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Chained eigenstructure assignment for constant-ratio proportional and derivative (CRPD) control law in controllable singular systems

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Abstract: This paper deals with chained eigenstructure assignment for controllable singular systems of the form $\hat{E}\dot{x}(t) = \hat{A}x(t) + \hat{B}u(t)$ with constant-ratio proportional and derivative (CRPD) control of the form $u(t) = \mu Kx(t) - K\dot{x}(t) + w(t)$. The closed-loop system exhibits the following features: regularity, impulse-free response, and arbitrary eigenvalue (except open-loop poles) assignment. This parametric characterization conveniently chooses the nonunique gain matrix K to modify the dynamic response of the system. An illustrative example is included to demonstrate our approach.

Keywords: Singular systems; standard forms; controllable; eigenstructure assignment; regularity; constant-ratio proportional and derivative feedback.

1. Introduction

One of the most popular methods to modify the dynamic response of a singular system is pole placement. To achieve this purpose, [6, 8] have used the geometric approach to investigate the state feedback control problems of singular systems. Kucera and Zagalak [5] have proposed the fundamental theorem of state feedback design for singular systems. The methods in [2, 8] need the restricted equivalent transformation first and then finding the state feedback gain for pole placement. Hence, the eigenspaces discussed are in the transformed coordinate instead of the original coordinate. For controllable singular systems, a method has been proposed in [4] for obtaining parametric gain matrices in the eigenstructure assignment problem. However, this algorithm can only assign the primary eigenvectors. Furthermore, the dead-beat eigenstructure control problem still needs some further investigation, since the zero eigenvalues assignment is not allowed in [4].

The problem of eigenvalue assignment via proportional and derivative feedback has been well studied (see e.g. [7, 11]). Combined proportional and derivative (PD) feedback can modify the system structure by replacing the pole locations to decrease the susceptibility to noise, and hence improve the performance. However, dealing with PD feedback, determination of two gain matrices, in general, needs a complicated computation procedure. In [9, 10, 14] a theoretical classification of generalized systems using constant-ratio proportional and derivative (CRPD) feedback was proposed. Compared to PD feedback, the CRPD design is easier, since only one control parameter needs to be determined.

The published papers ([1, 2, 4, 8]) have been focused on eigenstructure assignment with proportional state feedback. The use of CRPD and PD control law in singular systems has been intensively and widely studied for 10 years (see e.g. [10, 11]). This paper applies the CRPD control law to treat the eigenstructure assignment problem. The proposed method provides a superior feature, i.e., both the primary and chained eigenvectors can be assigned simultaneously. Also, the dead-beat control for placing zero values as the desired closed-loop poles are possible. The development of this

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method depends crucially on the properties of the standard form singular systems. The nonunique CRPD gain makes the closed-loop eigenvectors having adjustable degrees of freedom. Although we need to transform the given singular system from the general form to its standard one, the CRPD feedback gain matrix and the closed-loop eigenvectors of both forms remain invariant. We provide a convenient CRPD state feedback method such that the closed-loop system is regular, and the chained eigenvectors can be obtained. The computational procedures for determining gain matrices are also provided explicitly.

1.1. Problem formulation

This paper is devoted to the problem of eigenstructure assignment of controllable generalized state systems. We consider a controllable time-invariant singular system of the form

$$\Sigma:\ \hat{E}\dot{x}(t) = \hat{A}x(t) + \hat{B}u(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^r$ is the input vector, $\hat{E}$, $\hat{A}$ and $\hat{B}$, are real constant matrices of appropriate dimensions. We assume that $\hat{E}$ is singular whose rank g is less than n. Furthermore, we assume $s\hat{E} - \hat{A}$ is a regular pencil (i.e. $|s\hat{E} - \hat{A}| \not\equiv 0$) such that system (1) is solvable. $\{\hat{E}, \hat{A}\}$ is commonly called as the general form.

