CALIBRATION–FREE BEARING ESTIMATION FOR ARRAYS WITH RANDOMLY PERTURBED SENSOR LOCATIONS

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ABSTRACT

In this paper, calibration-free bearing estimation algorithms for linearly periodic arrays with the presence of sensor positioning errors are investigated. The Toeplitz Approximation Method (TAM) is used to cope with the two-dimensional (2-D) sensor positioning errors which are assumed to be independent identically distributed (i.i.d.) and Gaussian in each dimension. After a Toeplitz covariance matrix being reconstructed by the TAM, the eigenstructure-based bearing estimation algorithms, such as MUSIC, can be employed as though the array were linear and equally spaced without using any calibration process. To improve the effectiveness of the TAM, a modification for TAM is also described. Several examples are provided to illustrate the effectiveness of the proposed methods.

I. INTRODUCTION

High-resolution bearing estimation techniques are usually developed based on the assumption that the locations of array sensors are known precisely. However, if there are uncertainties in the sensor locations, the performance of these techniques tends to be deteriorated greatly. In general, this problem is solved by calibrating the sensor locations prior to performing bearing estimation. In [1], an approach based on Schmidt's MUSIC method [2] was proposed. It requires the deployment of at least two auxiliary sources at precisely known locations for calibration. Later [3] proposed a method to eliminate the requirement of deploying auxiliary sources at precise locations. However, this method needs no less than three auxiliary sources which must be spectrally (or temporally) disjoint to each other and targets [4]. Moreover, it cannot be used in the case of linear array configurations. Recently, an algorithm avoiding the need of calibration process was presented in [5]. It assumes that an array composed of m matched sensor doublets has elements translationally separated by a known constant displacement. This assumption may be impractical in some applications where the array suffers random sensor location perturbation.

In this paper, we propose a calibration-free algorithm for a nominal linearly periodic array with the presence of 2-D perturbations in sensor locations. Based on the assumptions that the nominal array is linearly periodic and the random perturbations have zero mean, we use a spatial averaging method, the TAM [6] to cope with the sensor positioning errors. Since the ideal covariance matrix of the nominal array is Toeplitz, we reconstruct a Toeplitz covariance matrix from the computed covariance matrix to approximate the ideal one. The reconstructed covariance matrix is then utilized for bearing estimation based on the existing eigenstructure algorithms. It is shown that the accuracy of the approximation depends on the number of sensor elements and the variance of the random perturbations. Therefore, the TAM method is suitable for the situations where the variance of random perturbations is small compared with the nominal distances between sensors. To improve the TAM method, a modified TAM, which appears to be superior to the TAM, is also presented in this paper.

II. FORMULATION OF THE PROBLEM

Consider a randomly perturbed linear array with L sensor elements and M incoherent narrow-band far-field sources. These sources have arrival angles θ_m, m =1,..., M relative to array broadside. Suppose that the ℓth sensor with nominal location vector $\hat{\underline{u}}_\ell = [\hat{x}_\ell,\ 0\ ]^t$ suffers a 2-D random perturbation $\Delta\underline{u}_\ell = [\Delta x_\ell,\ \Delta y_\ell]^t$ in its position. Thus, its actual location vector is given by

$$\underline{u}_\ell = \hat{\underline{u}}_\ell + \Delta\underline{u}_\ell \qquad ,\ \ell = 1, 2, ..., L \tag{1}$$

The received signal at the ℓth sensor $\nu_\ell(t)$ is given by

$$\nu_\ell(t) = \sum_{m=1}^{M} \alpha_m(t) \exp[\, j\{\, k_c\, \underline{u}_\ell^t\, \underline{\theta}_m + \omega t\,\}\,] + n_\ell(t), \qquad \ell = 1, 2, ..., L \tag{2}$$

where k_c is the wavenumber corresponding to the signal frequency ω, $\alpha_m(t)$ and $\underline{\theta}_m = [\sin\theta_m,\ \cos\theta_m]^t$ are waveform envelope and bearing vector of the mth source, respectively, and $n_\ell(t)$ is assumed to be the additive white noise with mean zero and variance $E[n_\ell(t)\ n_\ell^*(t)] = \sigma^2$. Rewriting (2) in vector form, we have

$$\underline{v}(t) = \sum_{m=1}^{M} \alpha_m(t)\ e^{j\omega t}\ \underline{S}_m + \underline{N}(t) \tag{3}$$

