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數位式離散分數訊號轉換及其應用(2/3)

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Digital Discrete Fractional Signal Transforms and its Applications (2

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摘要

本研究提出一新穎近似三行對角交換矩陣(Nearly Tridiagonal Commuting Matrices)， 其埃根向量更能逼近類比赫曼高斯函數(Hermite-Gaussian Functions)，可以應用到數位式離散分數傅立葉轉換及其應用(Discrete Fractional Fourier Transform)。

ABSTRACT

Based on discrete Hermite-Gaussian like functions, a discrete fractional Fourier transform (DFRFT) which provides sample approximations of the continuous fractional Fourier transform was defined and investigated recently. In this paper, we propose a new nearly tridiagonal matrix which commutes with the discrete Fourier transform (DFT) matrix. The eigenvectors of the new nearly tridiagonal matrix are shown to be better discrete Hermite-Gaussian like functions than those developed before. Furthermore, by appropriately combining two linearly independent matrices which both commute with the DFT matrix, we develop a method to obtain even better discrete Hermite-Gaussian like functions. Then, new versions of DFRFT produce their transform outputs more close to the samples of the continuous fractional Fourier transform, and their application is illustrated.

1. INTRODUCTION

The a^{th}-order continuous fractional Fourier transform (FRT) of *x(t)* is defined as [4]

$$X_a(u)=\int_{-\infty}^{+\infty} x(t)K_a(t,u)dt, \qquad (1)$$

where the transform kernel $K_a(t,u)$ is given by

$$K_a(t,u)=\sqrt{1-j\cot a}\cdot e^{jp(t^2\cot a-2tu\csc(a)+u^2\cot a)}, \qquad (2)$$

in which $\alpha = a\pi/2$. It is known that the transform kernel $K_a(t,u)$ can also be written as [4]

$$K_a(t,u)=\sum_{n=0}^{\infty}\exp(-jnap/2)\cdot\Psi_n(t)\Psi_n(u), \qquad (3)$$

where

$$\Psi_n(t)=\frac{2^{1/4}}{\sqrt{2^n n!}}H_n(\sqrt{2p}\cdot t)e^{-p\cdot t^2} \qquad (4)$$

is the n^{th}-order Hermite-Gaussian function with H_n being the n^{th}-order Hermite polynomial.

The $N\times N$ DFT matrix $\mathbf{F}$ is defined by

$$\mathbf{F}_{kn}=(1/\sqrt{N})\cdot e^{-j\frac{2p}{N}kn},\quad 0\le k,n\le N-1. \qquad (5)$$

In [5], Dickinson and Steiglitz introduced an $N\times N$ nearly tridiagonal matrix $\mathbf{S}$ whose nonzero entries are:

$$\begin{aligned}\mathbf{S}_{n,n}&=2\cos(\frac{2p}{N}\cdot n),\quad 0\le n\le(N-1)\\ \mathbf{S}_{n,n+1}&=\mathbf{S}_{n+1,n}=1,\quad 0\le n\le(N-2)\\ \mathbf{S}_{N-1,0}&=\mathbf{S}_{0,N-1}=1.\end{aligned} \qquad (6)$$

With $\mathbf{S}$ defined above, $\mathbf{S}$ commutes with $\mathbf{F}$, i.e., $\mathbf{SF}=\mathbf{FS}$. Therefore, the DFT matrix $\mathbf{F}$ and the above matrix $\mathbf{S}$ share a common eigenvector set and we can find the eigenvectors of $\mathbf{F}$ from those of the matrix $\mathbf{S}$ [3].

Analogous to the spectral expansion of the continuous FRT kernel $K_a(t,u)$ in (3), and from the fact that the eigenvectors of $\mathbf{S}$ can be used as the discrete Hermite-Gaussian like functions, in [2], Pei et al. defined the a^{th}-order DFRFT matrix $\mathbf{F}_{\mathbf{S}}^a$ by

$$\mathbf{F}_{\mathbf{S}}^a=\mathbf{V}\mathbf{D}^a\mathbf{V}^T=\begin{cases}\sum_{k=0}^{N-1}e^{-j\frac{p}{2}ka}\mathbf{v}_k\mathbf{v}_k^T, & \text{for } N \text{ odd}\\ \sum_{k=0}^{N-2}e^{-j\frac{p}{2}ka}\mathbf{v}_k\mathbf{v}_k^T+e^{-j\frac{p}{2}Na}\mathbf{v}_N\mathbf{v}_N^T, & \text{for } N \text{ even}\end{cases} \qquad (7)$$

where T denotes the matrix transpose, the matrix $\mathbf{V}=[\mathbf{v}_0\,|\,\mathbf{v}_1\,|\,\mathbf{L}\,|\,\mathbf{v}_{N-2}\,|\,\mathbf{v}_{N-1}]$ for odd N and $\mathbf{V}=[\mathbf{v}_0\,|\,\mathbf{v}_1\,|\,\mathbf{L}\,|\,\mathbf{v}_{N-2}\,|\,\mathbf{v}_N]$ for even N, $\mathbf{D}$ is a diagonal matrix with its diagonal entries corresponding to the eigenvalues for each column eigenvectors in $\mathbf{V}$, and $\mathbf{v}_k$ is the k^{th}-order discrete Hermite-Gaussian like function with k zero-crossings and is obtained from the corresponding normalized eigenvector of $\mathbf{S}$. The $\mathbf{S}$-based DFRFT of $\mathbf{x}$ can be easily obtained by $\mathbf{y}_a=\mathbf{F}_{\mathbf{S}}^a\mathbf{x}$.

2. A NEW NEARLY TRIDIAGONAL COMMUTING MATRIX T

In [1], Grünbaum introduced an exactly tridiagonal matrix commuting with the centered discrete Fourier transform matrix of even size. Inspired by the work of Grünbaum, we propose in this section a novel nearly tridiagonal matrix which commutes with the ordinary DFT matrix of any size, even or odd. Moreover, we will demonstrate that its eigenvectors approximate samples of the continuous Hermite-Gaussian functions better than those of