ON COMPUTING THE OPTIMAL WEIGHTS FOR ADAPTIVE ARRAY PROCESSING

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ABSTRACT

Based on the Hermitian and Toeplitz properties of the signal correlation matrix, we develop an approach to recursively compute the adaptive weights required for optinum processing of array signals. The presented approach finds the elements of the corresponding adaptive weight vector with size n from the elements of the adaptive weight vectors with size less than n. There are no matrix inversions required during the computation process. This results in a saving in the number of operations and storage locations. Simulation result demonstrating the proposed approach is given.

I. PROBLEM FORMULATION

Consider the case of a linearly periodic adaptive array with N isotropic receiving sensor elements. The signal data vector $\underline{X}$ received by the sensors has the correlation matrix R given by $R = E\{\underline{X}\ \underline{X}^H\}$. The ideal correlation matrix R has the properties of Hermitian and Toeplitz structure. Based on the $MMSE$ criterion, the problem of adaptive array processing can be described as follows [1]. It is desired to select the weight vector $\underline{W}$ to minimize the mean squared error between the desired array response d and the actual array output $\underline{W}^H\underline{X}$. That is,

$$\underset{\underline{W}}{\text{Minimize}} \quad E\{\,|\,d - \underline{W}^H \underline{X}\,|^2\} \tag{1}$$

To find the Wiener weight vector $\underline{W}_0$ from (1), any algorithm based on the direct matrix inverse (DMI) of R possesses more rapid convergence than the least-mean-square (LMS) or maximum signal-to-noise ratio (SNR) algorithms. However, the computational burden due to DMI method is about $(L^3/2 + 2L^2)$ complex multiplications [1], where L is the number of adaptive weights used. Hence the value of L dominates the overall computational requirement. Furthermore, DMI may cause computational difficulties when R is ill-conditioned.

II. WEIGHT RECURSION BASED ON IDEAL CORRELATIONS

Let the $\underline{X}_n = [x_1\ x_2\ \cdots\ x_n]^T$ denote the input signal vector of size n received by n successive elements of a linearly periodic array and the corresponding correlation matrix of size n be given by $R_n = E\{\underline{X}_n\ \underline{X}_n^H\}$. The optimal weight vector $\underline{W}_{on} =$

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$[w_{n1}\ w_{n2}\ \cdots\ w_{nn}]^T$ of size n is the solution of the following equation $R_n\ \underline{W}_{on} = \underline{P}_n$ where $\underline{P}_n = E\{d^* \underline{X}_n\} = [p_1\ p_2\ \cdots\ p_n]^T$ denotes the cross-correlation vector of the desired array response d and input signal vector $\underline{X}_n$. $*$ denotes the complex conjugate. The ith element of $\underline{P}_n$ is $p_i = E\{d^* x_i\}$, $i = 1, 2, \cdots n$. To derive the desired order-update formula for $\underline{W}_{on}$, we start with the case of $n = 3$. It follows that

$$R_3 \underline{W}_{03} = \begin{bmatrix} r(0) & r(1) & r(2) \\ r^*(1) & r(0) & r(1) \\ r^*(2) & r^*(1) & r(0) \end{bmatrix} \begin{bmatrix} w_{31} \\ w_{32} \\ w_{33} \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} \tag{2}$$

where $r(j) = E\ \{x_i\ x_{i+j}^*\}$, $j = 0, 1, 2$. Since the correlation matrix R possesses the property that R_n is Toeplitz and thus contains as subblocks all the lower order correlation matrices, we can write

$$R_3 \begin{bmatrix} w_{21} \\ w_{22} \\ 0 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ q_3 \end{bmatrix} \tag{3}$$

where q_3 is given by $q_3 = r^*(2)\ w_{21} + r^*(1)\ w_{22}$. From (2) and (3), we have

$$R_3 \left(\begin{bmatrix} w_{33}^* \\ w_{32}^* \\ w_{31}^* \end{bmatrix} - \begin{bmatrix} 0 \\ w_{22}^* \\ w_{21}^* \end{bmatrix} \right) = \begin{bmatrix} p_3^* - q_3^* \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

Next, consider the case of $n = 4$. From (4), it follows that

$$R_4 \left(\begin{bmatrix} w_{44}^* \\ w_{43}^* \\ w_{42}^* \\ w_{41}^* \end{bmatrix} - \begin{bmatrix} 0 \\ w_{33}^* \\ w_{32}^* \\ w_{31}^* \end{bmatrix} \right) = \begin{bmatrix} p_4^* - q_4^* \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

where q_4 is given by $q_4 = r^*(3)\ w_{31} + r^*(2)\ w_{32} + r^*(1)\ w_{33}$. From the results of (3) and (4), we may write

$$R_4 \left(\begin{bmatrix} w_{33}^* \\ w_{32}^* \\ w_{31}^* \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ w_{22}^* \\ w_{21}^* \\ 0 \end{bmatrix} \right) = \begin{bmatrix} p_3^* - q_3^* \\ 0 \\ 0 \\ \Delta_3^* \end{bmatrix} \tag{6}$$

where Δ_3 is given by $\Delta_3 = (w_{31} - w_{21})\ r(1) + (w_{32} - w_{22})\ r(2) + w_{33}\ r(3)$. From the matrix coefficients of (5) and (6), it is appropriate to relate them as follows

$$\begin{bmatrix} w_{44}^* \\ w_{43}^* \\ w_{42}^* \\ w_{41}^* \end{bmatrix} - \begin{bmatrix} 0 \\ w_{33}^* \\ w_{32}^* \\ w_{31}^* \end{bmatrix} = \alpha \left(\begin{bmatrix} w_{33}^* \\ w_{32}^* \\ w_{31}^* \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ w_{22}^* \\ w_{21}^* \\ 0 \end{bmatrix} \right) + \beta \left(\begin{bmatrix} 0 \\ w_{31} \\ w_{32} \\ w_{33} \end{bmatrix} - \begin{bmatrix} 0 \\ w_{21} \\ w_{22} \\ 0 \end{bmatrix} \right) \tag{7}$$

