U6.19

ESTIMATING TWO-DIMENSIONAL ANGLES OF ARRIVAL IN COHERENT SOURCE ENVIRONMENT

Chien-Chung Yeh, Ju-Hong Lee, and Yih-Min Chen

Department of Electrical Engineering
National Taiwan University
Taipei, Taiwan, Republic of China

ABSTRACT

In this work, estimating two-dimensional (2-D) angle of arrival for radiating sources in a coherent environment is studied. The concept of spatial smoothing is first extended to a rectangular planar array and a 2-D search function is formed to estimate the source directions. To avoid performing a 2-D search, an approach based on two one-dimensional (1-D) searches is also discussed. This approach uses rows and columns of the rectangular array to perform 1-D searches. To match the data obtained, a 2-D verification is then performed. Computer simulation results for both approaches based on the MUSIC method are presented.

1. INTRODUCTION

Eigenstructure methods for estimating the angles of arrival (AOA) of radiating sources have been studied extensively. For a non-coherent source environment, those methods use the property that the rank of the signal component of the covariance matrix equals the number of sources. In this case, the signal and noise subspaces can be obtained from the eigenstructure of the covariance matrix [1]. That property does not hold, however, for coherent sources. To resolve coherent sources, several methods have been discussed [2-5]. Among them the spatial smoothing approach groups an equally spaced array into subarrays. Covariance matrices of the subarrays are averaged to form the spatially smoothed covariance matrix. It was shown that the signal and noise subspaces can be obtained from the eignstructure of the smoothed covariance matrix [3-4].

In this paper, we extend that approach to estimate two-dimensional (2-D) AOAs of coherent sources using a rectangular planar array. We first describe a 2-D spatial smooth and search process for estimating the AOAs. Following this, we discuss the approach of performing two one-dimensional (1-D) spatial smoothes and searches and then carrying out a 2-D verification of the data obtained in 1-D processing. This avoids performing a 2-D search and therefore saves computation. Simulation results for both approaches are presented.

2. TWO-DIMENSIONAL SPATIAL SMOOTHING

Let us consider the problem of estimating the 2-D AOAs of K coherent narrow-band sources by the use of an M x N rectangular array. The element spacings along the X and Y axes are d_x and d_y, respectively. The AOA of the kth source is denoted by (θ_k, ϕ_k) as shown in Fig. 1. The input of the mnth array element, which is located at the intersection of the mth row and the nth column, can be written as

$$v_{mn}(t) = \sum_{k=1}^{K} \rho_k \, \alpha_1(t) \exp [\, jk_c \, (m d_x u_k + n \, d_y \, v_k)] + n_{mn}(t) \tag{1}$$

where $\alpha_1(t)$ is the complex envelope of the 1st source at the origin, ρ_k is a complex number representing the relative amplitude and phase between the kth and the 1st sources ($\rho_1 = 1$), k_c is the wavenumber of the carrier frequency, $u_k = \sin\theta_k \cos\phi_k$, $v_k = \sin\theta_k \sin\phi_k$, and $n_{mn}(t)$ is the complex noise of the mnth element. Using Eq. (1), the input vector of the mth row of the array, which has a size of $N \times 1$, can be written as

$$\underline{v}_m(t) = [\, v_{m1}(t), \ldots, v_{mN}(t)]^T = \sum_{k=1}^{K} \rho_k \, \alpha_1(t) \, \underline{s}_{km} + \underline{n}_m(t) \tag{2}$$

In Eq. (2) $\underline{s}_{km}$ is the phase vector of the kth source at the mth row,

$$\underline{s}_{km} = [\, \exp [jk_c (m d_x u_k + d_y v_k)], \ldots, \exp [jk_c (m d_x u_k + N d_y v_k)]\,]^T \tag{3}$$

and $\underline{n}_m(t)$ is the noise vector of the mth row,

$$\underline{n}_m(t) = [\, n_{m1}(t) \, n_{m2}(t), \ldots, n_{mN}(t)]^T \tag{4}$$

