Steady-State Response by Time-Reversal FD-TD Method with Lanczos Algorithm

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Abstract - Finite-difference time-domain (FD-TD) method combined with Lanczos algorithm and time-reversal technique is proposed for the steady-state response. With Lanczos algorithm, modes of FD-TD operator can be extracted efficiently. The steady-state response can then be calculated at any time by an analytic formula with a few of these modes. The existing FD-TD code needs not to be modified, except for an additionally time-reversal electric field update. An example of an air-filled cavity is demonstrated for the validation of this hybrid method.

Keywords: FD-TD method, Lanczos algorithm, model-order reduction, time reversal

I. INTRODUCTION

Steady-state response is desired in finite-difference time-domain (FD-TD) method[1] to increase the resolution in frequency domain. In order to satisfy the stability condition, the time step in conventional FD-TD simulation should be small enough to get the correct simulated solution. It will be time-consuming with small time step, especially for the steady-state response. In addition, only those fields at a few points are usually necessary. In conventional FD-TD method, however, fields at all the points in the computational domain need to be calculated at each time iteration. How to efficiently get the steady-state response at those field points concerned is therefore the subject of this study.

In late time, the electromagnetic sources fade to zero. The field response is believed to be composed by a few modal patterns. If these modes are extracted, the steady-state response can be obtained efficiently. The equivalent matrix of FD-TD operator is large and sparse. In order to efficiently obtain the modes of this operator, Lanczos algorithm is adopted[2]. The matrix is converted into a tridiagonal form by Lanczos algorithm. This tridiagonal matrix is much smaller in size compared to the original matrix, and has eigenvalues approximately equal to some eigenvalues of the original matrix. The corresponding eigenmodes of the original matrix can be also obtained by a

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transformation of eigenmodes of the small tridiagonal matrix. The original large system is therefore can be efficiently approximated by the reduced model.

Remis proposes a modified Lanczos algorithm for the computation of transient electromagnetic fields[3]. Accurate representation of the transient electromagnetic fields is obtained on a certain bounded interval in time. Lanczos algorithm is also employed in finite-element frequency-domain (FEFD) and finite-element time-domain (FETD) techniques[4]. In this study, FD-TD method combined with Lanczos algorithm to obtain the steady-state response is proposed. With Lanczos algorithm, modes for source-free FD-TD operator are extracted. The components of the electric field on each mode are found, and the steady-state response at any time can be obtained by an analytic formula. The existing FD-TD code needs not to be modified in the proposed technique, except for an additional time-reversal electric field update. Time reversal in FD-TD method was proposed by Sorrentino for the numerical synthesis of a microwave structure in 1993.[5] To our knowledge, this is the first paper to construct the steady-state response by FD-TD method with Lanczos algorithm and time-reversal technique.

II. OUTLINE OF THE THEORY

In inhomogeneous medium with permittivity $\varepsilon_r(\mathbf{r})$ and permeability $\mu_r(\mathbf{r})$, source-free Maxwell equation for FD-TD method can be written in the matrix form,

$$\begin{bmatrix} \eta_0 \mathbf{H}^{n+1/2} \\ \mathbf{E}^{n+1} \end{bmatrix} = \begin{bmatrix} \mathrm{I} & -c \cdot \Delta t \cdot \mu_r^{-1}(\mathbf{r}) \cdot \mathbf{D} \\ c \cdot \Delta t \cdot \varepsilon_r^{-1}(\mathbf{r}) \cdot \mathbf{D}^{T} & \mathrm{I} - (c \cdot \Delta t)^2 \cdot \varepsilon_r^{-1}(\mathbf{r}) \cdot \mathrm{D}^{T} \cdot \mu_r^{-1}(\mathbf{r}) \cdot \mathrm{D} \end{bmatrix} \cdot \begin{bmatrix} \eta_0 \mathbf{H}^{n-1/2} \\ \mathbf{E}^{n} \end{bmatrix} \quad (1)$$

where I is the identity matrix, D denotes the discretized curl operator, Δt is the time step, c denotes the velocity of light in free space, and the superscript n means the n-th time step. As equation (1) indicates, the equivalent matrix of FD-TD operator is large and sparse, but not symmetric. Although Lanczos algorithm for asymmetric systems is available[6], the symmetric form is preferred.

