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Design of IIR Eigenfilters with Linear Constraints

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中文摘要

此研究計劃針對使用埃根逼近法來設計 IIR 濾波器，傳統上只要考慮濾波器是否有符合頻率響應的規格，然而，在某些通訊應用下我們還需要加入特殊時域及頻域線性限制，例如要設計通訊系統用的 Nyquist filter ，在某個特定頻率需要較高的精確度，如凹谷道型濾波器等等，埃根逼近法是個很有效加入線性限制的設計方 法。

關鍵詞： IIR 埃根濾波器、FIR 埃根濾波器、時域及頻域線性限制

Abstract

An effective eigenfilter approach to design causal stable IIR filters with time and frequency domain constraints is presented. By minimizing a quadratic measure of the error in the passband and stopband, an eigenvector of an appropriate real symmetric and positive-definite matrix is computed to get the filter coefficients. Several IIR filters such as notch filters, Nyquist filters and partial response filters can be easily designed by this approach. Some numerical design examples are illustrated to show the effectiveness of this approach.

Keywords： IIR Eigenfilters 、 FIR Eigenfilters 、 Time and Frequency Domain Linear Constraints

I. Introduction

The eigenfilter approach has been recently used to effecitvely design linear phase FIR digital filters [1], FIR Hilbert transformers [2], and digital differentiators [3]. Also this approach has been applied to design complex FIR filters with arbitrary complex frequency response [4][5]. The design of 1-D and 2-D IIR eigenfilters in time domain has been studied in [6]. In this approach, we compute the filter coefficients by approximating an ideal desired impulse response. Recently, the eigenfilter approach is extended to design stable causal IIR filters in frequency domain with an arbitrary number of zeros and poles [7][8]. These methods work out in the frequency domain and allow to design filters with arbitrary prescribed magnitude frequency response.

Eigenfilter approach consists of expressing the error function between a desired target and actual filter response as a real, symmetric, positive-definite quadratic eigen_form in the filter coefficients. The eigenvector corresponding to the minimum eigenvalue minimizes the weighted error function and is the filter solution we need. The eigenfilter approach is widely used for its simplicity and easy implementation. Here we present another advantage of this approach in which adding the time domain or frequency domain constraints is quite flexible and easy. In this paper, the eigenfilter approach is used to design IIR notch filters, Nyquist filters, and partial response filters which have special frequency domain or time domain constraints.

In many applications of signal processing, it is necessary to eliminate narrowband or sinusoidal disturbance while leaving the broardband signal unchanged. Usually this work can be done by the notch filters characterized by a unit gain over the whole frequency domain except at some certain frequencies in which their gain are zero. So far, several methods to

effectively design IIR or FIR notch filters have been developed [9]. Adaptive notch filter design has been studied, too [10][11]. When the frequencies of narrowband interferences are known in advance, fixed notch filters can be used. In section 2, we study the properties of notch filters and formulate the design algorithm by adding the frequency-domain constraints to the eigenfilter approach. In section 3, we present the effectiveness of this method by showing some examples with arbitrarily chosen frequencies of notches.

Nyquist filters play an important role in digital data transmission for its intersymbol interference (ISI)-free property. Also they can be adopted in decimation or interpolation multirate systems. To achieve zero ISI, Nyquist filters must satisfy some criteria in time domain that they should have zeros equally spaced in the impulse response coefficients except one specified. There are two conventional methods, which are IIR and FIR filter forms, to implement Nyquist filters. Designing of FIR Nyquist filters are more straightforward than those of IIR filters because FIR filters have exact linear phase characteristic, are always stable and the filter coefficients are directly its impulse response. So far the design procedures on FIR Nyquist filters have been widely studied [12][13]. However, FIR Nyquist filters usually require higher filter order to meet the magnitude specification. In contrast to FIR filters, IIR filters have lower orders, but their impulse responses are more difficult to keep the zero-crossing time constrained property and the problem of filter stability should also be carefully concerned.

Recently, Nakayama and Mizukami have proposed a novel expression of transfer functions for IIR Nyquist filters that we can keep exact zero intersymbol interference [14]. They have used the iterative Chebyshev approximation procedure but without considering its filter stability [15]. After adopting the proposed transfer function, the frequency response of the filters can be optimized without taking care of its time domain constraints (zero-crossing property). In this paper, we study this transfer function of Nyquist filters and present a new process based on eigenfilter method to design a Nyquist filter satisfying the arbitrarily selected time and frequency criteria. With some adequate constraints added, we can solve the optimal solution by the eigen_method to get the equal-ripple, low-pass stable IIR Nyquist filter with well behaved group delay. Then we summarize the design procedures and conclude some remarks worthy to be noted. Some design examples to demonstrate the validity of the proposed method are also illustrated.

The eigenfilter approach can also be applied to design the partial response filters which have similiar zero-crossings as Nyquist filters but sinusoidal shape magnitude response over the passband [16]. Partial response filters have received considerable attention since they can be employed at an increased bit rate under a prescribed available bandwidth for data transmission [16][17]. They play an important role in binary data communication for that the intersymbol interference can be reduced by introducing rolloff around the Nyquist frequency. These filters can be classified for the different time domain or frequency domain responses. We introduce the filters in section 2.3 and formulate the design algorithm in this paper. The designing procedures of class 1 to class 5 partial response filters are mainly based on Nyquist filter design for we can take one partial response filter as the multiplication of one Nyquist filter and a polynomial which shapes the filter. In the section 3, some satisfactory experiment examples are presented.