The results shown in (7) can be straightforwardly generalized to any order n as follows

$$\begin{bmatrix} w_{nn}^* \\ w_{n,n-1}^* \\ \vdots \\ w_{n1}^* \end{bmatrix} = \begin{bmatrix} 0 \\ w_{n-1,n-1}^* \\ \vdots \\ w_{n-1,1}^* \end{bmatrix} + \alpha \begin{bmatrix} w_{n-1,n-1}^* \\ w_{n-1,n-2}^* - w_{n-2,n-2}^* \\ \vdots \\ w_{n-1,1}^* - w_{n-2,1}^* \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ w_{n-1,1} - w_{n-2,1} \\ \vdots \\ w_{n-1,n-2} - w_{n-2,n-2} \\ w_{n-1,n-1} \end{bmatrix} \tag{8}$$

and

$$\alpha \begin{bmatrix} p_{n-1}^* - q_{n-1}^* \\ 0 \\ \vdots \\ 0 \\ \Delta_{n-1}^* \end{bmatrix} + \beta \begin{bmatrix} \Delta_{n-1} \\ 0 \\ \vdots \\ 0 \\ p_{n-1} - q_{n-1} \end{bmatrix} = \begin{bmatrix} p_n^* - q_n^* \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{9}$$

where q_n and Δ_{n-1} are given by

$$q_n = \sum_{i=1}^{n-1} r^*(n-i)\, w_{n-1,i} \quad and \quad \Delta_{n-1} = \sum_{i=1}^{n-1} r(i)\,(w_{n-1,i} - w_{n-2,i})$$

respectively. By solving (9), the scalars α and β in (8) can be found as

$$\alpha = \frac{(p_{n-1} - q_{n-1})(p_n - q_n)^*}{|\, p_{n-1} - q_{n-1}\,|^2 - \,|\,\Delta_{n-1}\,|^2} \quad and \quad \beta = \frac{-\alpha\, \Delta_{n-1}^*}{p_{n-1} - q_{n-1}}$$

Therefore, the optimal weight vector with size n can be recursively computed from the optimal weight vectors with sizes $n-1$ and $n-2$ without the need of matrix inversion.

We now evaluate the computational complexity of the proposed recursive procedure in terms of the required multiplications. To initiate the recursion, computing w_{11} and $[\,w_{21}\; w_{22}]^T$ needs 6 complex multiplications (CM) and 3 real multiplications (RM). From q_n and Δ_{n-1}, we note that finding the q_n and Δ_{n-1} both require $(n-1)$ CM. To compute α requires 3 CM and 1 RM. It requires 2 CM and 1 RM to compute β. Furthermore, computing the nth-order weight vector from (8) needs $(2n-2)$ CM. Therefore, when using a linearly periodic array with N elements, the proposed order-update recursion requires about

$$6 + \sum_{n=3}^{N} (4n+1) = 2N^2 + 3N - 8 \quad CM \quad \text{and} \quad 2N-1 \quad RM \tag{10}$$

in order to obtain the Nth-order optimal weight vector. (10) indicates that the main computational burden is $O\,(N^2)$ complex multiplications. This represents a factor of N savings over solving the optimal weight vector by direct matrix inversion.

III. SIMULATION RESULTS

We examine the application of the proposed recursive approach in adaptive array beamforming. In the example under consideration, a linearly periodic array of 15 elements in which the weights are derived adaptively using the a priori information contained in the steering vector of steering angle = 0°. The optimal weight vector $\underline{W}_0$ satisfies [3] $R\,\underline{W}_0 = \underline{S}$, where $\underline{S}$ is the steering vector. There are two incoherent jammers of $(JNR) = 20\ dB$ and $30\ dB$ at 20° and 30° with respect to the broadside, respectively. The array noise is taken to be spatially white Gaussian. The interelement spacing is half wavelength.

Let the required correlation matrix be the ideal R. Then we compute the optimal weight vector using the DMI method and the proposed order-update recursion, respectively. The resulting array pattern is shown in Figure 1, for DMI method (dotted line), and order-update recursion (solid line). We note that these two patterns are almost the same. This confirms the theoretical work described in Section II.

REFERENCES

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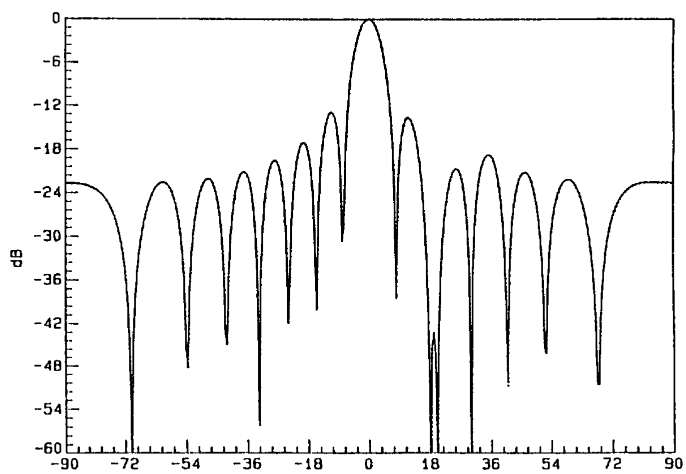

Figure 1