Our purpose is to find the matrix K in the CRPD control law, $u(t) = \mu K x(t) - K\dot{x}(t) + w(t)$, such that the closed-loop system

$$\Sigma_c:\ \hat{E}_c\dot{x}(t) = \hat{A}_c x(t) + \hat{B}w(t),$$

$$\text{where } \hat{E}_c \triangleq \hat{E} + \hat{B}K \quad \text{and} \quad \hat{A}_c \triangleq \hat{A} + \mu\hat{B}K, \tag{2}$$

satisfies the following requirements: there exist n finite eigenvalues (repeated roots are counted); their locations can arbitrarily be assigned to any places in complex conjugate pairs except for open-loop finite poles; and the regularity of the closed-loop system should be guaranteed. Since there exist no infinite poles in the closed-loop system, the responses caused by any initial conditions are impulse-free.

2. Properties of a standard pair

An important aspect of regular pencils is that they can be transformed into standard forms in which eigenstructure assignment problems with proportional control law are greatly simplified [1].

$\{E, A\}$ pair is a standard form if there exist some scalars α and β such that $\alpha E + \beta A = I$ [9]. If we premultiply (1) by $(\mu\hat{E} - \hat{A})^{-1}$, the singular system becomes

$$E\dot{x}(t) = Ax(t) + Bu(t), \tag{3}$$

where

$$E = (\mu\hat{E} - \hat{A})^{-1}\hat{E}, \qquad A = (\mu\hat{E} - \hat{A})^{-1}\hat{A},$$
$$B = (\mu\hat{E} - \hat{A})^{-1}\hat{B}. \tag{4}$$

The $\{E, A\}$ pair in (3) is a standard pair since $\mu E - A = I$. In the above procedure, we do not change the system or the 'state' variable x. In order to avoid numerical error, it is better to choose μ such that the number of conditions for $(\mu\hat{E} - \hat{A})$ is smaller. Some important properties of the standard form will be investigated in the following.

As already seen, a general form singular system can easily be transformed into a standard form. If the CRPD control law, $u(t) = \mu K x(t) - K\dot{x}(t) + w(t)$, is applied to (3), the closed-loop system becomes

$$E_c\dot{x}(t) = A_c x(t) + Bw(t),$$

$$\text{where } E_c \triangleq E + BK \quad \text{and} \quad A_c \triangleq A + \mu BK. \tag{5}$$

Lemma 2.1. *The general form singular system* (1) *and the associated standard form singular system* (3) *have the following properties:*

(i) *E and A commute, $EA = AE$.*

(ii) (1) *is controllable if and only if* (3) *is controllable.*

(iii) *$\{E, A\}$ and $\{\hat{E}, \hat{A}\}$ have the same eigenstructure.*

(iv) *If* (1) *and* (3) *use the same CRPD feedback with the gain matrix K and the scalar ratio μ then $\{E_c, A_c\}$ pair is in the standard forms and these closed-loop pairs $\{E_c, A_c\}$ and $\{\hat{E}_c, \hat{A}_c\}$ have the same eigenstructure.*

Proof. For (i) and (ii) see [1, 13]. For (iii) and (iv) it is obvious from (2), (4) and (5). □

It is well known [14] that the singular system (3) is controllable if and only if

$$\langle E|B\rangle = \sum_{i=0}^{n-1} \operatorname{Im} E^i B = \mathbb{R}^n.$$

The eigenvalues of the singular system (1) can be assigned arbitrarily (subject to complex conjugate with number up to n) by using CRPD feedback if and only if the system is controllable. The closed-loop system has n finite eigenvalues that can be assigned to the stable region. As a result, the impulsive and diverged responses are eliminated. To guarantee the regularity of the closed-loop system, there is no restriction on feedback gain matrix K, since $\det(sE + sBK - A - \mu BK) \neq 0$ holds for any K at $s = \mu$. In the following, we provide some other useful properties of a standard pair.

Lemma 2.2 (Chen and Chang [1]). *Let $\{E, A\}$ be in the standard form and k be any positive integer. Then*

(i) $E(sE - A)^k = (sE - A)^k E$.

(ii) $(sE - A)^{-k} E(sE - A)^{-1} = E(sE - A)^{-k-1}$.

(iii) $\dfrac{\mathrm{d}}{\mathrm{d}s}\{(sE - A)^k\} = kE(sE - A)^{k-1}$.

(iv) $\dfrac{\mathrm{d}}{\mathrm{d}s}\{(sE - A)^{-k}\} = -kE(sE - A)^{-k-1}$.