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where $\underline{v}(t) = [\nu_1(t),..., \nu_L(t)]^t$ denotes the signal vector, $\underline{S}_m = [e^{jk_c \underline{u}_1^t \underline{\theta}_m},...,e^{jk_c \underline{u}_L^t \underline{\theta}_m}]^t$ the direction vector of the mth source, and $\underline{N}(t) = [n_1(t), ..., n_L(t)]^t$ the noise vector. Therefore the covariance matrix R is formed by

$$R = E[\underline{v}(t)\ \underline{v}^+(t)] = \sum_{m=1}^{M} E[|\alpha_m(t)|^2]\ \underline{S}_m\ \underline{S}_m^+ + \sigma^2 I \quad (4)$$

where "+" and "E" denote the complex conjugate transpose and ensemble expectation, respectively, and I the L x L identity matrix.

To perform bearing estimation based on the eigenstructure of R given in (4), the well-known MUSIC of [2] can be employed. The basic idea of MUSIC is first to find the eigenvectors $\underline{q}_\ell$, ℓ=M+1,...,L which span the noise subspace. Then a search function is established as follows

$$P(\theta) = \frac{1}{\sum_{\ell=M+1}^{L} |\underline{q}_\ell^+ \underline{S}(\underline{\theta}, \underline{U})|^2} \quad (5)$$

for finding the bearings θ_i, i=1,...,M which correspond to the peaks of $P(\theta)$. In (5), $\underline{S}(\underline{\theta}, \underline{U})$ is the search vector given by

$$\underline{S}(\underline{\theta}, \underline{U}) = [e^{jk_c \underline{u}_1^t \underline{\theta}}, ..., e^{jk_c \underline{u}_L^t \underline{\theta}}]^t \quad (6)$$

and $\underline{\theta} \triangleq [\sin\theta, \cos\theta]^t$ is the bearing vector associated with θ, $\underline{U} \triangleq [\underline{u}_1^t, \underline{u}_2^t, ..., \underline{u}_L^t]^t$ is the sensor location vector. Hence, MUSIC requires the precise information of $\underline{U}$. In case of $\underline{U}$ having random perturbation, the performance of MUSIC will be degraded considerably because of the inconsistency between the search vector and the source direction vector. Recent solutions for this problem have been focused on the use of calibration process before estimating bearings.

III. THE TAM METHOD AND ITS MODIFICATION

In this section, we utilize a method without using calibration process to tackle the problem described above. For simplicity, the array is assumed to be linear and equally spaced with $\hat{\underline{u}}_\ell = [(\ell-1)d, 0]^t$, where d is the interelement spacing. The random perturbations of sensor locations Δx_ℓ, Δy_ℓ, ℓ = 1,2, ..., L are i.i.d. Gaussian random variables with mean zero and variance σ_e^2, respectively.

From (4), it is noted that $R = \hat{R} = [\hat{r}_{ij}]$ is Toeplitz and Hermitian if $\underline{u}_\ell = \hat{\underline{u}}_\ell$, ℓ=1,2,...,L, i.e., no sensor location errors. In this case, the element $\hat{r}_{ij}$ is given by

$$\hat{r}_{ij} = \sum_{m=1}^{M} E[|\alpha_m(t)|^2]\ e^{jk_c(\hat{\underline{u}}_i - \hat{\underline{u}}_j)^t \underline{\theta}_m} + \sigma^2\delta_{ij}$$

$$= \sum_{m=1}^{M} E[|\alpha_m(t)|^2]\ e^{jk_c(i-j)d\sin\theta_m} + \sigma^2\delta_{ij} \quad (7)$$

where δ_{ij} denotes the delta function. However, in case of random perturbations, the element is given by

$$r_{ij} = \sum_{m=1}^{M} E[|\alpha_m(t)|^2]\ e^{jk_c(\underline{u}_i - \underline{u}_j)^t \underline{\theta}_m} + \sigma^2\delta_{ij}$$

$$= \sum_{m=1}^{M} E[|\alpha_m(t)|^2]\ e^{jk_c(i-j)d\sin\theta_m} \cdot e^{jk_c(\Delta\underline{u}_i - \Delta\underline{u}_j)^t \underline{\theta}_m} + \sigma^2\delta_{ij} \quad (8)$$

