

## HYBRID PYRAMID IMAGE CODING AND DATA COMPRESSION

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### ABSTRACT

The hybrid pyramid coding scheme for image data compression are based on two coding techniques: sub-band pyramid and Laplacian pyramid. Source data are first decomposed into 7 bands by sub-band pyramid coding. Only the lowest band is further decomposed by Laplacian pyramid to reduce correlations between pixels. The advantage of hybrid pyramid is its efficiency as well as compactness. In coding process each component is PCM coded. The computer simulation results show that the quality of reconstructed image is good up to compression ratio of 10 to 1.

### INTRODUCTION

The basic idea of pyramid data structure is based on repeating the decomposition and decimation process of the low-pass filtered image [1]-[4]. In sub-band coding scheme input signal is decomposed into narrow bands where each band is then decimated to reduce data size. Fig. 1 shows the ideal location of subband images in frequency domain. To form pyramid structure the lowest band is further decomposed by the same manner. No matter whether the depth of the sub-band pyramid is, the total number of pixels in the pyramid is equal to that in the original image, thus the data structure itself is very compact. The advantage of such a coding scheme is that the quantization noise generated in a specific band is limited almost to that band in the reconstruction process. To compensate aliasing errors introduced in decomposition process, Woods [5] and Gharavi [6] apply Quadrature Mirror filter (QMF) to sub-band coding of images. Woods and Gharavi have chosen a type 32D and 16A QMF filter coefficients respectively [7]. In subband coding of image signals the 2-D QMF filters decompose the input signal into 4 bands. Woods repeats the above process to each sub-band signal and produces a tree-typed data structure. Instead of Woods' 4 band decomposition manner, only the lowest band is further decomposed in Gharavi's paper and sub-band

pyramid is formed in this case. We prefer Gharavi's subband pyramid, since it is observed that except for the lowest band the pixel to pixel correlation of upper band signals is very low. Theoretically, the decomposition process can be continued as far as possible, however in Woods and Gharavi's experiments they repeated the process just once again. In their subband coding system, QMF filtering is done in frequency transform domain, the filtered signals should be transformed back to spatial domain before decimation. It is time consuming to transform data back and forth between two domains, so Woods and Gharavi allowed only two-level subband decomposition. Another reason of their decision is due to imperfect reconstruction. Since the QMF filters they use are approximated by FIR filters, successive filtering will result in error accumulation. In their experiments pixels are highly correlated in the lowest band, therefore more elaborate quantization technique such as DPCM are required to reduce quantization error.

The Laplacian pyramid introduced by Burt [3] is efficiently obtained by fast hierarchical convolution. The most pronounced advantage of such a code is its reversibility, the original image can be perfectly reconstructed from its Laplacian pyramid representation. But the total number of pixels in Laplacian pyramid is greater than that in the original image by a factor about  $\frac{4}{3}$ .

The hybrid pyramid is a compromise between these two coding schemes. Input signal is first decomposed by two level subband pyramid coding scheme, then the lowest band is further decomposed into Laplacian pyramid to reduce pixel to pixel correlations. In subband coding process we choose short length QMF filter so we can apply fast hierarchical convolution technique as the same manner as Burt does. The technique allows filtering and decimation to be done simultaneously, which provides computational efficiency. The excess number of pixels in the lowest band Laplacian pyramid is just a little bit when compared with the original image.

Each component in the hybrid pyramid can be quantized by simple PCM code. The quality of reconstructed image is still good at high data compression ratio. The hybrid pyramid takes the advantage of data compactness and computational efficiency, there is some experiment results presented here to show its effectiveness.

The principle of QMF filter can be found in [5]-[7], in our software simulation we used a 8-tap filter designated as type 8A QMF filter in [7]. It has a normalized transitional bandwidth of 0.14 radian with an overall passband ripple of 0.06 dB and stopband attenuation of 31 dB. The same QMF lowpass filter can be used in Laplacian pyramid generation. The encoding and decoding schemes of hybrid pyramid are described briefly in Section II. Histogram analysis and quantization results are discussed in Section III. Section IV gives the computer simulation results.

