begin{bmatrix} \Delta\mathbf{q} \\ \sqrt{1 - |\Delta\mathbf{q}|} \end{bmatrix} \cong \begin{bmatrix} \Delta\mathbf{q} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\theta_x \\ \frac{1}{2}\theta_y \\ \frac{1}{2}\theta_z \\ 1 \end{bmatrix} \tag{18}$$

The reduced representation of state error vector can be defined by

$$\mathbf{x} = \begin{bmatrix} \theta \\ \mathbf{d} \end{bmatrix}. \tag{19}$$

where $\mathbf{d} = \hat{\mathbf{b}} - \mathbf{b}$ is the drift bias error. The dynamic equations associated to the reduced state error representation will be

$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{F}\mathbf{x}(t) + \mathbf{G}\mathbf{w}(t), \tag{20}$$

where

$$\mathbf{F} \equiv \begin{bmatrix} [|\mathbf{u} - \hat{\mathbf{b}}|] & \mathbf{I}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \end{bmatrix}, \mathbf{G} \equiv \begin{bmatrix} -\mathbf{I}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} \end{bmatrix}, \tag{21}$$

and $\mathbf{w}(t) = [\ \eta_1(t) \quad -\eta_2(t)\ ]^T$.

4 Measurement Equations

4.1 STR Observations

Assuming that there are one star tracker which provides the attitude measurement directly, the quaternion form of the measurement equations can be written as

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 0 \end{bmatrix} = \overline{\mathbf{q}} \otimes \begin{bmatrix} x_{inert} \\ y_{inert} \\ z_{inert} \\ 0 \end{bmatrix} \otimes \overline{\mathbf{q}}^* \tag{22}$$

where $\overline{\mathbf{q}}^* = [\ -\mathbf{q}^T \quad q_4\ ]^T$ is the conjugate of quaternion $\overline{\mathbf{q}}$, which is equal to its quaternion inverse $\overline{\mathbf{q}}^{-1}$. On the other hand, the STR measurement vector can be expressed as

$$\begin{bmatrix} x_m \\ y_m \\ z_m \\ 0 \end{bmatrix} = \Delta\overline{\mathbf{q}} \otimes \hat{\overline{\mathbf{q}}} \otimes \begin{bmatrix} x_{inert} \\ y_{inert} \\ z_{inert} \\ 0 \end{bmatrix} \otimes \hat{\overline{\mathbf{q}}}^* \otimes \Delta\overline{\mathbf{q}}^* \tag{23}$$

where $[\ x_m \quad y_m \quad z_m\ ]^T$ denotes the measurement star vector in the STR reference frame.

The measurement equation can be shown as

$$\mathbf{z} = \begin{bmatrix} X_m - RX_p \\ Y_m - RY_p \end{bmatrix} \tag{24}$$

$$= \begin{bmatrix} 0 & -(1+R) & (Y_m + RY_p) & \mathbf{0}_{1\times4} \\ (1+R) & 0 & -(X_m + RX_p) & \mathbf{0}_{1\times4} \end{bmatrix}\mathbf{x},$$

where $X_m = \frac{x_m}{z_m}$, $Y_m = \frac{y_m}{z_m}$, $X_p = \frac{x_p}{z_p}$, $Y_p = \frac{y_p}{z_p}$ and

$$R = \sqrt{\frac{X_m^2 + Y_m^2 + 1}{X_p^2 + Y_p^2 + 1}}. \tag{25}$$

4.2 GPS Observations

The quaternion form of the predicted difference of satellite direction vector can be written as

$$\begin{bmatrix} s_x^p \\ s_y^p \\ s_z^p \\ 0 \end{bmatrix} = \hat{\overline{\mathbf{q}}} \otimes \begin{bmatrix} s_x^j \\ s_y^j \\ s_z^j \\ 0 \end{bmatrix} \otimes \hat{\overline{\mathbf{q}}}^* \quad (26)$$