The factor $e^{jk_c(\Delta\underline{u}_i - \Delta\underline{u}_j)^t \underline{\theta}_m}$ in (8) represents the effect of random perturbations in sensor locations. Moreover, it destroys the Toeplitz structure of R and hence degrades the performance of eigenstructure techniques for bearing estimation. To reduce the effect of $e^{jk_c(\Delta\underline{u}_i - \Delta\underline{u}_j)^t \underline{\theta}_m}$ and thus restore the effectiveness of eigenstructure approaches may be achieved by reconstructing the Toeplitz structure for R.

The TAM of [6] was proposed for finding directions of coherent sources. In the following, we describe the use of TAM for eliminating the effect of random perturbations. After obtaining the r_{ij} of (8) from the received signal, we construct the following matrix $\tilde{R}$ with size L x L.

$$\tilde{R} = \begin{bmatrix} \tilde{r}(0) & \tilde{r}(-1) & \tilde{r}(-2) \cdots \\ \tilde{r}(1) & \tilde{r}(0) & \tilde{r}(-1) \\ \tilde{r}(2) & \tilde{r}(1) & \tilde{r}(0) \\ \vdots & & & \ddots \end{bmatrix}$$

The elements of $\tilde{R}$ are given as

$$\tilde{r}(-n) = \frac{1}{L-n} \sum_{\ell=1}^{L-n} r_{\ell(\ell+n)}, \quad 0 \leqslant n < L,$$

$$\tilde{r}(n) = \tilde{r}^*(-n) \quad (9)$$

and "*" denotes complex conjugate. From (8) and (9), we have

$$\tilde{r}(-n) = \sum_{m=1}^{M} E[|\alpha_m(t)|^2]\ e^{-jk_c nd\sin\theta_m} \cdot g(n, \theta_m) + \sigma^2\delta_{on} \quad (10)$$

where

$$g(n, \theta_m) = \frac{1}{L-n} \sum_{\ell=1}^{L-n} e^{jk_c(\Delta\underline{u}_\ell - \Delta\underline{u}_{\ell+n})^t \underline{\theta}_m} \quad (11)$$

The $g(n, \theta_m)$ of (11) represents the random factor due to the random perturbations in sensor locations. Taking the expectation of $g(n, \theta_m)$, we obtain

$$E[g(n, \theta_m)] = E[e^{jk_c(\Delta\underline{u}_\ell - \Delta\underline{u}_{\ell+n})^t \underline{\theta}_m}] = \begin{cases} e^{-\sigma_\ell^2 k_c^2}, & 1 \leqslant n \leqslant L-1 \\ 1, & n = 0 \end{cases} \quad (12)$$

The variance of $g(n, \theta_m)$ is given as

$$\mathrm{Var}[g(n, \theta_m)] = E[|g(n, \theta_m)|^2] - |E[g(n, \theta_m)]|^2$$

$$
= \begin{cases} 0 & , n = 0 \\ \frac{1}{(L-n)^2}\,[(L-n) + 2(L-2n)A^3 + (5n-3L)A^2] & , 1 \leq n < \frac{L}{2} \\ \frac{1}{(L-n)}\,(1-A^2) & , \frac{L}{2} \leq n \leq L-1 \end{cases} \tag{13}
$$

While the variance of $e^{jk_c(\Delta \underline{u}_\ell - \Delta \underline{u}_{\ell+n})^t \underline{\theta}_m}$ is given as

$$
\mathrm{Var}\,[e^{jk_c(\Delta \underline{u}_\ell - \Delta \underline{u}_{\ell+n})^t \underline{\theta}_m}] = \begin{cases} 0 & , n = 0 \\ 1-A^2 & , 1 \leq n \leq L-1 \end{cases} \tag{14}
$$

where $A = e^{-\sigma_e^2 k_c^2}$. From (12), we note that the expectation of $g(n, \theta_m)$ in $\tilde{r}(-n)$ is the same as that of $e^{jk_c(\Delta \underline{u}_\ell - \Delta \underline{u}_{\ell+n})^t \underline{\theta}_m}$ in $r_{\ell(\ell+n)}$. However, (13) shows that the variance due to random perturbation is reduced in $\tilde{r}(-n)$. Furthermore (10) can be approximately written as

$$
\tilde{r}(-n) = \sum_{m=1}^{M} E[\,|\alpha_m(t)|^2\,]\, e^{-jk_c n d \sin\theta_m} \cdot A + \sigma^2 \delta_{on} \tag{15}
$$

when L is large enough. Comparing (7) and (15), we note that the signal subspace of $\tilde{R}$ is therefore the same as that of $\hat{R}$. Thus any existing technique for bearing estimation based on the use of eigenstructure of $\tilde{R}$ can be utilized to resolve the directions of the M sources.