#### THE HYBRID PYRAMID

##### Encoding

There are two steps to generate hybrid pyramid, the first step is to construct a subband pyramid, and the second step is to generate the Laplacian pyramid of the lowest band. For the sake of fast computation, the QMF filter we choose is short in length, a type 8A QMF filter is used in this paper. If "K" refers to the number of levels in the subband pyramid, in general we let  $K \leq 3$  to reduce accumulated error. Suppose the source image is represented by the array  $G_0$ , let  $kB_{ij}$  be the component located at  $ij$  band in level  $k$  of the subband pyramid we define  $oB_{00}$  to be equal to  $G_0$ . thus:

$$\text{for } 1 \leq k \leq K \text{ and } 0 \leq i, j \leq 1$$

$$kB_{ij}(m,n) = \sum_{p=0}^7 \sum_{q=0}^7 k^{-1} oB_{00}(2m-p, 2n-q) hij(p,q) \quad (1)$$

where  $hij$  is the 2-D QMF filter. Separability characteristics of  $hij$  can reduce 2-D QMF filtering problem to 1-D filtering, and it can be written as  $hij(p,q) = h_i(p)h_j(q)$ , where  $h_i(p)$  is the type 8A QMF filter. The lowest band of the subband pyramid is baseband component  $kB_{00}$ .

To form a Laplacian Pyramid of  $kB_{00}$  we can follow Burt's method [1] [3] by substitute  $kB_{00}$  for initial image. The only difference is replaced Burt's kernel function  $W(m,n)$  by lowpass QMF filter  $h_{00}(m,n)$ . Fig.2 is the block diagram of hybrid pyramid coding scheme.

##### Decoding

The reconstruction of Laplacian

pyramid can be found in [3]. We only deal with the recovery of subband pyramid. Suppose the reconstruction task of Laplacian pyramid is fulfilled, in subband pyramid level  $K$ , there are 4 sub-images band :  $kB_{00}$ ,  $kB_{01}$ ,  $kB_{10}$  and  $kB_{11}$ .

To recover the baseband component  $k^{-1}B_{00}$  in pyramid level  $K-1$ , the decimated signals are interpolated to the same size as  $k^{-1}B_{00}$  and filtered by a similar set of QMF filters before summing together. Let  $k^{-1}B_{ij}$  be the interpolated and filtered version of  $kB_{i,j}$ , then

$$k^{-1}B_{ij} = 4 \sum_{p=0}^7 \sum_{q=0}^7 k B_{ij} \left( \frac{m-p}{2}, \frac{n-q}{2} \right) hij(p,q) \quad (2)$$

Only terms for which  $\frac{m-p}{2}$  and  $\frac{n-q}{2}$  are integers are included in this sum. The recover baseband image  $k^{-1}B_{00}$  can be written as

$$k^{-1} \hat{B}_{00}(m,n) = \sum_{i=0}^1 \sum_{j=0}^1 k^{-1} B_{ij}(m,n) \quad (3)$$

Starting at  $k^{-1} \hat{B}_{00}$ ,  $k^{-1}B_{01}$ ,  $k^{-1}B_{10}$  and  $k^{-1}B_{11}$ , we can repeat the above process until  $oB_{00}$  is obtained. The little difference between  $oB_{00}$  and  $G_0$  is invisible to the human eyes.

#### HISTOGRAM ANALYSIS AND PCM QUANTIZATION

The histogram analysis of an image is useful for the decision of quantization. The uniform quantized range, the number of quantum level  $L$  and the size of quantum step all should be decided before quantization. By the distribution of image histogram, one can make a good choice of these values. We study the histograms of the hybrid pyramid of a source image "Girl".

It is observed that most of the pixels in subband pyramid are concentrated about the origin, thus we may choose fewer quantization levels to encode them.

Since the dispersive histograms of the lower level Laplacian pyramid, the quantizer would be finer than that of upper levels. The reduced dimension make it without detriment to compression ratio. In hybrid pyramid, pixel to pixel correlations are largely reduced, so we can apply simple PCM quantizations with different number of quantum levels and quantum steps to each component.