where $[\ s_x^p \quad s_y^p \quad s_z^p\ ]^T$ denotes the predicted difference of satellite direction vectors in the spacecraft frame and $[\ s_x^j \quad s_y^j \quad s_z^j\ ]^T$ denotes the difference of satellite direction vectors in the reference frame. The difference of measurement satellite direction vectors in the spacecraft frame can be expressed as

$$\begin{bmatrix} s_x \\ s_y \\ s_z \\ 0 \end{bmatrix} = \Delta\overline{\mathbf{q}} \otimes \hat{\overline{\mathbf{q}}} \otimes \begin{bmatrix} s_x^j \\ s_y^j \\ s_z^j \\ 0 \end{bmatrix} \otimes \hat{\overline{\mathbf{q}}}^* \otimes \Delta\overline{\mathbf{q}}^*. \quad (27)$$

The measurement equation can be shown as

$$\psi_i^j(t_k) - a_i^T A(\hat{\overline{\mathbf{q}}}) s^j = [\ -\mathbf{a}^T[|s^p|] \quad \mathbf{0}_{1\times 4}\ ]\mathbf{x}, \quad (28)$$

4.3 Fusion Processes

It is assumed that the star tracker frame and the spacecraft frame are the same.

Applying Kalman filtering method, the local information systems of STR-gyro process and GPS-gyro process estimate the same target. the fusion problem is to decide how to combine the corresponding state estimates. Figure 1 shows the fusion architecture to obtain the optimal estimation of state.

Let $\hat{\mathbf{x}}^{STR}$ be the error state estimate by STR. Assume that for the same time one has an error state estimate $\hat{\mathbf{x}}^{GPS}$ of a target from GPS, the fused estimate is expressed as [15]

$$\hat{\mathbf{x}}^{Fusion} = \mathbf{P}^{STR}(\mathbf{P}^{GPS} + \mathbf{P}^{STR})^{-1}\hat{\mathbf{x}}^{GPS} + \mathbf{P}^{GPS}(\mathbf{P}^{GPS} + \mathbf{P}^{STR})^{-1}\hat{\mathbf{x}}^{STR}, \quad (29)$$

where $\mathbf{P}^{STR}$ and $\mathbf{P}^{GPS}$ are the error covariances corresponding to $\hat{\mathbf{x}}^{STR}$ and $\hat{\mathbf{x}}^{GPS}$, respectively.

5 Simulations

To verify the filtering algorithm for attitude determination, several simulations are performed to assess the performance of our method. In this Simulations, the initial quaternion attitude representation is assumed to be $\overline{\mathbf{q}}_{init} = [\ 0.0756 \quad -0.9966 \quad 0.0034 \quad -0.0328\ ]^T$.

Assume that there are two stars in sight, which directions are fixed in the STR reference frame. The default measured stars directions are $S_1 = [\ 0.1107 \quad 0.1107 \quad 0.9877\ ]^T$ and $S_2 = [\ -0.1107 \quad -0.1107 \quad 0.9877\ ]^T$ in the STR reference frame. This corresponds to an angular separation between the two stars about FoV/2, where FoV is the STR field of view.

The main assumptions of the taken into account for the attitude and drift errors are the following:

(1)Angular velocity: $\omega = [\ 0 \quad 0 \quad 10^{-3}\ ]^T$ rad/sec.

(2)GPS output data :1 Hz
- baseline: 1m
- carrier phase noise: $\lambda/100$=0.19cm.

(3)STR output data: 1 Hz
- constant bias not taken into account.
- noise (1σ): 9 arcsec on axis_x and axis_y. 18 arcsec on axis_z.

(4)Gyro output data: - data rate: 16 Hz.
- rate flicker error B: 0.002 deg/hr.
- rate white noise N: 0.0006deg/sqrt(hr).
- angle white noise Φ: 0.019"/sqrt(Hz).