The input vector of the rectangular array can be defined as the MN x 1 vector given by

$$\underline{v}(t) = [\underline{v}_1^T(t), \ldots, \underline{v}_M^T(t)]^T = \alpha_1(t) \, (\sum_{k=1}^{K} \rho_k \underline{s}_k) + \underline{n}(t) \tag{5}$$

where $\underline{s}_k$ is the phase vector of the kth source

$$\underline{s}_k = [\underline{s}_{k1}^T, \ldots, \underline{s}_{kM}^T]^T \tag{6}$$

and $\underline{n}(t)$ is the noise vector. The covariance matrix of $\underline{v}(t)$ is defined as

$$R = E\, [\underline{v}^*(t) \, \underline{v}^T(t)] \tag{7}$$

where 'E', '*' and 'T' denote expectation, conjugation and transposition respectively. With the assumption that noises are uncorrelated and have equal power, Eq. (7) becomes

CH2561-9/88/0000-2909 $1.00

$$R = E[\,|\alpha_1(t)|^2\,]\;(\sum_{k=1}^{K}\rho_k\,\underline{s}_k)^*\;(\sum_{k=1}^{K}\rho_k\,\underline{s}_k)^T + \sigma^2 I \quad (8)$$

where σ^2 is the noise power. Therefore, the signal component of R has a rank of one.

To reconstruct the signal subspace, the approach of spatial smoothing can be used. Let us use a simple example to demonstrate the spatial smoothing scheme for a 2-D array. The M x N rectangular array shown in Fig. 1. can be grouped into four (M−1) x (N−1) subarrays as shown in Fig. 2. Let $\underline{y}_{(i,j)}(t)$, i=1, 2 and j=1,2, be the input vector of the subarray which consists of the ith to the (M−2 +i)th rows and the jth to the (N−2+j)th columns of the array. From Eq. (5), $\underline{y}_{(1,1)}$ can be written as

$$\underline{y}_{(1,1)}(t) = \alpha_1(t)\,[\,\sum_{k=1}^{K}\rho_k\,\underline{s}_{k(1,1)}\,] + \underline{n}_{(1,1)}(t) \quad (9)$$

where $\underline{s}_{k(1,1)}$ is the phase vector of the kth source at the (1, 1)th subarray and $\underline{n}_{(1,1)}(t)$ is the noise. From Eq. (6), $\underline{s}_{k(1,1)}$ is given by

$$\underline{s}_{k(1,1)} = [\,\tilde{\underline{s}}_{k1}^T, \ldots, \tilde{\underline{s}}_{k(M-1)}^T\,]^T \quad (10)$$

where $\tilde{\underline{s}}_{km}$, m = 1, 2, ..., M−1, is the (N−1) x 1 vector which contains the first (N−1) entries of $\underline{s}_{km}$ given by Eq. (3).

In general, $\underline{y}_{(i,j)}(t)$ is given by

$$\underline{y}_{(i,j)}(t) = \alpha_1(t)\sum_{k=1}^{K}\rho_k\exp[\,j\phi_{k(i,j)}\,]\,\underline{s}_{k(1,1)} + \underline{n}_{(i,j)}(t) \quad (11)$$

where

$$\phi_{k(i,j)} = k_c\,[\,(i-1)d_x u_k + (j-1)d_y v_k\,] \quad (12)$$

The covariance matrix of the (i,j)th subarray can be expressed as

$$R_{(i,j)} = E[\,|\alpha_1(t)|^2\,]\{\sum_{k=1}^{K}\rho_k\exp[\,j\phi_{k(i,j)}\,]\,\underline{s}_{k(1,1)}\}^*\,\{\sum_{k=1}^{K}\rho_k\exp[\,j\phi_{k(i,j)}\,]\,\underline{s}_{k(1,1)}\}^T + \sigma^2 I \quad (13)$$