II. Problem Formulation and Design Procedures

A. Notch Filter

Assume the transfer function of a notch filter is defined by

$$H(z) = \frac{N(z)}{D(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_M z^{-M}} \tag{1}$$

If we need this filter to have a notch at frequency μ_i, i.e. the ideal desired frequency response is unity everywhere in the frequency domain except $\mu = \mu_i$. The transfer function (1) should have a pair of zeroes at $z = e^{\pm j\mu_i}$, that is,

$$(1 - 2 \cdot \cos(\mu_i) \cdot z^{-1} + z^{-2}) \mid N(z) \cdot \qquad (2)$$

Eq.2 means that if we need *K* different notch positions in the frequency domain, the order of numerator polynomial should be at least *2K*.

When the positions of the notches of the desired filter are given, from (1) and (2), the transfer function of this filter can be written as

$$H(z) = \frac{N(z)}{D(z)} = \frac{b_0 \cdot C(z)}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{2K} z^{-2K}}, \qquad (3)$$

where

$$C(z) = \prod_{i=1}^{K} (1 - 2 \cdot \cos(\mu_i) \cdot z^{-1} + z^{-2}). \qquad (4)$$

Clearly, $C(z)$ is known when we have chosen the position of the notches. Then we will use the eigenfilter approach to find the optimal b_0 and denominator coefficients.

Let $H^d(\mu)$ denotes the desired target function representing the notch filter with the bandwidth $0.001 \cdot \pi$ at each chosen notch position in our examples. With *K* notches, we can divide the frequency domain into *3K+1* intervals to express different desired frequency response in each interval. The response inside the notch consists two transition bands: one decreasing function from 1 to 0 and the other increasing function from 0 to 1, respectively, and is unity elsewhere.

To proceed with the formulation of notch filters, consider a cost function related to the error of the difference between the calculated response $H(\mu)$ and target response $H^d(\mu)$:

$$\begin{aligned} E(\mu) &= (error) D(\mu) \\ &= (H^d(\mu) - H(\mu)) D(\mu) \end{aligned} \qquad (5)$$

Let $E_k(\mu)$ be the cost function in the frequency interval I_k, so,

$$E_k(\mu) = D(\mu) H_k^d(\mu) - b_0 \prod_{i=1}^{K} (1 - 2 \cdot \cos(\mu_i) \cdot e^{-j\mu} + e^{-2j\mu}). \qquad (6)$$

To minimize $E(\mu)$ somehow means a weighted minimization of the error, in which $D(\mu)$ acts as the weighting function. Consider the global square cost function given by $\Phi = \sum_k \lambda_k \varepsilon_k$, where λ_k is a positive constant that weights the *k-th* band cost function. And ε_k is given by

$$\varepsilon_k = \int_{I_k} |E_k(\mu)|^2 W_k(\mu) d\mu, \qquad (7)$$

where $W_k(\mu)$ denotes the positive weighting function. Consider the vectors

$$\begin{aligned} C_k(\mu) = [&H_k^d(\mu) \quad H_k^d(\mu) e^{-j\mu} \quad \cdots \quad H_k^d(\mu) e^{-j2K\mu} \\ &- \prod_{i=1}^{k} (1 - 2 \cdot \cos(\mu_i) \cdot e^{-j\mu} + e^{-j2\mu})]^T \end{aligned} \qquad (8)$$

and

$$A = [a_0 \quad a_1 \quad \cdots \quad a_{2K} \quad b_0]^T, \qquad (9)$$

The cost function ε_k can be expressed as

$$\varepsilon_k = \int A^T C_k^*(\mu) C_k^T(\mu) A W_k(\mu) d\mu, \qquad (10)$$

where superscript T and $*$ denote the transposition and conjugation operations, respectively. (T $*$ since the filter coefficients are assumed real). Therefore, is given by

$$A^T R_k A, \qquad (11)$$

and

$$R_k = \int C_k^*(\mu) C_k^T(\mu) W_k(\mu) d\mu \qquad (12)$$

is a real, symmetric, positive-definite $(2K+2) \times (2K+2)$ matrix. The global square cost function

now can be expressed as

$$\Phi = \sum_k {}_k \Phi_k = A^T (\sum_k P_k) A = A^T P A . \tag{13}$$

Applying the eigenfilter approach, the optimum filter coefficients, which minimize the cost function, are the elements of the eigenvector of the matrix P corresponding to the minimum eigenvalue. After some calculation iterations and computing the corresponding eigenvector, we obtain the solution of

$A = [a_0 \quad a_1 \quad \cdots \quad a_{2K} \quad b_0]^T$.

Then we can get the transfer function coefficients of numerator and denominator by (3) and (4).

Experimentally, some remarks need to be mentioned.

1. Updating of the target function phase: Reference to [1], here we only approximate the magnitude response of $H^d(\mu)$. Yet the cost function $E(\mu)$ also depends on the phase of $H^d(\mu)$. In absence of any information, we can initially assume the desired phase response linear, i.e. $\Phi(\mu) = M\mu$, where M is a given constant. Reference to [1][8], to have a well-behaved response and fast convergence, an iterative phase updating can be adopted. Here we describe this method in brief. At the *n-th* iteration, let $A^{(n)}$ be the coefficient vector obtained and $H^{(n)}(\mu)$ the corresponding frequency response. Assume that $\Phi^{(n+1)}(\mu)$ be the phase of $H^d(\mu)$ at the *(n+1)-th* iteration. Assign $\Phi^{(n+1)}(\mu) = \angle H^{(n)}(\mu)$, and redo the design procedure until some criterion is met.
2. Choice of weighting function $W_k(\mu)$: As discussed previously, to compute the quadratic matrix P in (12)(13), we need to decide the weighting function. Let $W^{(n)}(\mu)$ be the weighting function in the *n-th* iteration. We adopt the recommendation of [8] in our experiments, let

$$W^{(n)}(\mu) = \frac{1}{|D^{(n-1)}(\mu)|}, \tag{14}$$

   where $D^{(n)}(\mu)$ is the denominator of $H^{(n)}(\mu)$. So at each iteration, we can give higher weights to the regions where $D^{(n-1)}(\mu)$ is small and the error will be smaller in the regions.
3. Stability check: The stability of the filters we designed should be carefully concerned. To ensure the stability of the IIR filters, we need to calculate the roots of denominator polynomial to find unstable poles and substitute them with their inverse conjugate. So that we can make this filter always stable without changing its magnitude response.