Consider the performance of the proposed method. From (15), we note that the expectation A of $g(n, \theta_m)$ determines the effectiveness of this technique when L is large enough. Hence the performance turns out to be dependent of the variance σ_e^2 of sensor location errors. It will be degraded if σ_e^2 is large because the effective signal-to-noise ratio (SNR) is reduced. On the other hand, when L is not large enough and σ_e^2 is small, the variance of $g(n, \theta_m)$ dominately determines the performance.

From the above description, it can be seen that the accuracy of the Toeplitz approximation depends upon $E[g(n, \theta_m)]$ and $\mathrm{Var}\,[g(n, \theta_m)]$. Small $E[g(n,\theta_m)]$ or large $\mathrm{Var}\,[g(n, \theta_m)]$ makes the approximation inadequate and thus deteriorates the performance of the TAM. To circumvent this problem, we present a modification for the TAM. The elements of $\tilde{R}$ are replaced by

$$
\tilde{r}_M(-n) = \beta(-n)\, e^{j\alpha(-n)} \tag{16}
$$

where $\beta(-n) \triangleq \frac{1}{L-n} \sum_{\ell=1}^{L-n} |r_{\ell(\ell+n)}|$

and $\alpha(-n) \triangleq \mathrm{ARG}\,(\tilde{r}(-n))$ represents the phase of $\tilde{r}_M(-n)$. From (9) and (16), it can be shown that the magnitude of $\tilde{r}_M(-n)$ is greater than that of $\tilde{r}(-n)$. Hence the modified TAM (MTAM) tends to reduce the effect of $E[g(n, \theta_m)]$ and $\mathrm{Var}[g(n, \theta_m)]$ on the approximation made in (15).

IV. SIMULATION RESULTS

In this section, we present several computer simulations to illustrate the effectiveness of the decribed approaches. A linear array with L = 20 and $d = \lambda/2$ (λ = wave length) was used. The random perturbations $\{\Delta x_i, \Delta y_i\}$, i = 1, 2, ..., L were generated from a Gaussian random number generator. 100 snapshots were taken to estimate the covaiance matrix R. The size of $\tilde{R}$ used for the simulations is 10 x 10. All sources are equally powered with SNR = 10dB. Figure 1 shows the results of the MUSIC based on R using nominal sensor locations for searching, the MUSIC based on R using actual sensor locations for searching, and the proposed TAM for $\sigma_e^2 = 0.01\lambda^2$. In this case, two incoherent sources with 10° separation were used. Obviously, TAM has better performance. In the next example, two coherent sources had 10° difference in direction and the variance σ_e^2 of random perturbation was equal to $0.01\lambda^2$. The above three approaches were used again. Figure 2 shows the simulation results. We observe that TAM possesses the capability to resolve coherent sources. To compare the performances of TAM and MTAM, we present the third example. In this case, the separation of the two incoherent sources was reduced to 5°. Figure 3 shows the result for MUSIC, TAM and MTAM. It can be seen that MTAM shows better performance than TAM. Finally, we increased the σ_e^2 to examine the effectiveness of MTAM. The simulation results are shown in Figure 4. In this case, σ_e^2 was set to $0.05\lambda^2$ and the two incoherent sources had 10° separation. Again, we observe that MTAM is more effective than TAM.

V. CONCLUSIONS

In this paper, we have presented two approaches, i.e., the TAM and the modified TAM for bearing estimation using linearly periodic arrays. The proposed approaches do not require calibration process when arrays suffer from 2-D random perturbation in sensor locations. Simulation examples show that the approaches are capable of estimating bearings for coherent and incoherent sources. Furthermore, the modified TAM appears to be more effective than the TAM. More detailed characteristics of the TAM and the MTAM for bearing estimation with the presence of random perturbation in sensor locations are currently under investigation.