The special case of K = 2 is considered below for discussion. In this case, we can see from Eq. (13) that the signal subspaces of $R_{(i1,j1)}$ and $R_{(i2,j2)}$ are different if

$$\det\begin{bmatrix}\rho_1\exp[\,j\phi_{1(i_1,j_1)}\,] & \rho_1\exp[\,j\phi_{1(i_2,j_2)}\,]\\ \rho_2\exp[\,j\phi_{2(i_1,j_1)}\,] & \rho_2\exp[\,j\phi_{2(i_2,j_2)}\,]\end{bmatrix} \neq 0 \quad (14)$$

where 'det' denotes determinant. Under such circumstance, the spatially smoothed covariance matrix computed from $R_{(i1,j1)}$ and $R_{(i2,j2)}$, which is given by

$$\bar{R} = \tfrac{1}{2}\,[\,R_{(i1,j1)} + R_{(i2,j2)}\,], \quad (15)$$

has a signal subspace equal to the subspace spanned by $\{\underline{s}_{1(1,1)}, \underline{s}_{2(1,1)}\}$. Therefore, the noise subspace of $\bar{R}$ is orthogonal to $\underline{s}_{1(1,1)}$ and $\underline{s}_{2(1,1)}$. When the MUSIC method described in [1] is used to perform estimation, a 2-D search function can be formed as

$$E(\theta,\phi) = \frac{1}{[\,N_{\bar{R}}\,\underline{s}^*(\theta,\phi)]^T\,[\,N_{\bar{R}}\,\underline{s}^*(\theta,\phi)]^*} \quad (16)$$

where $N_{\bar{R}}$ contains the noise eigenvectors of $\bar{R}$ and $\underline{s}(\theta,\phi)$ is the search vector.

In the above discussion, it is assumed that the determinant shown in Eq. (14) is not equal to zero. It can be shown, however, that the determinant equals zero when

$$c\,[\,\sin\theta_1\,(\cos\zeta\cos\phi_1 + \sin\zeta\sin\phi_1) - \sin\theta_2\,(\cos\zeta\cos\phi_2 + \sin\zeta\sin\phi_2)\,] = n \quad (17)$$

where n is an integer,

$$c = \left\{ [\frac{(i_1-i_2)\,d_x}{\lambda}]^2 + [\frac{(j_1-j_2)\,d_y}{\lambda}]^2 \right\}^{1/2}, \quad (18)$$

$$\cos\zeta = \frac{(i_1-i_2)\,d_x}{\lambda c} \quad (19)$$

and

$$\sin\zeta = \frac{(j_1-j_2)\,d_y}{\lambda c} \quad (20)$$

For $\sin\theta_1 \geqslant \sin\theta_2$, Eq. (17) is satisfied with n = 0 when

$$\phi_1 = \zeta \pm \cos^{-1}[\,\frac{\sin\theta_2}{\sin\theta_1}\cos(\phi_2-\zeta)\,] \quad (21)$$

In this case, $R_{(i1,j1)}$ and $R_{(i2,j2)}$ have the same eigenspace. Therefore, there is no smoothing effect in computing $\bar{R}$ from these two covariance matrices. In order to perform spatial smoothing for all combinations of (θ_1, ϕ_1) and (θ_2, ϕ_2), at least three covariance matrices, for example $R_{(1,1)}$, $R_{(2,1)}$ and $R_{(2,2)}$, are required. This phenomenon does not appear in the 1-D bearing estimation problem.

In general, the M x N rectangular array can be grouped into P^2 subarrays, each with size (M−P+1) x (N−P+1), and spatial smoothing can be performed to reconstruct the signal subspace of K coherent sources for $K \leqslant P^2$.

3. ONE-DIMENSIONAL SIGNAL PROCESSING

The technique discussed in Section II involves a 2-D search with a search vector length of order MN. Therefore, considerable computation is required. In this section, we discuss the approach of using two 1-D searches together with a 2-D verification to estimate the AOAs. This avoids performing a 2-D search.