To summarize the method to design IIR multiple notch filters, we can take the following steps:

1) Decide the positions of notches and calculate polynomial $C(z)$ according to (4). The initial state can be set $\Phi^{(0)}(\mu) = K\mu$ (we use K denotes the number of notches we need) and $W_k^{(0)}(\mu) = 1 \quad \forall k$.
2) Compute the matrix P by using (12), (13), and (14), where P is a function of $\Phi^{(n)}(\mu)$ and $W_k^{(n)}(\mu)$. Let $A^{(n)}$ be the eigenvector corresponding to the minimum eigenvalue of P, and let be the relative transfer function; find unstable poles and substitute them with their inverse conjugate.
3) Update the phase of the target function by using $\Phi^{(n+1)}(\mu) = \angle H^{(n)}(\mu)$; update the weighting function $W_k^{(n+1)}(\mu)$ according to (14).
4) Compute the poles outside the unit circle and substitute them with their inverse conjugate.
5) Repeat step.2 to step.4 until some stop criterion is met. In our experiments, we take fifty iterations in each result for that the error converges within twenty iterations.
6) Using (1) and (2) to get the desired transfer function coefficients.

B. Nyquist Filter

As mentioned in the introduction, to obtain the zero intersymbol interference, Nyquist filter

$H(z)$ has the impulse response $h(n)$ with the time domain constraints, i.e.,

$$h(K+kN)=\begin{cases} c\neq 0, & if\ k=0 \\ 0, & otherwise \end{cases} \tag{15}$$

, where K and N are integers. Then the impulse response crosses the time axis every N samples. According to [15], the conditions in (15) can be rearranged and written as

$$h(K+kN)=\begin{cases} \frac{1}{N}\neq 0, & if\ k=0 \\ 0, & otherwise \end{cases} \tag{16}$$

, where K and N are integers. It is known that the transfer function of an IIR Nyquist filter can be expressed in the form

$$H(z)=b_K z^{-K}+\frac{\sum_{i=0,i\neq kN+K}^{N_n} b_i z^{-i}}{\sum_{i=0}^{N_d/N} a_{iN} z^{-iN}},\quad b_K=\frac{1}{N}, \tag{17}$$

where N_n, N_d are integers, and all the filter coefficients a_i, b_i are real, $a_0=1$. N_d is a multiple of N. Hence, only frequency-response optimization should be considered when the above transfer function is employed. The specification of Nyquist filters in frequency domain should be a lowpass filter with passband and stopband cutoff frequency μ_p and μ_s expressed as

$$\mu_p=\frac{1-\circ}{N}® \tag{18a}$$

$$\mu_s=\frac{1+\circ}{N}® \tag{18b}$$

, where N is the interpolation ratio and $\circ$ is the rolloff rate.

Now we can formulate the design procedure of Nyquist filters with the zero-crossing time domain constraints of Nyquist filters. According to (17), assume the calculated frequency response of a Nyquist filter is given by

$$H(e^{j\mu})=\frac{1}{N}e^{-jK\mu}+\frac{\sum_{i=0,i\neq kN+K}^{N_n} b_i e^{-ijw}}{\sum_{i=0}^{N_d/N} a_{iN}(e^{-jw})^{iN}} \tag{19}$$

and the desired frequency response is $H^d(e^{j\mu})$,a lowpass filter described by (18). Then the current error function is

$$\pounds(\mu)=H^d(e^{j\mu})-H(e^{j\mu}). \tag{20}$$

Let the cost function be

$$\begin{aligned} E(\mu)&=\pounds(\mu)\sum_{i=0}^{N_d/N} a_{iN}(e^{-j\mu})^{iN} \\ &=\left(H^d(e^{j\mu})-\frac{1}{N}e^{-jK\mu}\right)\sum_{i=0}^{N_d/N} a_{iN}(e^{-j\mu})^{iN}-\sum_{i=0,i\neq kN+K}^{N_n} b_i e^{-ij\mu} \end{aligned} \tag{21}$$

and the global cost function of the whole frequency range will be

$$\Phi(\mu)=\int_{\Gamma}\left|E(\mu)\right|^2 W(\mu)d\mu \tag{22}$$

$$\Phi(\mu)=A^T B \tag{23a}$$

where

$$A^T=\left(H^d(e^{j\mu})-\frac{1}{N}e^{-jK\mu}\right)[1\quad e^{-j\mu}\quad e^{-jN\mu}\quad e^{-j2N\mu}\cdots e^{-jN_d\mu}] \tag{23b}$$

$$B=\left[-1 \quad -e^{-j\mu} \quad -e^{-j2\mu}\ldots-e^{-ji\mu}\ldots-e^{-jN_d\mu}\right] \quad , \quad i\neq kN+K, \tag{23c}$$

and $$A=\left[a_0 \quad a_N \quad a_{2N}\ldots a_{N_d} \quad b_0 \quad b_1 \quad b_2\ldots b_i\ldots b_{N_n}\right]^T \quad , \quad i\neq kN+K \tag{24}$$

so we can rewrite the cost function to be

$$\Phi(\mu)=\int_{\mu} A^T C^*(\mu)C(\mu)A\,W(\mu)d\mu = A^T PA \tag{25}$$

$$P=\int_{\mu} C^*(\mu)C(\mu)W(\mu)d\mu \tag{26}$$

In (25) and (26), $W(\mu)$ is the weighting function of each frequency grid. Since Eq.25 matches the eigenform, we can solve the minimization problem by eigenfilter approach to obtain the optimal filter coefficients.

