

## BEARING ESTIMATION IN THE PRESENCE OF NEAR-FIELD SOURCES

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### **Abstract:**

Most high-resolution bearing estimation algorithms were proposed based on the assumption of far-field source environment. When this assumption is violated, i.e., some sources are in the near-field, these algorithms generally show unsatisfactory performances. In this paper, a new algorithm is proposed to tackle this problem using a uniform linear array. The observed signal covariance matrix of near-field sources is equivalently viewed as a corrupted measurement of the ideal signal covariance matrix of far-field sources located at the same bearings. Taking this viewpoint, we reconstruct a far-field approximation (FFA) covariance matrix from the observed covariance matrix to reduce the corruptions. Therefore, existing high-resolution bearing estimation techniques, e.g., MUSIC [1], can be directly applied to estimate the bearings of the near-field sources. Simulation results demonstrating the effectiveness of this approach are included.

### **I. Introduction**

High-resolution methods for estimating the bearings of multiple sources radiating narrow-band signals generally assume that all sources are in the far-field [1-5]. Under this assumption, each signal received by the array is modeled as having a planar wavefront. In the circumstances that some sources are in the near-field, the wavefronts of the signals emitted from these sources become spherical, and the existing methods generally show unsatisfactory performance in resolving source bearings because the modeling error.

In this paper, the near-field source problem is equivalently viewed as far-field signals being received by an array of non-ideal sensors, in which each sensor distinctly possesses an unknown complex and non-omnidirectional gain pattern. Then, we present a method to cure the problem of bearing estimation in the presence of near-field sources using a uniform linear array. The approach is to reduce the effect of the range- and bearing-dependent complex non-omnidirectional gain pattern of each sensor. Utilizing the characteristics of the wavefront shape of near-field signals and the signal covariance matrix of incoherent far-field sources, we construct a far-field approximation (FFA) covariance matrix from the measured covariance matrix to approximate the ideal covariance matrix associated with the equivalent far-field signals, whose bearings are the same as those of the near-field signals. It is shown that the effect caused by the mismatch between the true phase vector and the assumed far-field phase vector can be alleviated when the FFA covariance matrix is used instead of the original measured covariance matrix. Therefore, conventional high-resolution methods can be utilized to estimate the bearings of near-field sources based on the FFA covariance matrix. Moreover, the proposed method causes no effect on the original far-field sources.

Consequently, it can be employed to deal with the case in which some of signal sources are in the near-field while the others are in the far-field.

## II. The Near-Field Source Problem

Consider  $M$  near-field, narrow-band, and incoherent radiating sources observed by a uniform linear array of  $N$  sensors with interelement spacing  $d$ . The observed covariance matrix is given as

$$\mathbf{R} = \sum_{m=1}^M E[|a_m(t)|^2] \mathbf{P}_m \hat{\mathbf{S}}_m \hat{\mathbf{S}}_m^\dagger \mathbf{P}_m^\dagger + \sigma^2 \mathbf{I} \quad (1)$$

where

$$\mathbf{P}_m = \text{diag}(p_{m1}, p_{m2}, \dots, p_{mN}) \quad (2)$$

$$p_{mn} = \frac{d_m}{d_{mn}} \exp(-j\kappa \frac{x_n \cos \theta_m}{2d_m}) \quad (3)$$

$$\hat{\mathbf{S}}_m = [e^{j\kappa x_1 \sin \theta_m}, e^{j\kappa x_2 \sin \theta_m}, \dots, e^{j\kappa x_N \sin \theta_m}]^T, \quad (4)$$

$a_m(t)$  and  $d_m$  denote the signal amplitude and range of the  $m$ th source observed at the array center, respectively,  $\kappa$  is the wavenumber,  $\theta_m$  is the bearing of the  $m$ th source from the array broadside measured at the array center,  $x_n = (n-c)d$  (where  $c = (N+1)/2$ ) is the location of the  $n$ th sensor,  $d_{mn} = \sqrt{(d_m \sin \theta_m - x_n)^2 + (d_m \cos \theta_m)^2}$  indicates the range of the  $m$ th source referring to the  $n$ th sensor, and  $\sigma^2$  is the noise power. From (1)–(4), it is noted that the signal subspace of  $\mathbf{R}$  is spanned by  $\mathbf{P}_m \hat{\mathbf{S}}_m$ ,  $m=1, 2, \dots, M$ , which are the actual phase vectors of the near-field sources. Therefore, the performances of most bearing estimation algorithms based on the far-field assumption are deteriorated because the assumed far-field signal model is  $\hat{\mathbf{S}}_m$ , not  $\mathbf{P}_m \hat{\mathbf{S}}_m$ .

## III. The FFA Approach

As noted from (1), the observed signal covariance matrix of the near-field sources can be considered as a corrupted measurement of the ideal signal covariance matrix  $\hat{\mathbf{R}}$  of far-field sources with the same bearings.  $\hat{\mathbf{R}}$  is obtained from (1) by letting  $\mathbf{P}_m = \mathbf{I}$ ,  $m=1, 2, \dots, M$ , which has Toeplitz structure. The approach is to reconstruct a Toeplitz covariance matrix  $\bar{\mathbf{R}}$  approximating  $\hat{\mathbf{R}}$  from  $\mathbf{R}$ . After surveying each signal component in the corresponding entries of  $\mathbf{R}$  and  $\hat{\mathbf{R}}$ , we find those entries nearest to the cross-diagonal in  $\mathbf{R}$  contain least corruption. Consequently, we use these entries to reconstruct  $\bar{\mathbf{R}}$ , which is termed the far-field approximation (FFA) covariance matrix. The detailed analysis of the proposed approach has been presented in [6]. The procedure of the proposed approach is summarized as follows:

- (1) Compute the observed covariance matrix by

$$\mathbf{R} = \frac{1}{K} \sum_{k=1}^K \mathbf{V}(kT) \mathbf{V}^\dagger(kT) = [r_{ij}] \quad (5)$$

where  $T$  denotes the sampling period and  $K$  the number of snapshots.