This approach is based on the fact that the AOA of the kth source (θ_k, ϕ_k) can be solved from u_k and v_k if they are properly estimated. In the following, we discuss the estimation of v_k and u_k by using rows and columns of the array, respectively. Let us first consider searching v_k, k=1,2,...,K, using rows of the array. The input vector of the mth row of the array is given by Eq. (2). Using Eq. (3), it can be written as

$$\underline{y}_m(t) = \alpha_1(t)\sum_{k=1}^{K}\rho_k\exp(\,jk_c m d_x u_k)\,\underline{s}_{kr} + \underline{n}_m(t) \quad (22)$$

where

$$\underline{s}_{kr} = [\exp(jk_c d_y v_k), \ldots, \exp(jk_c N d_y v_k)]^T \quad (23)$$

The covariance matrix of $\underline{y}_m(t)$ is given by

$$R_{mr} = E[\,|\alpha_1(t)|^2\,]\,[\sum_{k=1}^{K} \rho_k \exp(jk_c m d_x u_k)\,\underline{s}_{kr}]^* \, [\sum_{k=1}^{K} \rho_k \exp(jk_c m d_x u_k)\,\underline{s}_{kr}]^T + \sigma^2 I \quad (24)$$

Therefore, when $K \leqslant M$ and $u_k \neq u_\ell$ for $k \neq \ell$, a spatially smoothed covariance matrix, $\overline{R}_r$, can be computed by averaging over at least K R_{mr}'s. In this way, the signal and noise subspaces of $\overline{R}_r$ are the same as and orthogonal to, respectively, the subspace spanned by $\{\underline{s}_{1r}, \underline{s}_{2r}, \ldots, \underline{s}_{Kr}\}$. The search function of the MUSIC method can be formed as

$$E_r(v) = \frac{1}{[N_{\overline{R}_r}\, \underline{s}_r^*(v)]^T\, [N_{\overline{R}_r}\, \underline{s}_r^*(v)]^*} \quad (25)$$

where $N_{\overline{R}_r}$ contains the noise eigenvectors of $\overline{R}_r$ and $\underline{s}_r(v)$ is the search vector with $-1 \leqslant v \leqslant 1$. Peaks of $E_r(v)$ will indicate v_k, k=1, 2, . . . K.

Similarly, when $K \leqslant N$ and $v_k \neq v_\ell$ for $k \neq \ell$, u_k can be determined by using columns of the antenna array. After obtaining v_k and u_k, k = 1, 2, . . . K, we can use a 2-D verification to match these two data sets. Here, K^2 combinations of (v_k, u_ℓ) are substituted into the search vectors $\underline{s}(\theta, \phi)$ in Eq. (16). The K largest outcomes of the search function indicate the correct combinations of (v_k, u_k), k = 1, 2, ..., K. Then, θ_k and ϕ_k can be computed from (v_k, u_k).

In the above discussion, it is assumed that $u_k \neq u_\ell$ and $v_k \neq v_\ell$ for $k \neq \ell$. For the circumstance that the above assumption does not hold, there will be no smoothing effect between the kth and the ℓth sources in performing spatial smoothing for row or column search. Therefore, the searches will fail to identify the parameters of the two sources. This phenomenon can be eliminated by grouping subrows in performing row search (and similarly grouping subcolumns in column search) as shown in Fig. 3. An extra degree of smoothing is provided by using subrows or subcolumns. Therefore the two sources can be resolved even with $u_k = u_\ell$ or $v_k = v_\ell$.

4. SIMULATION RESULTS

Computer simulations are carried out based on a 5 x 5 antenna array with half wavelength interelement spacing in both axes. Two coherent sources with equal power of 10 dB and AOAs of (45°, 65°) and (45°, 25°) are used for simulations. The covariance matrix is computed by using four hundred samples. We first simulate the 2-D search method described in Section II. Figure 4a plots the simulation results of the MUSIC method without 2-D spatial smoothing. The two coherent sourced can not be resolved as expected. In Fig. 4b, a spatially smoothed covariance matrix computed by averaging over the covariance matrices of three 4 x 4 subarrays, $R_{(1,1)}$, $R_{(2,1)}$ and $R_{(2,2)}$, is used. The two sources are successfully resolved with peaks located at (44.9°, 64.9°) and (45.0°, 25.0°). We also perform simulation using a smoothed covariance matrix computed from the covariance matrices of two subarrays, $R_{(1,1)}$ and $R_{(2,2)}$. In this particular case, the determinant given by Eq. (14) equals zero. Therefore, there is no smoothing effect and the two sources can not be identified as shown in Fig. 4c.