As the notch filter design discussed in the previous section, we need a weighting function $W(\mu)$ when computing the matrix *P*. Here we can use the weighting function prescribed in (14) to get a non-equiripple response. If the equiripple response is desired, another kind of weighting function should be employed. In the previous works proposed [8], a recursive updating of the weighting function was introduced to obtain an almost equiripple solution. Also we adopted the recursive procedure to get a convergent solution satisfying the given specification. An adequate choice of the recursively updating weighting function is given below.

Let $A^{(n)}$ be the solution vector at the *n-th* iteration and $H^{(n)}(\mu)$ the corresponding frequency response. The magnitude error is

$$e^{(n)}(\mu)=\left\|H^d(\mu)\right|-\left|H^{(n)}(\mu)\right\|, \tag{27}$$

and the weighting function used in the *(n+1)* iteration will be

$$W^{(n+1)}(\mu)=W^{(n)}(\mu)env(e^{(n)}(\mu)), \tag{28}$$

where $env(e^{(n)}(\mu))$ is the envelope of the positive function $e^{(n)}(\mu)$. Then the resulting solution will be almost equiripple.

The Nyquist filters are lowpass filters and have no constraints of the phase response. Usually we use a linear phase as the initial target phase response. So that we can initially set $\angle H(\mu)=-M\mu$, in which M is a constant, as the desired phase response when we calculated $H_d(\mu)$. Experimentally we use the given integer K to be M, then

$$H_d(\mu)=\left|H_d(\mu)\right|e^{-jK\mu}. \tag{29}$$

We can conclude the design procedure into some steps:

1) Give the desired numerator and denominator order N_n and N_d, two integers K, N, rolloff rate ^{o}. And we will need the initial weighting function to control the amplitude ratio of the ripples of each band.
2) Calculate $H_d(\mu)$ according to (18a) and (18b). Calculate matrix *C* and *P* according to (23) and (26), respectively.
3) Calculate the eigenvalue of matrix P and obtain the eigenvector corresponding to the minimum eigenvalue and the vector is the coefficients we need.
4) Compute the frequency response $H(\mu)$ according to the coefficients obtained and decide the weighting function used in the next iteration. We adopt (14) or (28) to be the weighting

function in the non-equiripple case or equiripple case respectively.

5) Find unstable poles and substitute them with their inverse conjugate to ensure the stability of filters.
6) Update the phase of the target function by using $\theta^{(n+1)}(\mu)=\angle H^{(n)}(\mu)$; update the weighting function $W_k^{(n+1)}(\mu)$.
7) Repeat the step.4 to step.6 until a stop criterion is met. In our experiments, we stop the iteration until the maximum of the error of each frequency grid is smaller than 0.005.

C. Partial Response Filters

The partial response filters can be thought as a modification of Nyquist filters but have some specified magnitude and impulse response. These filters can be classified to five classes and summarized in Table.A [16].

In Table.A, each binary symbol C_n is chosen to be a prescribed superposition of n successive transmitted impulses $b_1, b_2, \ldots b_n$ as $C_n = k_1 b_n + k_2 b_{n-1} + \cdots + k_n b_1$ [16]. Partial response filters can achieve high data rates with better error rate performance.
And the class of binary in Table.A is the original Nyquist filter.

Table.A The Properties of class 1 to class 5partial response filters [16].

Class	Impulse response $h(n)$ k_1 k_2 k_3 k_4 k_5	$H(f)$ over the passband $[0 < f < F]$
Binary	1	1
1	1 1	$2\cos(\pi f / 2F)$
2	1 2 1	$4\cos^2(\pi f / 2F)$
3	2 1 -1	$2 + \cos(\pi f / F) - \cos(2\pi f$ $j[\sin(\pi f / F) - \sin(2\pi f /$
4	1 0 -1	$2\sin(\pi f / F)$
5	-1 0 2 0 -1	$4\sin^2(\pi f / F)$

The transfer function of partial response filters is assumed as $H(z) = Num(z) / Den(z)$, to obtain the impulse and magnitude response prescribed, the numerator polynomial of the transfer function must be divided by some specified polynomial. That is,

$$R(z) \mid Num(z), \tag{30}$$

which

$$R(z) = \begin{cases} 1 + z^{-N} & class(1) \\ 1 + 2z^{-N} + z^{-2N} & class(2) \\ 2 + z^{-N} - z^{-2N} & class(3) \\ 1 - z^{-2N} & class(4) \\ -1 + 2z^{-2N} - z^{-4N} & class(5) \end{cases} \tag{31}$$