(v) $\dfrac{\mathrm{d}^k}{\mathrm{d}s^k}\{(sE - A)^{-1}\} = (-1)^k \cdot (k!) \cdot E^k (sE - A)^{-k-1}$.

3. Eigenstructure assignment

It has been shown [14] that there are n finite eigenvalues that can be assigned arbitrarily with CRPD control law for the controllable singular system (1). From (5), the closed-loop characteristic polynomial is given as

$$\Delta_c(s) \triangleq |s(E + BK) - (A + \mu BK)| = |sE_c - A_c|.$$

Lemma 3.1. *Let $\Delta_c(s) \triangleq |s(E + BK) - (A + \mu BK)|$ be the closed-loop characteristic polynomial, $\Delta_o(s) \triangleq |sE - A|$ be the open-loop characteristic polynomial, and $\phi_o(s) \triangleq (sE - A)^{-1}$; then*

$$\Delta_c(s) = \Delta_o(s)|I_r - (\mu - s)K\phi_o(s)B|.$$

Proof. The closed-loop characteristic polynomial can be expressed as $\Delta_c(s) = |(sE - A) - (\mu - s)BK| = |(sE - A)||I_n - (\mu - s)BK(sE - A)^{-1}| = |(sE - A)||I_r - (\mu - s)K(sE - A)^{-1}B| = \Delta_o(s)|I_r - (\mu - s)K\phi_o(s)B|$. □

Let the set S contain h distinct finite eigenvalues of $(sE_c - A_c)$, $S_f = \{\lambda_1, \ldots, \lambda_h\}$, in which each eigenvalue λ_i has algebraic multiplicity m_i and geometric multiplicity q_i. Note that $\sum_{i=1}^{h} m_i = n$, and $q_i =$ nullity $(\lambda_i E_c - A_c)$. Furthermore, if p_{ij}, $j = 1, \ldots, q_i$, denotes the length of those chained eigenvectors, then $\sum_{j=1}^{q_i} p_{ij} = m_i$, $p_{i1} \geq p_{i2} \geq \cdots \geq p_{iq_i}$.

As described in [6], the closed-loop system (5) has the set of eigenvectors associated with n finite eigenvalues. Let us define the V matrix as follows:

$$V \triangleq V_f \triangleq [V_1(\lambda_1) \quad V_2(\lambda_2) \quad \ldots \quad V_h(\lambda_h)], \tag{6}$$

where V_f are the finite eigenvectors, $V_i(\lambda_i) \triangleq [V_{i1} \quad V_{i2} \quad \ldots \quad V_{iq_i}]$, $i = 1, 2, \ldots, h$. and $V_{ij} \triangleq [v_{ij}^{(0)} \quad v_{ij}^{(1)} \quad \ldots \quad v_{ij}^{(p_{ij}-1)}]$, $j = 1, 2, \ldots, q_i$. Their relationships are as follows [6]:

$$(\lambda_i E_c - A_c)v_{ij}^{(0)} = 0 \quad \text{for } j = 1, 2, \ldots, q_i, \tag{7a}$$

$$(\lambda_i E_c - A_c)v_{ij}^{(k)} = -Ev_{ij}^{(k-1)} \quad \text{for } k = 1, 2, \ldots, (p_{ij} - 1), \tag{7b}$$

where $v_{ij}^{(0)}$ and $v_{ij}^{(k)}$ (for $k \neq 0$) are primary and chained eigenvectors, respectively.

The closed-loop minimum polynomials $\Delta_m(s)$ and the characteristic polynomials $\Delta_c(s)$ are $\Delta_m(s) = k(s - \lambda_1)^{p_{11}}(s - \lambda_2)^{p_{21}} \ldots (s - \lambda_h)^{p_{h1}}$ and

$$\Delta_c(s) = k(s - \lambda_1)^{m_1}(s - \lambda_2)^{m_2} \ldots (s - \lambda_h)^{m_h}. \tag{8}$$

