

GENERALIZED INTERPRETATION AND PREDICTION IN MICROWAVE  
IMAGING INVOLVING FREQUENCY AND ANGULAR DIVERSITY

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## I. INTRODUCTION

In microwave imaging involving frequency and angular diversity, the range-corrected field data (or the Fourier space data) are accessed in a polar format if the object is rotating while the transmitter and receiver are kept stationary (hereafter this imaging is referred as monostatic imaging); and are accessed in an Ewald sphere format if the object is kept stationary while the bistatic angle between the transmitter and receiver is changed (this imaging is referred as bistatic imaging). The microwave image is then obtained by Fourier transforming the Fourier space data arranged in the mentioned format. Either the above Fourier transform (FT) method or the back-projection method can be employed to reconstruct the image. In this paper we will prove that the above two methods are equivalent in both the monostatic and bistatic cases. However, the back-projection method provides more physical insight to the image formation process. Microwave images can then be predicted from the steps involved in the back-projection method.

## II. Frequency / Angular Diversity Imaging and Ewald Sphere Construction

A metallic object is illuminated by a plane wave with wavenumber  $k$ . Denote  $\hat{l}_i$  the unit vector directed from the transmitting antenna to the reference point,  $\hat{l}_r$  the unit vector directed from the reference point to the receiving antenna;  $\beta$  the bistatic angle;  $\hat{l}_p$  the unit vector bisecting the transmitter and receiver;  $\hat{n}(\vec{r}')$  the unit vector normal to the surface at point  $\vec{r}'$ ;  $\hat{e}_i$  the unit vector of the incident magnetic field;  $\hat{e}_r$  the polarization of the receiving antenna. Under the far field and the physical optics (PO) approximation the P-space data or the Fourier space data  $\Gamma(\vec{p})$  is

$$\Gamma(\vec{p}) = \Gamma(\hat{l}_r, \hat{l}_i, k) = \int_{S_{ill}} \gamma(\vec{r}') e^{j\vec{p} \cdot \vec{r}'} d\vec{r}' \quad (1)$$

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where

$$\gamma(\vec{r}') = [\hat{n}(\vec{r}') \times \hat{e}_i] \cdot \hat{e}_r^* \quad (2)$$

$$\vec{p} = k(\hat{l}_r - \hat{l}_i) = 2k \cos \beta/2 \hat{l}_p \quad (3)$$

$S_{ill}$  indicates the illuminated surface region of the object and  $*$  represents the complex conjugate.

The accessed Fourier space data can be graphically represented by an Ewald sphere. For a given  $k l_i = \vec{k}_i$ , the incident vector, the Ewald sphere is the sphere intersecting the origin of the Fourier space with the radial vector  $\vec{k}_i$ . For the monostatic diversity imaging, rotating the object is equivalent to rotating the transmitting / receiving antenna; while frequency diversity is equivalent to varying the wavenumber  $k$ . Distribution of the sampled point in the Ewald sphere representation is shown in Fig. 1. For the bistatic diversity imaging, changing the direction of the receiving antenna will change the direction of  $\vec{p}$ . For a given bistatic angle  $\beta$  the equivalent frequency becomes  $k \cos(\beta/2)$ , and the sampled Fourier space data are distributed on the radial line along the direction of  $\hat{l}_p$  as shown in Fig. 1.

By the fact that the Fourier space data  $\Gamma(\vec{p})$  and the scattering characteristic function  $\gamma(\vec{r})$  have an approximate (but not exact) Fourier transform relationship [1], the microwave image of an object is usually obtained by Fourier transforming the Fourier space data directly.

### III. Equivalence Between the FT Method and the Back-projection Method

Let  $\Psi(k, \phi)$  denotes the range-corrected field at wavenumber  $k$  and rotation angle  $\phi$ . The frequency coverage is from  $k_{min}$  to  $k_{max}$ , and the angular coverage is from  $\phi_{min}$  to  $\phi_{max}$ . The image  $f(r, \theta)$  reconstructed from the specified frequency and angular windows can be written by

$$f(r, \theta) = \int_{\phi_{min}}^{\phi_{max}} \left[ \int_{k_{min}}^{k_{max}} k \Psi(k, \phi) e^{-j2kr \cos(\theta-\phi)} dk \right] d\phi \quad (4)$$

$$= \int_{\phi_{min}}^{\phi_{max}} B[r \cos(\theta-\phi)] d\phi \quad (5)$$

where  $B[\ ]$  is the Fourier transform of the product of the wavenumber  $k$  and the range-corrected field with respect to  $2k$ ; and  $|f(r, \theta)|^2$  is the image intensity at the image pixel  $(r, \theta)$ . The physical meaning of  $B[\ ]$  is the range profile of the object observed at the aspect angle  $\phi$ , while the integration of Eq. (5) with respect to  $\phi$  is a process of back-projection. From the above derivation, one can easily see that the FT method and the back-projection method are equivalent in the monostatic diversity imaging.