So the transfer functions of the classified partial response filters can be thought as Nyquist filters cascade with some specified $R(z)$. We can assume the calculated frequency response of a partial response filter as

$$H(e^{j\mu}) = \left[\frac{1}{N} e^{-jK\mu} + \frac{\sum_{l=0,N,2N,\ldots,N_n} b_l e^{-jlw}}{\sum_{i=1}^{N_d/N} a_{iN} (e^{-jw})^{iN}} \right] R(e^{j\mu}) \tag{32}$$

and the desired frequency response $H^d(e^{j\mu})$ is given in the table, then the current error is

$$\varepsilon(\mu) = H^d(e^{j\mu}) - H(e^{j\mu}). \tag{33}$$

Let the cost function to be

$$E(\mu) = \pounds(\mu) \sum_{i=0}^{N_d/N} a_{iN} (e^{-j\mu})^{iN} \tag{34}$$
$$= \left(H^d(e^{j\mu}) - \frac{1}{N} e^{-jK\mu} D(e^{j\mu}) \right) \sum_{i=0}^{N_d/N} a_{iN} (e^{-j\mu})^{iN} - R(e^{j\mu}) \sum_{i=0, i \neq kN+K}^{N_n} b_i e^{-ij\mu}$$

and the total error function of the whole frequency range will be

$$\Phi(\mu) = \int_{\mu} |E(\mu)|^2 W(\mu) d\mu \,. \tag{35}$$

If

$$C(\mu) = [A \;\; B] \tag{36a}$$

where

$$A = \left(H^d(e^{j\mu}) - \frac{1}{N} e^{-jK\mu} R(e^{j\mu}) \right) \begin{bmatrix} 1 & e^{-j\mu} & e^{-jN\mu} & e^{-j2N\mu} \ldots e^{-jN_d\mu} \end{bmatrix} \tag{36b}$$

$$B = R(e^{j\mu}) \begin{bmatrix} -1 & -e^{-j\mu} & -e^{-j2\mu} \ldots -e^{-ji\mu} \ldots -e^{-jN_d\mu} \end{bmatrix} \;, \quad i \neq kN+K \tag{36c}$$

and $A = \begin{bmatrix} a_0 & a_N & a_{2N} \ldots a_{N_d} & b_0 & b_1 & b_2 \ldots b_i \ldots b_{N_n} \end{bmatrix}^T \;, \quad i \neq kN+K$ (37)

then we can rewrite the cost function as:

$$\Phi(\mu) = \int_{\mu} A^T C^*(\mu) C(\mu) A W(\mu) d\mu = A^T P A \tag{38}$$

and

$$P = \int_{\mu} C^*(\mu) C(\mu) W(\mu) d\mu \tag{39}$$

In (38) and (39), $W(\mu)$ is the weighting function of each frequency grid.

Since Eq.(38) results an eigen_form, we can solve the minimization problem by eigenfilter approach to obtain the optimal solution. At last, according to (32), we should multiply the calculated numerator by $R(z)$ specified by (31) and obtain the desired transfer function coefficients. The resulting numerator order will be $N_n + \deg[R(z)]$. The same as designing the Nyquist filters, we can also adopt the suitable weighting function in (39) to obtain equiripple or non-equiripple solutions by iterative procedures. The design procedure of partial response filters is summarized as below:

1) Give the desired numerator and denominator order N_n and N_d. And we will need the initial weighting function to control the amplitude ratio of the ripples of each band.
2) Find $H_d(\mu)$ according to table.1. Calculate matrix C and P according to (36) and (39), respectively.
3) Calculate the eigenvalue of matrix P and obtain the eigenvector corresponding to the minimum eigenvalue and the vector is the desired coefficients we need.
4) Compute the frequency response $H(\mu)$ based on the coefficients obtained and decide the weighting function used in the next iteration. We adopt (14) or (28) to be the weighting function in the non-equiripple case or equiripple case, respectively.
5) Find unstable poles and substitute them with their inverse conjugate to ensure the stability of filters.
6) Normalize the numerator polynomial by its unit DC gain.
7) Update the phase of the target function by using $\angle H^{(n+1)}(\mu) = \angle H^{(n)}(\mu)$; update the weighting function $W^{(n+1)}(\mu)$.

8) Repeat the step.3 to step.7 until a stop criterion is met. We take 50 iterations in our experiments to obtain satisfactory results.

It is worthy to notice that we need to normalize the calculated coefficients by unit DC gain in each iteration to ensure the convergence of solutions for designing class 1 to class 3 partial response filters. For DC equals to zero in class 4 and class 5 partial response filters, normalization is not used then.

III. Experimental Results

A. Notch Filter Design

Here we present some design examples using our proposed method. First for the notch filters. We show the effectiveness of the proposed approach with arbitrarily prescribed notch frequencies and bandwidth.

Example 1: A notch filter with one notch at μ=0.25® after 100 iterations. The magnitude response and pole-zero plotting are given in Fig.1, and the computed coefficients are given in Table.1.

Example 2: A notch filter with two notches at μ=0.3® and 0.7® and plotted in Fig.2, and the coefficients are given in Table.2.

Example 3: A notch filter with three different notches at μ=0.3®, 0.6® and 0.9®. Besides that, we can arbitrarily specify the notch bandwidth. Here the widths of three notches are 0.002®, 0.02®, and 0.04®, respectively. The responses are shown in Fig.3, and Table.3 gives the resultant coefficients.

Designing the notch filter using the eigenfilter approach with constraints can obtain exact zeroes at the desired locations and very flat pasband elsewhere, and the solution converges after several iterations.

B. Nyquist Filter Design

Example 4: A Nyquist filter with with N_n=15, N_d=4, ρ=0.3, N=4, K=9 and the weighting of ripple ratio $\phi_p / \phi_s = 1000$. To express the properties of Nyquist filters completely, the magnitude response, impulse response, and pole-zero plotting are given in Fig.4. The resultant coefficients are given in Table.4.

Example 5: We design the filter with same specification as example 4 but equiripple is presented in Fig.5. The resultant coefficients are given in Table.5.

The specification of the last two examples is the same as illustrated in [14]. Compared with the attenuation 38 dB in [14], an improved minimum stopband attenuation of almost 40 dB is obtained but the ripples are not quite equal. The coefficients of example 5 are also listed in Table.5.