Next, we simulate the estimation of AOAs using the 1-D search approach. The spatially smoothed covariance matrix for row search is computed from three subrows as shown in Fig. 3 (and similarly for column search). The search results are plotted in Fig. 5. For each search, two peaks, indicating v_k or u_k, k=1,2, are obtained. The correct combinations of v_k and u_k are determined by performing a 2-D verification with results shown in Table 1. The AOAs computed from the two combinations having the highest two verification outputs are (45.1°, 65°) and (45.2°, 24.9°).

REFERENCES

[1] R. Schmidt, "Multiple Emitter Location and Signal Parameter Estimation," Proc. RADC Spectral Estimation Workshop, 1979, pp. 243-258.

[2] S.S. Reddi, "Multiple Source Location-A Digital Approach," IEEE Trans. Aerosp. Electron. Syst. vol. AES-15, pp. 95-105, Jan. 1979.

[3] J.E. Evans, J.R. Johnson and D.F. Sun, "High Resolution Angular Spectrum Estimation Techniques for Terran Scattering Analysis and Angle of Arrival Estimation," Proc. of the First ASSP Workshop on Spectral Estimation, Aug. 1981.

[4] T.J. Shan, M. Wax and T. Kailath, "On Spatial Smoothing for Direction-of-Arrival Estimation of Coherent Signals," IEEE Trans. Acoust., Speech, Signal Processing, vol. ASSP-33, no. 4, pp. 806-811, Aug. 1985.

[5] C.C. Yeh, "Simple Computation of Projection Matrix for Bearing Estimations," IEE Proc., Part F, vol. 134, no. 2, pp. 146-150, Apr. 1987.

Table 1

2-D Verification of the Data Obtained From 1-D Processing ($v = \sin\theta \sin\phi$, $u = \sin\theta \cos\phi$)

Combination	v = 0.30, u = 0.30	v = 0.64, u = 0.30	v = 0.30, u = 0.64	v = 0.64, u = 0.64
Verification Output (dB)	-20.09	16.11	13.02	-16.99

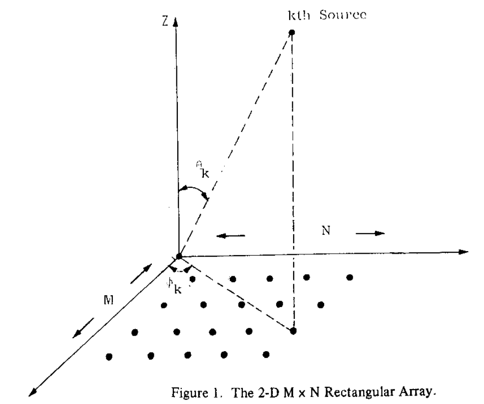

Figure 1. The 2-D M x N Rectangular Array.