In the proposed method, it is assumed that S_f does not include any open-loop eigenvalue ($\Delta_o(\lambda_i) \neq 0$). To assign the finite eigenvalue spectrum $\{\lambda_i\}$ to the closed-loop system, we have

$$\Delta_c(\lambda_i) = 0, \quad i = 1, 2, \ldots, h, \tag{9}$$

and

$$\frac{\mathrm{d}^k}{\mathrm{d}s^k}(\Delta_c(\lambda_i)) = 0, \quad k = 1, 2, \ldots, (m_i - 1). \tag{10}$$

If the open-loop system is controllable, we can find a nonunique feedback gain matrix K, such that (9) and (10) are satisfied [14]. By Lemma 3.1, (9) and (10) are equivalent to $|I_r - (\mu - s)K\phi_o(s)B| = 0$ and $\mathrm{d}^k/\mathrm{d}s^k|I_r - (\mu - s)K\phi_o(s)B| = 0$, since λ_i is not the root of the open-loop system. Let us define

$$S(s) \triangleq (\mu - s)\phi_o(s)B = (\mu - s)(sE - A)^{-1}B. \tag{11}$$

Lemma 3.2. *Let $\{E, A\}$ be the standard form and k be any positive integer. Then*

(i) $\dfrac{\mathrm{d}}{\mathrm{d}s} S(s) = -(sE - A)^{-2} B.$

(ii) $\dfrac{\mathrm{d}^k}{\mathrm{d}s^k} S(s) = (-1)^k \cdot (k!) \cdot E^{k-1} (sE - A)^{-k-1} B$

for $k \geq 2$.

Proof.

(i) $\dfrac{\mathrm{d}}{\mathrm{d}s} S(s) = -(sE - A)(sE - A)^{-2} B - (\mu - s) E (sE - A)^{-2} B = -(sE - A)^{-2} B.$

(ii) From Lemma 2.2(v) and Lemma 3.2(i), the higher-order derivative of $S(s) (k \geq 2)$ can be found as

$$\frac{\mathrm{d}^k}{\mathrm{d}s^k} S(s) = \frac{\mathrm{d}^{k-1}}{\mathrm{d}s^{k-1}} \left\{ \frac{\mathrm{d}}{\mathrm{d}s} [(\mu - s)(sE - A)^{-1} B] \right\}$$

$$= (-1)^k \cdot (k!) \cdot E^{k-1} (sE - A)^{-k-1} B. \qquad \square$$

The determinant in (9) vanishes if and only if the columns of the matrix $(I_r - KS(\lambda_i))$ are linearly dependent, i.e., for some nonnull r-dimensional vectors $f_{ij}^{(0)}$ satisfying

$$KS(\lambda_i) f_{ij}^{(0)} = f_{ij}^{(0)}. \tag{12a}$$

Considering (9)–(11) and Lemma 3.2 and recalling the rule of differentiating a determinant [3], we add $(m_i - q_i)$ equations as follows:

$$K \left[\frac{\mathrm{d}}{\mathrm{d}s} S(\lambda_i) f_{ij}^{(0)} + S(\lambda_i) f_{ij}^{(1)} \right] = f_{ij}^{(1)}, \tag{12b}$$

$$\vdots$$

$$K \left[\frac{1}{(p_{ij} - 1)!} \frac{\mathrm{d}^{(p_{ij}-1)}}{\mathrm{d}s^{(p_{ij}-1)}} S(\lambda_i) f_{ij}^{(0)} + \cdots + \frac{1}{k!} \frac{\mathrm{d}^k}{\mathrm{d}s^k} S(\lambda_i) f_{ij}^{(p_{ij}-k-1)} + \cdots + S(\lambda_i) f_{ij}^{(p_{ij}-1)} \right] = f_{ij}^{(p_{ij}-1)},$$

$$j = 1, 2, \ldots, q_i \quad \text{and} \quad 0 \leq k \leq (p_{ij} - 1). \tag{12c}$$