In the bistatic case the data points are distributed along the radial line of  $\phi = \beta/2$  for a given bistatic angle  $\beta$ , and the effective frequency coverage has been reduced from  $(k_{\min}, k_{\max})$  to  $(k_{\min} \cos \beta/2, k_{\max} \cos \beta/2)$ . Assume the bistatic angular coverage is from  $\beta_{\min}$  to  $\beta_{\max}$ , then the bistatic image  $\hat{f}(r, \theta)$  at the image point  $(r, \theta)$  is expressed by

$$\hat{f}(r, \theta) = \int_{\beta_{\min}}^{\beta_{\max}} \left[ \int_{k_{\min} \cos \phi/2}^{k_{\max} \cos \phi/2} k \Psi'(k, \phi) e^{-j2kr \cos(\theta - \phi/2)} dk \right] d\phi \quad (6)$$

$$= \int_{\beta_{\min}}^{\beta_{\max}} B'[r \cos(\theta - \phi/2)] d\phi \quad (7)$$

where  $\Psi'(k, \phi)$  is the range-corrected field at wavenumber  $k$  and bistatic angle  $\phi$ , and  $B'[\ ]$  is the bistatic range profile at the bistatic angle  $\phi$ . From the above derivation one can conclude that the bistatic image is obtained in two steps: first obtaining the bistatic range profile for each bistatic angle, and then back-projecting the range profile to the image plane along the directions normal to the bisection line  $\hat{l}_p$ .

In a summary, the FT method and the back-projection method are equivalent, but the back-projection method gives more physical insight. To predict the microwave image of an object given an imaging scheme and specified frequency and angular windows, one can first predict the range profile of each aspect, and then back-project the range profile along the corresponding directions into the image plane graphically.

## V. Examples

Consider an infinite metallic cylinder with radius 20cm. The cylinder's axis is normal to the azimuthal plane. The transmitting and receiving antennas have identical linear polarization which is parallel to the axis of cylinder.

The specular point is located at the point with its surface normal vector parallel to the incident wave in the monostatic case and is at the point with its surface normal vector parallel to the bisection line in the bistatic case. The scattering strength of the specular point is proportional to  $[\cos(\beta/2)]^{1/2}$ , where  $\beta$  is the bistatic angle [2]. Sketches of the back-projection for these two cases are shown in Figs. 2(a) and (b) respectively. Images reconstructed by the FT method from the data collected over the angular window  $1(-180^\circ$

to  $180^\circ$ ) and window  $2(-80^\circ \text{ to } 80^\circ)$  for the monostatic case and from data collected from window  $3(\beta_{\min}/2 = -80^\circ \text{ to } \beta_{\max}/2 = 80^\circ)$  for the bistatic case are shown in Figs. 2(c), (d) and (e) respectively. It is seen that Fig. 2(d) and Fig. 2(e) have similar appearance, however, the monostatic image has better resolution. Shown in Fig. 2(f) is the bistatic image reconstructed by the back-projection method using the same data as that of Fig. 2(e). Figures 2(e) and 2(f) show the equivalence between the FT method and the back-projection method.

#### REFERENCE

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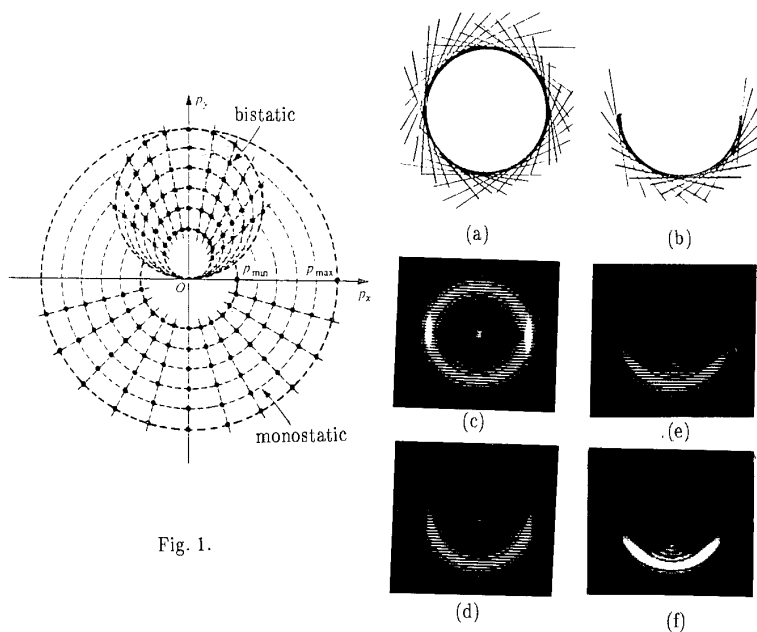


Fig. 1.

Fig. 2