Example 6: A Nyquist filter with N_n=17, N_d=4, ρ=0.2, N=4, K=10 and shown in Fig.6. The resultant coefficients are given in Table.6.

In these examples, we plot the frequency response, impulse response, and pole-zero positions. Since we formulate the design procedure of Nyquist filters according to the zero-crossing time domain constraints, we can obtain exact zeroes in the specific positions. Also the magnitude responses are wellbehaved.

C. Partial Response Filter Design

At last we design the partial response filters with class 1 to 5 as example 7 to example 11, respectively. Both equiripple and non-equiripple cases are displayed.

Example 7: Class 1 partial response filter with the specifications: N_n=10, N_d=2, N=2, K=6. The magnitude responses and impulse responses of both equiripple and non-equiripple cases are

given in Fig.7. To express the cosine shape of the magnitude response of the class 1 partial response filter, the magnitude response of the non-equal ripple is given in linear scale in Fig.7.1. Other magnitude responses are given in log scale to show the attenuation of the stopband. The coefficients of equiripple case of class 1 filter are given in Table.7.

Example 8: Class 2 partial response filter with specification: N_n=12, N_d=2, N=2, K=7. The responses of both equiripple and non-equiripple cases are given in Fig.8.

Example 9: Class 3 partial response filter with specification: N_n=13, N_d=2, N=2, K=5. The responses are given in Fig.9.

Example 10: Class 4 partial response filter with specification: N_n=12, N_d=2, N=2, K=7. The responses are given in Fig.10.

Example 11: Class 5 partial response filter with specification: N_n=12, N_d=2, N=2, K=9. The responses are given in Fig.11.

Both equiripple and non-equiripple cases are presented. The impulse responses and magnitude responses are very satisfactory.

IV. Conclusion

In this paper, we have presented a new method to design digital IIR notch filters, Nyquist filters, and partial response filters. To design these filters, many methods have been proposed before. However, designing these filters with time-domain or frequency-domain constraints, IIR eigenfilter approach is a better choice and easier to handle. Here we first formulated the properties of these IIR filters and extended the existing IIR eigenfilter approach to design these filters. The effectiveness of eigenfilter approach has been revealed for adding the time-domain or frequency-domain constraints. We have employed an iteration process to obtain an equiripple, stable solution of the desired IIR filters.

V. References

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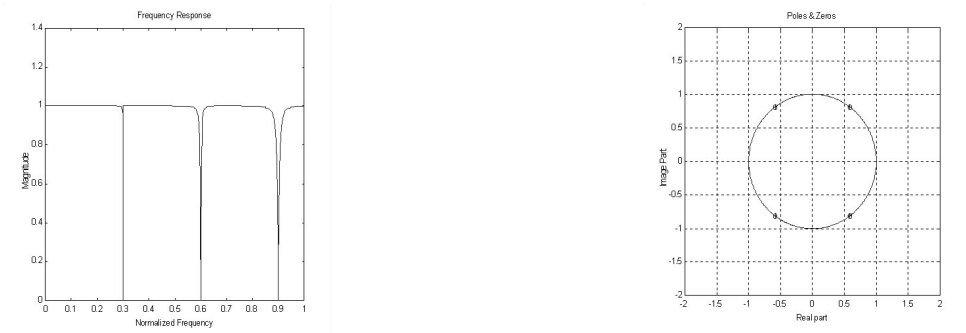


Fig.1.1 Magnitude response of Example 1.

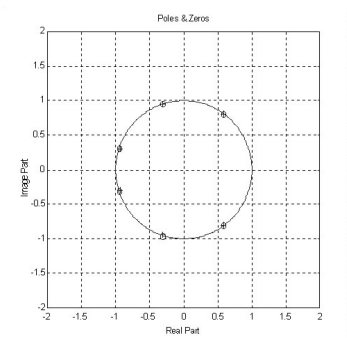


Fig.1.2 Pole-zero plotting of Example 1.

Table 1 The resulting coefficients of Example 1.

b_0	0.44721416257553	a_0	0.44748014854380
b_1	-0.63245633399964	a_1	-0.63245085376586
b_2	0.44721416257553	a_2	0.44695293602563

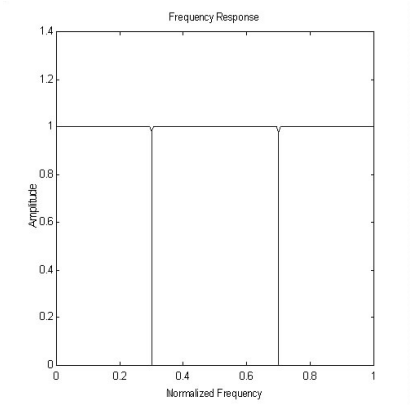


Fig.2.1 Magnitude response of Example 2.

Fig.2.2 Pole-zero plotting of Example 2.

Table.2 The resulting coefficients of Example 2.

b_0	0.54377099289015	a_0	0.54440003292807
b_1	0	a_1	0.00000001968196
b_2	0.33606895570239	a_2	0.33607333526488
b_3	0	a_3	-0.00000002024473
b_4	0.54377099289015	a_4	0.54313573327771

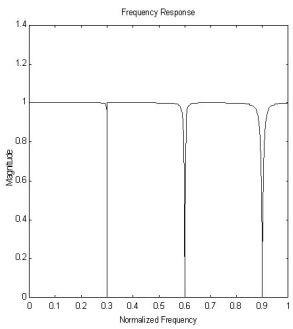


Fig.3.1 Magnitude response of Example 3.