Applying Lemma 3.2 in (12), we have

$$K[(\mu - \lambda_i) \phi_o(\lambda_i) B f_{ij}^{(0)}] = f_{ij}^{(0)}, \tag{13a}$$

$$K[(-1) \phi_o^2(\lambda_i) B f_{ij}^{(0)} + (\mu - \lambda_i) \phi_o(\lambda_i) B f_{ij}^{(1)}] = f_{ij}^{(1)}, \tag{13b}$$

$$\vdots$$

$$K[(-1)^{p_{ij}-1} E^{p_{ij}-2} \phi_o^{p_{ij}}(\lambda_i) B f_{ij}^{(0)} + \cdots + (-1)^k E^{k-1} \phi_o^{k+1}(\lambda_i) B f_{ij}^{(p_{ij}-k-1)} + \cdots - \phi_o^2(\lambda_i) B f_{ij}^{(p_{ij}-2)} + (\mu - \lambda_i) \phi_o(\lambda_i) B f_{ij}^{(p_{ij}-1)}] = f_{ij}^{(p_{ij}-1)},$$

$$j = 1, 2, \ldots, q_i \quad \text{and} \quad 0 \leq k \leq (p_{ij} - 1). \tag{13c}$$

Before proceeding to the subsequent development, these r-tuple $f_{ij}^{(k)}$ are in fact design parameter vectors. Hence, we define

$$F \triangleq F_f \triangleq [F_1 \quad F_2 \quad \ldots \quad F_h], \tag{14}$$

where

$F_i \triangleq [F_{i1} \quad F_{i2} \quad \ldots \quad F_{iq_i}]$, $i = 1, 2, \ldots, h$, and $F_{ij} \triangleq [f_{ij}^{(0)} \quad f_{ij}^{(1)} \quad \ldots \quad f_{ij}^{(p_{ij}-1)}]$, $j = 1, 2, \ldots, q_i$.

Next, we try to build up the possible closed-loop finite eigenvectors $v_{ij}^{(k)}$ according to some preselected r-tuple vectors $f_{ij}^{(k)}$ as follows:

$$v_{ij}^{(0)} = (\mu - \lambda_i) \phi_o(\lambda_i) B f_{ij}^{(0)}, \tag{15a}$$

$$v_{ij}^{(1)} = -\phi_o^2(\lambda_i) B f_{ij}^{(0)} + (\mu - \lambda_i) \phi_o(\lambda_i) B f_{ij}^{(1)}, \tag{15b}$$

$$\vdots$$

$$v_{ij}^{(p_{ij}-1)} = (-1)^{p_{ij}-1} E^{p_{ij}-2} \phi_o^{p_{ij}}(\lambda_i) B f_{ij}^{(0)} + \cdots + (-1)^k E^{k-1} \phi_o^{k+1}(\lambda_i) B f_{ij}^{(p_{ij}-k-1)} + \cdots - \phi_o^2(\lambda_i) B f_{ij}^{(p_{ij}-2)} + (\mu - \lambda_i) \phi_o(\lambda_i) B f_{ij}^{(p_{ij}-1)},$$

$$j = 1, 2, \ldots, q_i \quad \text{and} \quad 0 \leq k \leq (p_{ij} - 1). \tag{15c}$$

Furthermore, we define $V \triangleq V_f$ as in (6).

Theorem 3.3. *The closed-loop assignable chains of eigenvectors, $v_{ij}^{(k)}$, can be written as*

$$KV = F \tag{16}$$

under the conditions of

(a) *(space spanned by V) $= \mathbb{R}^n$.*

(b) *$f_{ij}^{(k)} \in \mathbb{R}^r$ for a real eigenvalue λ_i, whereas $f_{i_2 j}^{(k)} = f_{i_1 j}^{(k)*} \in \mathscr{C}^r$ for a complex conjugate pair of eigenvalues $\lambda_{i_1}, \lambda_{i_2} = \lambda_{i_1}^*$.*

Proof. Condition (a) is necessary due to the requirement that eigenvectors be independent.

Condition (b) is necessary because of the requirement that K be a real matrix. We now prove that these $v_{ij}^{(k)}$ are indeed the eigenvectors of the closed-loop system.