Eliminating the magnetic field in equation (1), the FD-TD updating equation which only electric fields are involved is

$$\mathbf{E}^{n+1} = 2 \cdot \left[\mathrm{I} - \tfrac{1}{2}(c \cdot \Delta t)^2 \cdot \varepsilon_r^{-1}(\mathbf{r}) \cdot \mathrm{D}^{T} \cdot \mu_r^{-1}(\mathbf{r}) \cdot \mathrm{D}\right] \cdot \mathbf{E}^{n} - \mathbf{E}^{n-1}. \quad (2)$$

Obviously, the updating matrix in (2) is symmetric. It is theoretically equivalent to the updating matrix in [7], and therefore has eigenvalues of which the magnitudes are no larger than unity. Expand the electric field $\mathbf{E}^{n}$ in terms of the eigenvectors of the updating matrix, and let $a_{n,j}$ denote the expansion coefficient corresponding to the j-th eigenvector. Substitution of the expansion coefficients to (2) yields the recurrence relation

$$a_{n+1,j} - 2\lambda_j a_{n,j} + a_{n-1,j} = 0 \tag{3}$$

where λ_j is the corresponding j-the eigenvalue. Once $a_{n,j}$, $a_{n-1,j}$ and λ_j are given for some n-th time step, the corresponding expansion coefficient $a_{k,j}$ at any k-th time step ($k \geq n-1$) is quite easy to obtained by solving the recurrence relation in (3). The steady-state response at any time step can therefore be obtained analytically.

Lanczos algorithm is applied to obtain the approximated eigensolutions of the updating matrix in (2) instead of the original FD-TD operator in (1). In Lanczos algorithm, a Krylov subspace of order m is established[2]:

$$\kappa^m(\mathbf{A},\mathbf{b}) \equiv \text{span}\{\mathbf{b}, \mathbf{A}\mathbf{b}, \mathbf{A}^2\mathbf{b}, \cdots, \mathbf{A}^{m-1}\mathbf{b}\} = \text{span}\{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \cdots, \mathbf{q}_m\}. \tag{4}$$

When the electromagnetic sources fade to zero and the modal patterns appear, Lanczos algorithm is employed by initially setting **b** vector to be the electric field $\mathbf{E}^n$ and **A** matrix to be the updating matrix in (2). However, $\mathbf{A}\mathbf{q}_{j-1}$ instead of $\mathbf{A}^{j-1}\mathbf{b}$ is used in the Lanczos j-th iteration. Updating equation in (2) is neither preferred because the existing FD-TD code needs to be modified. As (2) implies, $\mathbf{A}\mathbf{q}_j$ is the average of the 'virtual' electric fields at the next and the previous time steps. The former can be obtained by the standard time-forward FD-TD method, while the later will be available from the time-reversal FD-TD method. The existing FD-TD codes are therefore preserved.

III. NUMERICAL RESULTS

An air-filled cavity of dimension 10 cm × 10 cm × 1 cm is first considered. The size of spatial division Δ in each direction is 1 cm and the normalized time step $c\cdot\Delta t$ is set to be 0.5 cm, where c is the velocity of light in free space. An electric field excitation in the form of Gaussian pulse $\exp(-(c\cdot t-50.5)^2/100)$ (volt/m) is applied at the center of the cavity. The late-time responses obtained by FD-TD method and the hybrid method are depicted in Fig. 1. As shown in Fig. 1, the steady-state response can be faithfully obtained with only two modes in the hybrid method. That is, only two Lanczos iterations are necessary in this case and the CPU time for Lanczos iterations is therefore negligible. Table 1 lists the computed resonant frequencies and the exact solution as a comparison.

IV. CONCLUSION

FD-TD method combined with Lanczos algorithm and time-reversal technique is proposed for the steady-state response calculation. Only extra one electric field updating equation is necessary to the existing FD-TD code. With only two modes, the steady-state response of electric field in an air-filled cavity can be faithfully obtained at any time step. More examples are under studied.

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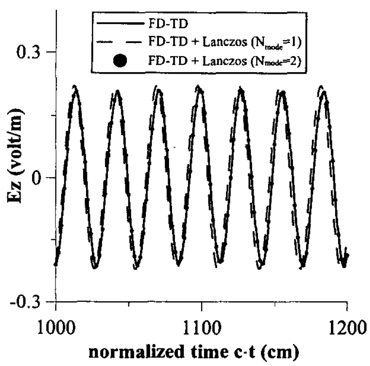

Fig. 1. steady-state electric field of an air-filled cavity excited by a Gaussian pulse at the center. N_{mode} is the number of modes extracted by Lanczos algorithm.

Resonant frequency (GHz)	Mode 1 (TM_{110})	Mode 2 (TM_{310} /TM_{130})
Exact	2.121	4.743
Lanczos (N_{mode}=1)	2.124	-
Lanczos (N_{mode}=2)	2.117	4.631

Table 1. Comparison of the calculated resonant frequencies with the exact solution.