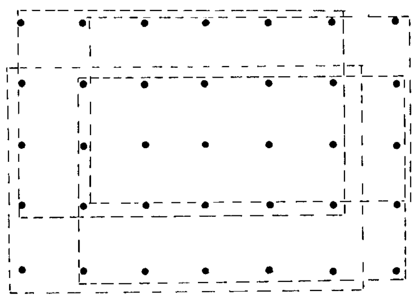

Figure 2. 2-D Subarray Grouping

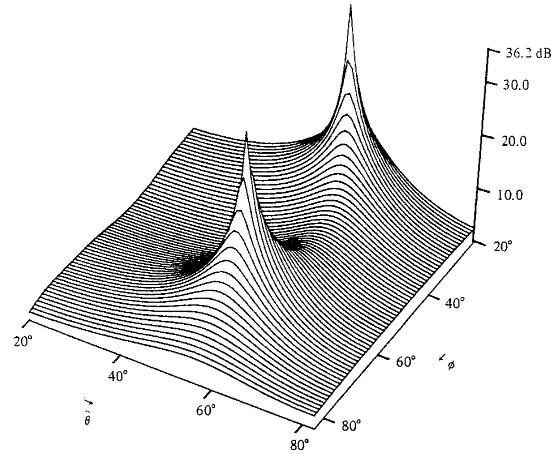

Figure 4(b). Smoothed covariance matrix computed from $R_{(1,1)}$, $R_{(2,1)}$ and $R_{(2,2)}$.

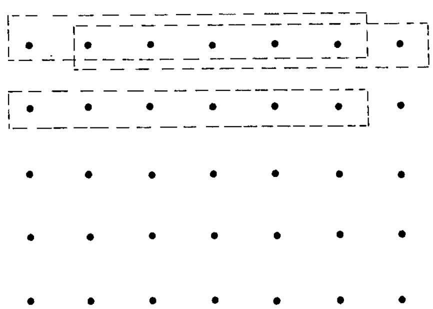

Figure 3. 1-D Subarray Grouping

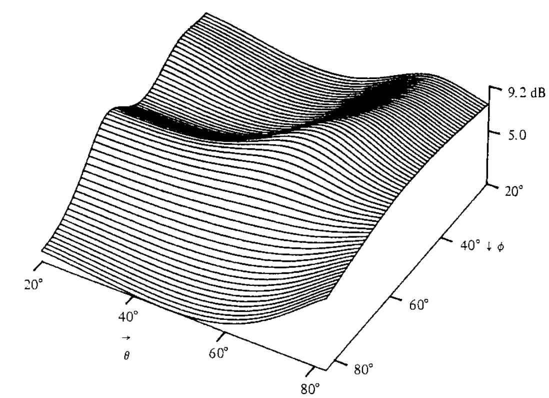

Figure 4(c). Smoothed covariance matrix computed from $R_{(1,1)}$ and $R_{(2,2)}$.
Figure 4. 2-D Search Output Source AOAs (45°, 65°), (45°, 25°)

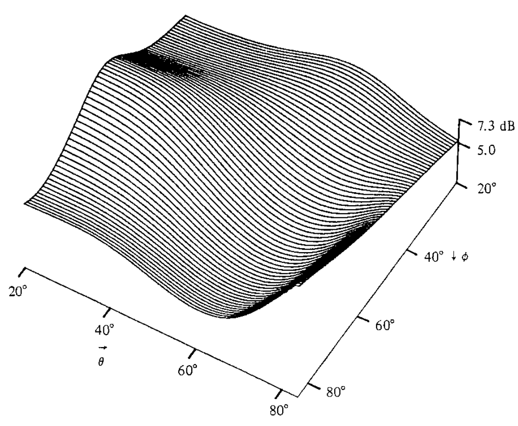

Figure 4(a). MUSIC without spatial smoothing

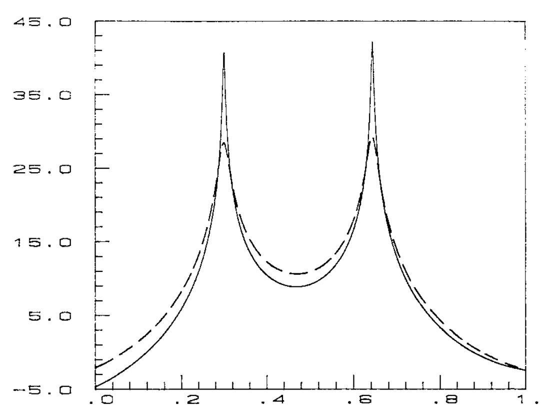

Figure 5. 1-D Search Output
Source AOAs (45°, 65°), (45°, 25°)
—— Search Output of u ($\sin\theta \cos\phi$)
– – – Search Output of v ($\sin\theta \sin\phi$)