Figure 2 and Figure 3 show the attitude errors of using STR-gyro estimator and GPS-gyro estimator, respectively. Figure 4 shows the fusion results by combining STR-GPS-gyro estimator. Its well known that the accuracy of GPS attitude sensors heavily depends on the length of its baseline. The baseline is defined by the separated antennae of GPS receivers. The accuracy of long baseline configuration be better than that of short configuration. The simulations of Figure 3 and Figure 4 are associated to the GPS sensors whose baselines are 1 m. If the length of baseline is extended to 100 m, the accuracy of GPS sensor will be compatible to STR. The associated fusion results will be improved of course, the simulated errors shown in Figure 5.

6 Conclusion

STR, GPS and gyro measurements are fused to determine the attitude of spacecraft. Thanks to gyros, the propagation can be performed in a high rate. Thanks to STR and GPS, the state update will be accurate in a lower rate. For small size spacecraft, the accuracy of GPS sensors are limited due to the length of baseline. However, the reliability of the attitude system is improved if 3 sensors (instead of 2) are used. For large

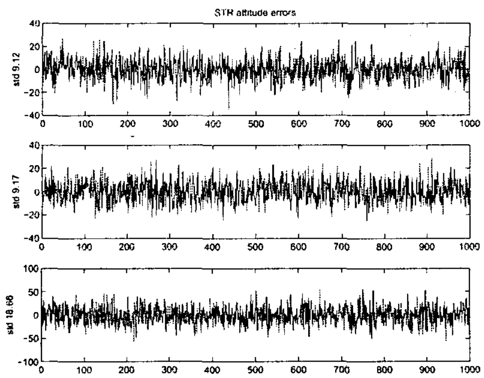

Figure 2: Attitude errors of using STR-gyro estimator

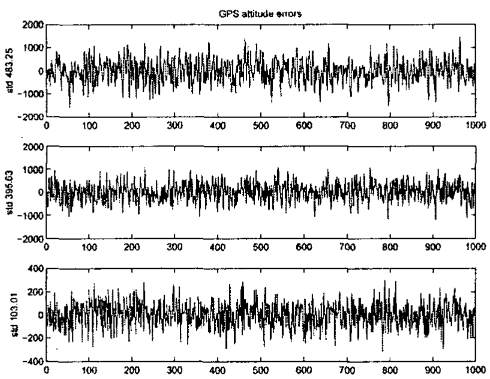

Figure 3: Attitude errors of using GPS (1 m baseline) -gyro estimator

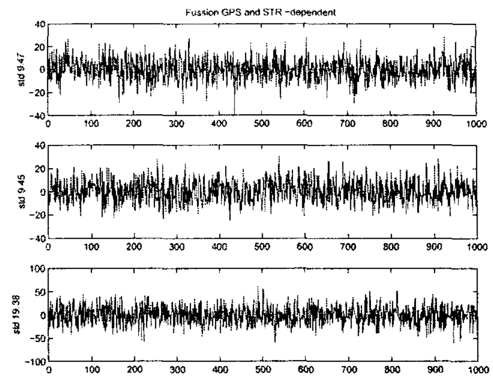

Figure 4: Attitude errors of fusion estimator of STR-GPS (1 m baseline)-gyro

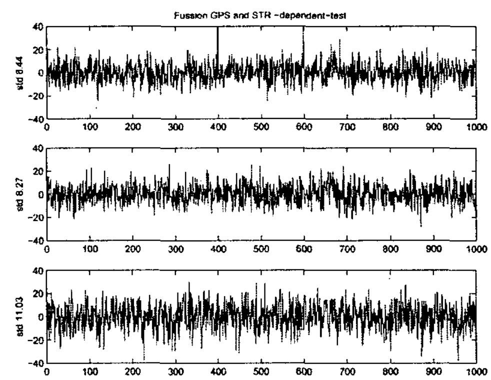

Figure 5: Attitude errors of fusion estimator of STR-GPS (100 m baseline)-gyro

size spacecrafts, such as space stations, the performance of GPS sensors can be upgraded to the level similar to STR. In this case, the 3 sensors attitude system can improve not only reliability but also accuracy.

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