- (2) Reconstruct the FFA covariance matrix  $\bar{\mathbf{R}}$  from  $\mathbf{R}$  by

$$\bar{\mathbf{R}} = [\bar{r}_{ij}] = [\bar{r}_{(i-j)}] \quad (6)$$

where

$$\bar{r}_n = \frac{1}{2}(r_{k(n)k(-n)} + r_{h(n)h(-n)}) \quad (7)$$

$$k(n) = \lfloor c - \frac{n}{2} \rfloor \quad (8)$$

$$h(n) = \lfloor c - \frac{n-1}{2} \rfloor \quad (9)$$

and  $\lfloor x \rfloor$  denotes the largest integer not greater than  $x$ .

- (3) Apply the method of [7] based on  $\mathbf{R}$  to estimate the number of sources  $M$ , and then use MUSIC based on  $\bar{\mathbf{R}}$  to estimate the source bearings.

#### IV. Simulation Results

A uniform linear array with  $N=11$  and  $d=\lambda/2$  ( $\lambda$ =wavelength) was used. First, the case of two near-field sources with  $(\theta_1, d_1)=(12^\circ, 100\lambda)$  and  $(\theta_2, d_2)=(15^\circ, 200\lambda)$  was considered. The sources are of equal power with common SNR=10dB, and 500 snapshots were taken to estimate the covariance matrix

$\mathbf{R}$ . Figure 1 shows the bearing spectra obtained by MUSIC based on  $\mathbf{R}$  and  $\bar{\mathbf{R}}$  in 5 independent simulations. Next, we present the results for the case of a near-field source accompanied by a far-field source. The near-field source is located at  $(\theta_1, d_1)=(12^\circ, 100\lambda)$  and the bearing of the far-field source is that  $\theta_2=15^\circ$ . Again, 500 snapshots were taken to estimate the covariance matrix  $\mathbf{R}$  and the array configuration is the same as that in the first case. Both sources are equally powered with SNR=10dB. The results of 5 independent experiments using MUSIC based on  $\mathbf{R}$  and  $\bar{\mathbf{R}}$  are shown in Figure 2. It is noted that the FFA approach has better performance in both cases.

#### References:

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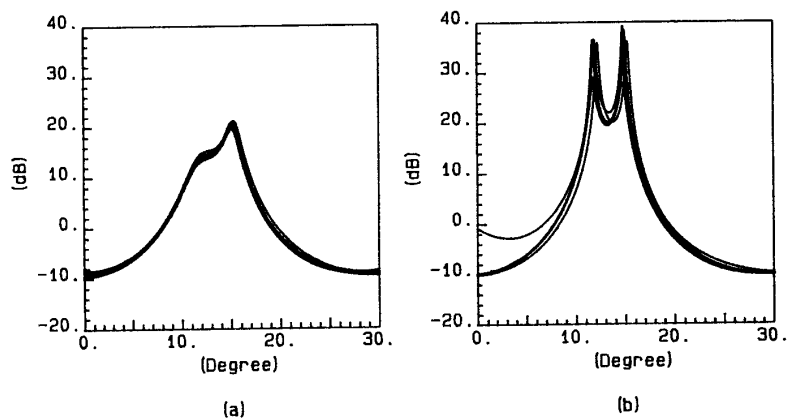


Figure 1. Bearing spectra of MUSIC for the case of two near-field sources with  $(\theta_1, d_1) = (12^\circ, 100\lambda)$  and  $(\theta_2, d_2) = (15^\circ, 200\lambda)$  (a) based on  $\mathbf{R}$ . (b) based on  $\bar{\mathbf{R}}$ .

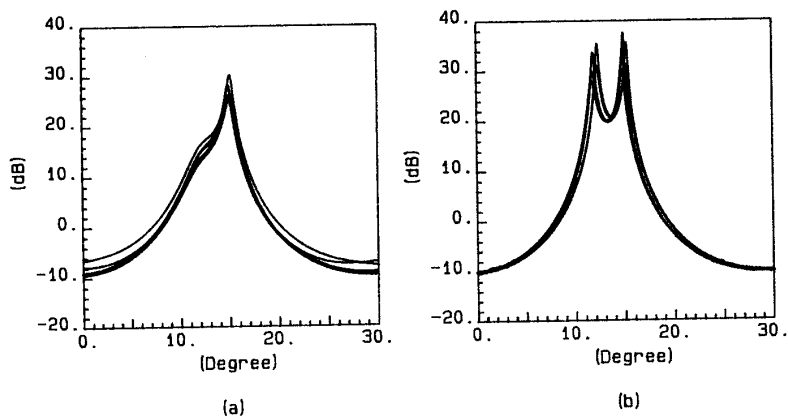


Figure 2. Bearing spectra of MUSIC for the case of a near-field source accompanied by a far-field source with  $(\theta_1, d_1) = (12^\circ, 100\lambda)$  and  $\theta_2 = 15^\circ$  (a) based on  $\mathbf{R}$ . (b) based on  $\bar{\mathbf{R}}$ .