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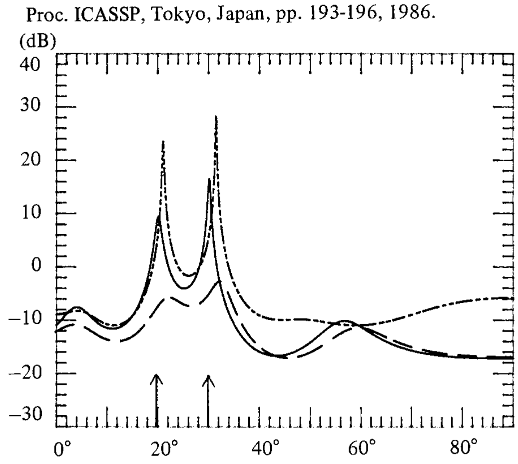

Figure 1. Bearing Spectra of Incoherent Sources with $\sigma_e^2 = 0.01\ \lambda^2$

—— ·· TAM Method

—— MUSIC Using Actual Sensor Locations

— — MUSIC Using Nominal Sensor Locations

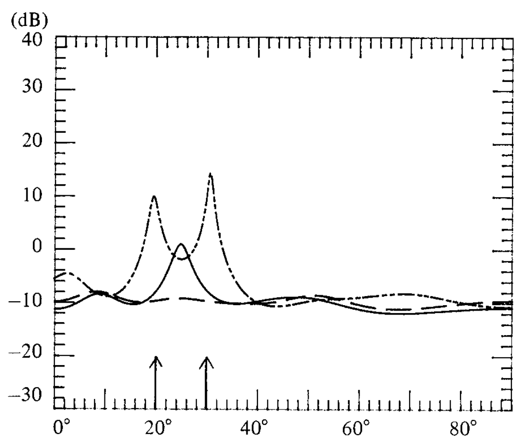

Figure 2. Bearing Spectra of Coherent Sources with $\sigma_e^2 = 0.01\ \lambda^2$

—— ·· TAM Method

—— MUSIC Using Actual Sensor Locations

— — MUSIC Using Nominal Sensor Locations

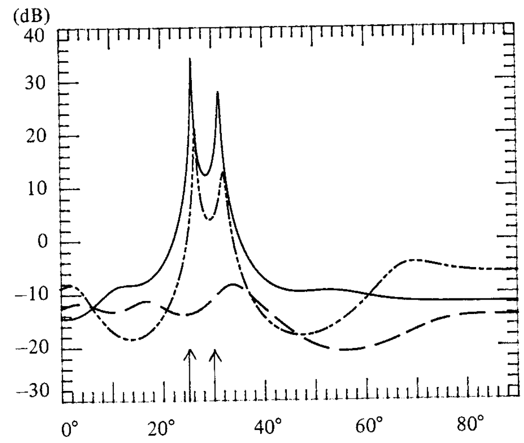

Figure 3. Bearing Spectra of Incoherent Sources with $\sigma_e^2 = 0.01\ \lambda^2$

—— ·· TAM Method

—— Modified TAM Method

— — MUSIC Using Nominal Sensor Locations

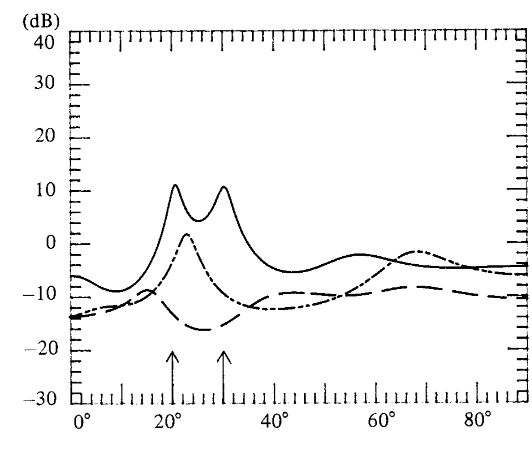

Figure 4. Bearing Spectra of Incoherent Sources with $\sigma_e^2 = 0.05\ \lambda^2$

—— ·· TAM Method

—— Modified TAM Method

— — MUSIC Using Nominal Sensor Locations