By (14) and (6), (16) is equivalent to

$$Kv_{ij}^{(k)} = f_{ij}^{(k)}.$$

We wish to show that $v_{ij}^{(0)}, \ldots, v_{ij}^{(p_{ij}-1)}$ are the chained eigenvectors of the closed-loop system. By (15), $\mu E - A = I$, and the fact of $E\phi_o(\lambda_i) = \phi_o(\lambda_i)E$, we have

$$\begin{aligned} v_{ij}^{(k)} &= -E\phi_o(\lambda_i)v_{ij}^{(k-1)} - \phi_o^2(\lambda_i)Bf_{ij}^{(k-1)} \\ &\quad + (\mu - \lambda_i)E\phi_o^2(\lambda_i)Bf_{ij}^{(k-1)} \\ &\quad + (\mu - \lambda_i)\phi_o(\lambda_i)Bf_{ij}^{(k)} = -E\phi_o(\lambda_i)v_{ij}^{(k-1)} \\ &\quad + (\mu - \lambda_i)\phi_o(\lambda_i)Bf_{ij}^{(k)} - \phi_o(\lambda_i)Bf_{ij}^{(k-1)}, \end{aligned} \tag{17}$$

where $v_{ij}^{(-1)} = 0$ and $f_{ij}^{(-1)} = 0$. By (7), we have

$$A_c v_{ij}^{(0)} - \lambda_i E_c v_{ij}^{(0)} = 0 \quad \text{for } k = 0,$$

$$A_c v_{ij}^{(k)} - \lambda_i E_c v_{ij}^{(k)} = E_c v_{ij}^{(k-1)} \quad \text{for } k \geq 1,$$

since the left-hand side is equal to

$$\begin{aligned} &(A + \mu BK)v_{ij}^{(k)} - \lambda_i(E + BK)v_{ij}^{(k)} \\ &= -(\lambda_i E - A)v_{ij}^{(k)} + (\mu - \lambda_i)Bf_{ij}^{(k)} \\ &= Ev_{ij}^{(k-1)} - (\mu - \lambda_i)Bf_{ij}^{(k)} + Bf_{ij}^{(k-1)} \\ &\quad + (\mu - \lambda_i)Bf_{ij}^{(k)} \\ &= (E + BK)v_{ij}^{(k-1)} = E_c v_{ij}^{(k-1)}. \end{aligned} \tag{17}$$

Hence, $v_{ij}^{(k)}$ is an eigenvector of rank k. □

From (14) and (6), the CRPD feedback gain can easily be obtained

$$K = F_f V_f^{-1} = FV^{-1}. \tag{18}$$

Just as mentioned in Lemma 2.1, the closed-loop eigenstructure of the general and standard form systems will be the same if the control law $u(t) = \mu Kx(t) - K\dot{x}(t) + w(t)$ is applied. In our derivation, zeroes are allowed as closed-loop eigenvalues. Hence, the dead-beat control is possible in our design. Also, the chained eigenstructure assignment is considered.

Remark. (i) Note that CRPD control law can assign n finite eigenvalues for the controllable singular system, the proportional control law can assign $n - \text{rank}(\hat{E})$ finite eigenvalues. Furthermore, to guarantee the regularity of the closed-loop system, there is no restriction on CRPD gain matrix K.

(ii) The properties of closed-loop invariant subspaces with CRPD and PD control law have been examined by geometric idea in [10] and [11], respectively. However, the explicit numerical methods for eigenstructure assignment in singular systems with CRPD control law are still lacking in existing results.

(iii) Some inefficiency can be found in our method. We need to choose another set of free parameters $f_{ij}^{(k)}$ if V is singular. The computation complexity of our methods is similar to that of [3], in which the state feedback eigenstructure assignment in regular systems is investigated. To improve this computational inefficiency is a topic worth further investigation.

4. Numerical example

Consider the controllable continuous-time singular system of (1), where

$$\hat{E} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\hat{A} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}.$$

We will find the CRPD feedback gain matrix K satisfying the design requirements described in the problem formulation.

Solution. We take $\mu = 2$ so that $|\mu\hat{E} - \hat{A}| \neq 0$. The general form triple $\{\hat{E}, \hat{A}, \hat{B}\}$ is left-multiplied by $(\mu\hat{E} - \hat{A})^{-1}$ and transformed to the standard form triple $\{E, A, B\}$.