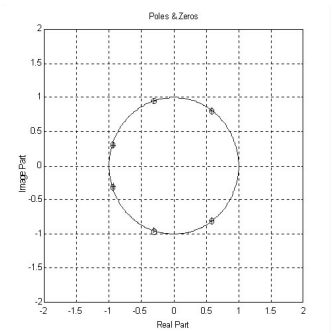
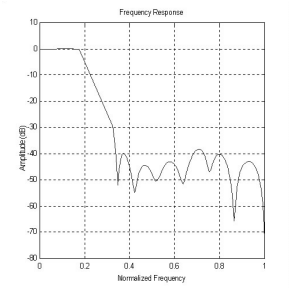


Fig.3.2 Pole-zero plotting of Example 3.

Table.3 The resulting coefficients of Example 3.

b_0	0.29801430260418	a_0	0.30692859574000
b_1	0.40070303293878	a_1	0.39958611590026
b_2	0.36147942821254	a_2	0.35985811773165
b_3	0.38956042881218	a_3	0.38939406634299
b_4	0.36147942821254	a_4	0.36292519208517
b_5	0.40070303293878	a_5	0.40128160491205
b_6	0.29801430260418	a_6	0.28887025331247



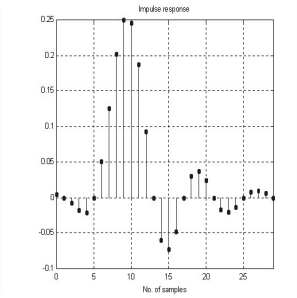


Fig.4.1 Magnitude response of Example 4.

Fig.4.2 Impulse response of Example 4.

Nyquist filter with specification

$N_n = 15, N_d = 4,\ ° = 0.3, N = 4, K = 9$

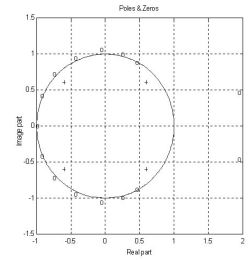
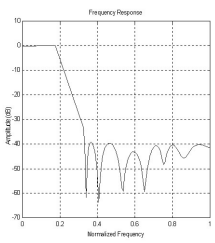


Fig.4.3 Pole-zero plotting of Example 4.

Table.4 The resulting coefficients of Example 4.

b_0	0.00458461115022	b_{11}	0.25239597909338
b_1	0	b_{12}	0.19754485652388
b_2	-0.00682683757339	b_{13}	0.12961566918558
b_3	-0.01740969380899	b_{14}	0.06790172230435
b_4	-0.01811987645841	b_{15}	0.02489208206431
b_5	0	a_0	1.00000000000000
b_6	0.04727636125958	a_1	0
b_7	0.11634386187788	a_2	0
b_8	0.19151334697879	a_3	0
b_9	0.25000000000000	a_4	0.51846267674232
b_{10}	0.27231999915970		



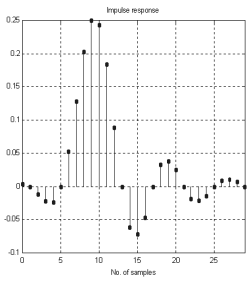


Fig.5.1 Magnitude response of Example 5 :

Equiripple case of example 4.

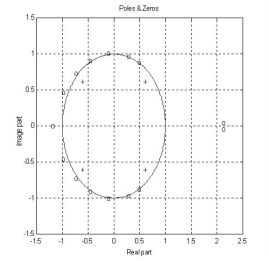
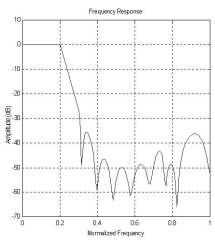


Fig.5.3 Pole-zero plotting of Example 5.

Fig.5.2 Impulse response of Example 5.

Table.5 The resulting coefficients of Example 5.

b_0	0.00373701676675	b_{11}	0.25294719373311
b_1	0	b_{12}	0.19736549464621
b_2	-0.01124615922963	b_{13}	0.13402787767668
b_3	-0.02174419712868	b_{14}	0.06879712348272
b_4	-0.02126344099409	b_{15}	0.02727697661389
b_5	0	a_0	1.0
b_6	0.04660624892311	a_1	0
b_7	0.11623394636996	a_2	0
b_8	0.19049647956317	a_3	0
b_9	0.25	a_4	0.53611151070671
b_{10}	0.27100557266568		



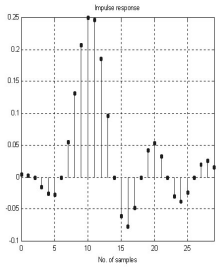


Fig.6.1 Magnitude response of Example 6.

Fig.6.2 Impulse response of Example 6.

Nyquist filter with specification

$N_n = 17, N_d = 4, \ ° = 0.2, N = 4, K = 10$

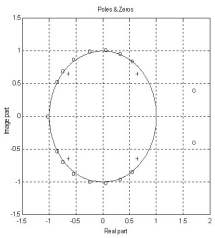
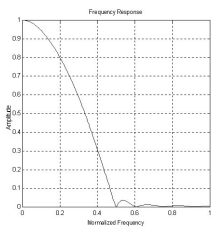


Fig.6.3 Pole-zero plotting of Example 6.