#### EXPERIMENTS

In the experiment of computer simulation, a source image "Girl" is decomposed into 2-level subband pyramid and 4-level Laplacian pyramid, the total number of bands is 10. The lowest band in the Laplacian pyramid is a low-pass component

of dimension 8 by 8. It contains the general brightness area and since pixel to pixel correlation of this subimage is high, it will not be applied to quantizer and remains unchanged. As mentioned earlier, the small size of this component makes little contribution to average bit rate. The quantization data of the Girl's hybrid pyramid are listed on Table 1.

Since the diagonal component of the subband images contain least information, the quantizer may be chosen as coarse as possible. In Laplacian pyramid we use finer quantizer to reduce quantization error of the lower frequency subimages. We compute the image data compression ratio through quantization alone. This may be specified as the average number of bits of information needed for each pixel to represent the quantized hybrid pyramid structure. Let there be  $N^2$  nodes in the original image, Level 1 will have 3 subband images with  $\frac{N^2}{4}$  nodes in each component. If the number of quantum levels of subimages in Level 1 are all equal to 3, their contribution to average bit rate is the sum of their sample density multiplying by their entropy which is given by  $(3 \times \frac{N^2}{4} \times \log_2 3) / N^2$  or 1.19 bits per pixel.

As the same manner, we can get the estimated average bit rate from 2 bits/pixel to 0.7 bit/pixel in our experiments. Since the original image is 8 bits/pixel, the compression ratio is defined as the ratio of 8 to average bit rate, we get the compression ratios from 3.8 to 10.7 in our experiments. Further data compression can be achieved by variable-length codeword such as Huffman code. The experiment results are shown in Fig.3. Fig.3(c) is obtain from (b) by eliminating Level 1 subimages. The simulation results show the effectiveness of hybrid pyramid coding scheme, in addition to the computational efficiency make it suitable for image data compression.

#### REFERENCES

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Table 1. The quantization data used in the hybrid pyramid of image "Girl".

Sub-band pyramid	no. of quantum levels	Laplacian pyramid	no. of quantum levels
1Bo1	3	L2	17
1B1o	3	L3	33
1B11	3	L4	65
2Bo1	5		
2Bo1	5		
2B11	3		

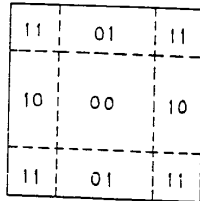


Fig.1. idealband segmentation of sub-band coding.

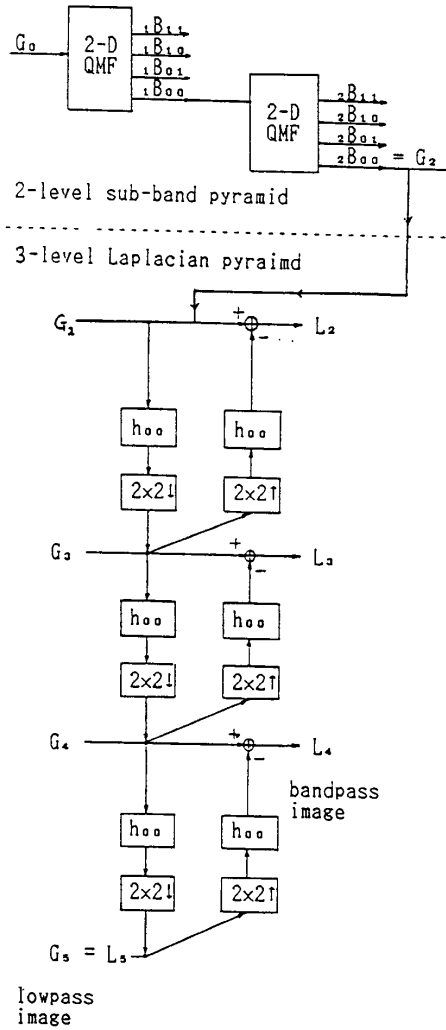


Fig.2. The block diagram of hybrid pyramid coding system.



(a) Source image — "Girl".



(b) 4.1 compression ratio.



(c) 10.7 compression ratio.

Fig.3. Experiment results.