We set the desired eigenvalues to be $\{\lambda_1 = -1, \lambda_2 = 0\}$; the associated algebraic and geometric multiplicities are $m_1 = 4$, $q_1 = 2$; $m_2 = 2$, $q_2 = 1$. The lengths of the Jordan chains associated with $\lambda_1 \lambda_2$ are $p_{11} = 3$, $p_{12} = 1$ and $p_{21} = 2$. We choose the design parameters

$$F_f = [f_{11}^{(0)} \ f_{11}^{(1)} \ f_{11}^{(2)} \ f_{12}^{(0)} \ f_{21}^{(0)} \ f_{21}^{(1)}] = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}.$$

The possible closed-loop finite eigenvectors are V_f (see (15))

$$V_f = [v_{11}^{(0)} \ v_{11}^{(1)} \ v_{11}^{(2)} \ v_{12}^{(0)} \ v_{21}^{(0)} \ v_{21}^{(1)}] = \begin{bmatrix} 0.0 & 1.50 & -1.25 & 1.50 & 0.00 & 0.00 \\ 0.0 & -1.50 & -0.25 & -1.50 & 0.00 & 2.00 \\ -3.0 & -0.50 & -0.25 & -1.50 & -2.00 & -1.00 \\ 3.0 & -1.00 & 0.00 & 3.00 & 0.00 & -4.00 \\ -3.0 & -2.00 & -2.00 & -3.00 & -4.00 & 0.00 \\ 0.0 & -3.00 & 4.00 & -3.00 & 0.00 & 0.00 \end{bmatrix}.$$

Since $V = V_f$ is nonsingular, we know that our choice of $f_{ij}^{(k)}$ is admissible. Finally, we have

$$K = FV^{-1} = F_f V_f^{-1} = \begin{bmatrix} 0.5648 & 0.0833 & -0.3889 & -0.1111 & -0.0556 & 0.3796 \\ 0.0278 & 0.2500 & 0.5000 & -0.0000 & -0.5000 & -0.1944 \end{bmatrix}.$$

To check the result, we apply the control law $u(t) = \mu Kx(t) - K\dot{x}(t) + w(t)$ to the general form system $\hat{E}\dot{x}(t) = \hat{A}x(t) + \hat{B}u(t)$. We have $\hat{E}_c = \hat{E} + \hat{B}K$, $\hat{A}_c = \hat{A} + \hat{B}K$ and

$$\hat{E}_c = \begin{bmatrix} 1.5648 & 0.0833 & -0.3889 & -0.1111 & -0.0556 & 0.3796 \\ 1.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.5926 & 0.3333 & 0.1111 & -0.1111 & -0.5556 & 0.1852 \\ 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix},$$

$$\hat{A}_c = \begin{bmatrix} 1.1296 & 0.1667 & 0.2222 & -0.2222 & -0.1111 & 0.7593 \\ 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 1.1852 & 0.6667 & 0.2222 & -0.2222 & -0.1111 & 0.3704 \\ 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}.$$

Then we transform $(s\hat{E}_c - \hat{A}_c)$ to its Weierstrass form by [6], $\hat{W}^{-1}(s\hat{E}_c - \hat{A}_c)\hat{V} = (s\hat{E}_w - \hat{A}_w)$, where $\hat{W} = \hat{E}_c V_f$, and $\hat{V} = V_f$. The computed

$$\hat{E}_w = \left[\begin{array}{ccc|c|cc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right],$$

$$\hat{A}_w = \left[\begin{array}{ccc|c|cc} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & -1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

Note that, the closed-loop system has no infinite eigenvalues and infinite eigenvectors to avoid any impulsive response. It is obvious that the closed-loop chained eigenstructure is exactly assigned to the desired pattern.

5. Conclusions

In this paper we investigate eigenstructure assignment problems for CRPD control law in controllable singular systems. Using nice properties of standard form singular systems we have developed a computational method for CRPD feedback gains. We can obtain nonunique CRPD feedback gain K to assign n arbitrary finite eigenvalues and associated eigenvectors in which the chained structure is allowed. The designed closed-loop system is stable, regular and impulse-free.

The proposed method has the following advantages. (a) This paper presents a CRPD control law for eigenstructure assignment. (b) The computed gain matrix and assigned eigenvectors are in the original coordinate; no further transformations are needed. (c) The closed-loop finite eigenvectors can possess the chained structure. (d) Furthermore, zero values can be assigned as the closed-loop eigenvalues also. Hence, this method can be applied to dead-beat control for discrete-time singular systems.

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