Table.6 The resulting coefficients of Example 6.

b_0	0.00510870405594	b_{12}	0.27723994576975
b_1	0.00302751851227	b_{13}	0.24109149576972
b_2	0	b_{14}	0.17449621243145
b_3	-0.01424096402269	b_{15}	0.11185237075775
b_4	-0.02125305506918	b_{16}	0.05274927478049
b_5	-0.02406533221737	b_{17}	0.01960823484487
b_6	0	a_0	1.0
b_7	0.04505898586477	a_1	0
b_8	0.11388103120759	a_2	0
b_9	0.18865812448367	a_3	0
b_{10}	0.25	a_4	0.69798484972580
b_{11}	0.28535192595900		



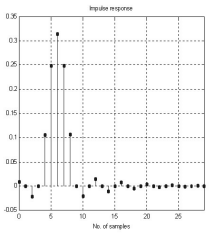
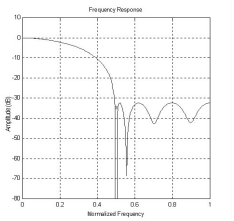


Fig.7.1 Linear magnitude response of Example 7 : Class 1 partial response filter.

Fig.7.2 Impulse response of Example 7.



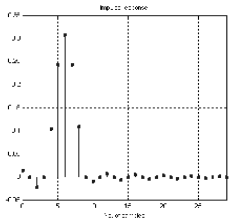
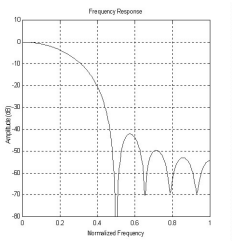


Fig.7.3 Magnitude response of equiripple case of Example 7.

Fig.7.4 Impulse response of equiripple case of Example 7.

Table.7 The resulting coefficients of equiripple case of Example 7.

b_0	0.01446798102295	b_7	0.45110472197426
b_1	0	b_8	0.37240643153797
b_2	-0.00961818862706	b_9	0.20708477391566
b_3	0	b_{10}	0.08488537433446
b_4	0.08634022042579	a_0	1.0
b_5	0.24401994805860	a_1	0
b_6	0.39794744727930	a_2	0.84863870992193



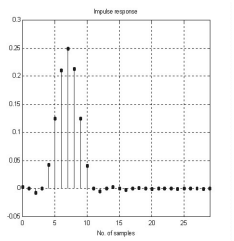
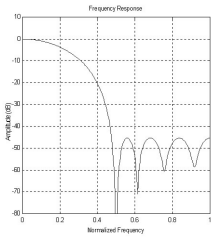


Fig.8.1 Magnitude response of Example 8: Class 2 partial response filter.

Fig.8.2 Impulse response of Example 8.



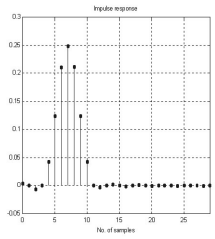
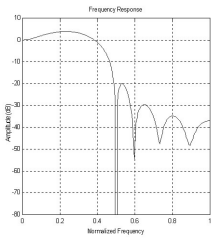


Fig.8.3 Magnitude response of equiripple case of Example 8.

Fig.8.4 Impulse response of equiripple case of Example 8.



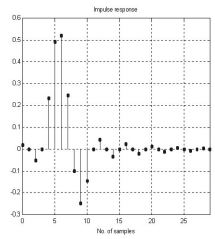


Fig.9.1 Magnitude response of Example 9:

Class 3 partial response filter.

Fig.9.2 Impulse response of Example 9.

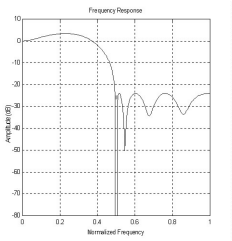


Fig.9.3 Magnitude response of equiripple case of Example 9.

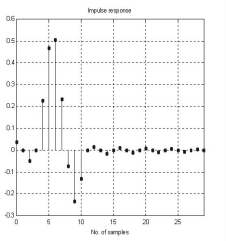


Fig.9.4 Impulse response of equiripple case of Example 9.

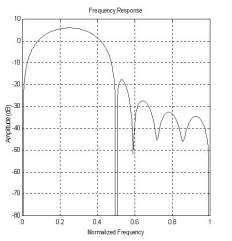


Fig.10.1 Magnitude response of Example 10:
Class 4 partial response filter.

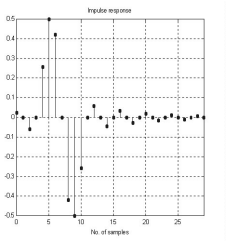
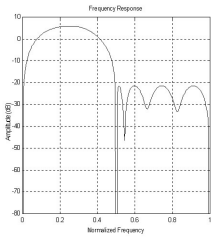


Fig.10.2 Impulse response of Example 10.



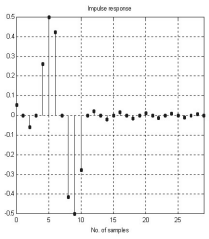
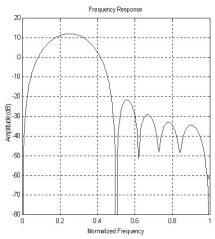


Fig.10.3 Magnitude response of equiripple case of Example 10.

Fig.10.4 Impulse response of equiripple case of Example 10.



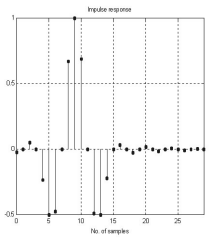
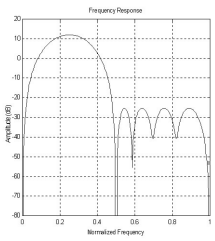


Fig.11.1 Magnitude response of Example 11:

Class 5 partial response filter.

Fig.11.2 Impulse response of Example 11.



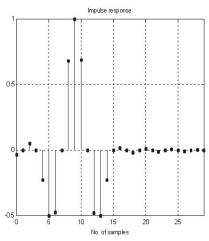


Fig.11.3 Magnitude response of equiripple case of Example 11.

Fig.11.4 Impulse response of equiripple case of Example 11.