

# Brief Papers

## Variable Structure-Based Nonlinear Missile Guidance/Autopilot Design With Highly Maneuverable Actuators

Fu-Kuang Yeh, Kai-Yuan Cheng, and Li-Chen Fu

**Abstract**—In this brief, we propose a variable structure based nonlinear missile guidance/autopilot system with highly maneuverable actuators, mainly consisting of thrust vector control and divert control system, for the task of intercepting of a theater ballistic missile. The aim of the present work is to achieve bounded target interception under the mentioned 5 degree-of-freedom (DOF) control such that the distance between the missile and the target will enter the range of triggering the missile's explosion. First, a 3-DOF sliding-mode guidance law of the missile considering external disturbances and zero-effort-miss (ZEM) is designed to minimize the distance between the center of the missile and that of the target. Next, a quaternion-based sliding-mode attitude controller is developed to track the attitude command while coping with variation of missile's inertia and uncertain aerodynamic force/wind gusts. The stability of the overall system and ZEM-phase convergence are analyzed thoroughly via Lyapunov stability theory. Extensive simulation results are obtained to validate the effectiveness of the proposed integrated guidance/autopilot system by use of the 5-DOF inputs.

**Index Terms**—Attitude control, bounded target interception, nonlinear system, quaternion, variable structure.

### NOMENCLATURE

$a$	Acceleration vector.
$d$	Disturbances vector.
$d_p$	Pitch angle of nozzle.
$d_y$	Yaw angle of nozzle.
$F$	Thrust vector.
$g$	Gravitational acceleration vector.
$J$	Moment of inertia matrix.
$J_0$	Nominal part of $J$ .
$\Delta J$	Variation of $J$ .
$l$	Distance between nozzle and c.g.
$L_b$	$= [-l \ 0 \ 0]^T$ Displacement vector.
$m$	Mass of the missile.
$\dot{m}$	Derivative of $m$ .
$N$	Magnitude of thrust.
$q$	Quaternion.
$r$	Position vector.

$\hat{r}$	Unit vector of $r$ .
$ r $	Magnitude of $r$ .
$t$	Present time.
$T$	Torque.
$v$	Velocity vector.
$v_{ex}$	Exhaust velocity.
$\omega$	Angular velocity vector.

### A. Subscripts

$b$	Body coordinate frame.
$D$	Disturbances.
$d$	Desired.
$e$	Error.
$g$	Gravity.
$M$	Missile.
$0$	Initial time.
$p$	Perpendicular to line-of-sight (LOS).
$S$	Divert control.
$T$	Target/Thrust.

### I. INTRODUCTION

GENERALLY speaking, there are two principal phases for missiles that try to intercept theater ballistic missiles. One is the midcourse guidance [1], the stage before the missile can lock onto the target by its own sensor, whose task is to deliver the missile to some place near the target subject to some additional conditions, such as suitable velocity or appropriate attitude. On the other hand, the homing guidance [2], i.e., the terminal guidance, will refer to the period after the distance between the center of the missile and that of the target is less than some prespecified value, which often leads to a situation where the sensor on the missile can lock onto the target. Based on the concept of the proportional navigation (PN) guidance law, constant bearing guidance is often employed on the intercept missiles. Ha and Chong developed a new command to line-of-sight (CLOS) guidance law for the short-range surface-to-air missile via feedback linearization [3], and then its modified version [4] with improved performance. As for Moon *et al.* [9], they proposed the missile guidance law using variable structure control, where the input command is derived under the condition wherein the target acceleration is treated as an uncertainty. An adaptive sliding-mode guidance of a homing missile was presented by Zhou *et al.* [10] to online estimate some necessary parameters so as to provide robustness to the disturbances.

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Besides guidance, attitude control is another important issue to be addressed for successful missile's operation. Quaternions are used in navigation and guidance algorithm to eliminate the singularities present with direction cosine matrices. It is quite often that quaternion representation has been adopted to describe the attitude of a spacecraft [5], because it is recognized as a kind of global attitude representation. To cope with the non-ideal factors surrounding the spacecraft under attitude control and to enhance the robustness property of the system, sliding mode control has been employed by Chen and Lo [6], which was then followed by a smooth version [7] incorporating a boundary layer as has been proposed by [8] to avoid the chattering phenomenon, but at the price of slightly degrading the tracking accuracy. A missile equipped with thrust vector control (TVC) can effectively control its acceleration direction [1] when the missile's fin fails, which in turn implies that the maneuverability/controllability of the missile can be greatly enhanced during the stage when the speed of the missile is slow and/or the air density surrounding the missile is low. In this brief, we investigate the variable structure (VS)-based missile guidance/autopilot problem for a missile equipped with TVC and divert control system (DCS) so that the intercepting missile is able to fulfill the purpose of successful interception of an inbound target missile in a single intercepting phase.

## II. PRELIMINARIES

### A. Equations of Motion for Missiles with TVC

The motion of a missile can be described in two parts as follows:

*Translation:*

$$\dot{v}_M = \frac{F_M}{m} + \frac{F_g}{m} + \frac{F_D}{m}, \quad \dot{r}_M = v_M. \quad (1)$$

*Rotation:*

$$J\dot{\omega} = -\dot{J}\omega - \omega \times (J\omega) + T_b + d \quad (2)$$

where  $F_M = (dm/dt)v_{ex}$ , and all the variables are defined in the nomenclature.

After referring to Fig. 1(a)–(c), the force and torque exerted on the missile can be, respectively, expressed in the body coordinate frame as

$$F_{Mb} = F_{Sb} + F_{Tb} = [0 \quad F_{sy} \quad F_{sz}]^T + N[\cos d_p \cos d_y \quad \cos d_p \sin d_y \quad \sin d_p]^T \quad (3)$$

$$T_b = L_b \times F_{Tb} + M_b = lN \left[ \frac{M_{bx}}{lN} \quad \sin d_p \quad -\cos d_p \sin d_y \right]^T \quad (4)$$

where  $M_b = [M_{bx} \quad 0 \quad 0]^T$  is the aforementioned variable moment in the axial direction of the missile. Let the rotation matrix  $B_b$  denote the transformation from the body coordinate frame to the inertial coordinate frame. From (1) to (4), the motion model of the missile can then be written as

$$\dot{v}_M = \frac{(B_b F_{Mb})}{m} + g_M + d_M \quad (5)$$

$$J\dot{\omega} = -\dot{J}\omega - \omega \times (J\omega) + lN \left[ \frac{M_{bx}}{lN} \quad \sin d_p \quad -\cos d_p \sin d_y \right]^T + d \quad (6)$$

where  $g_M = F_g/m$  and  $d_M = F_D/m$ .

### B. Zero-Effort-Miss Phase

Assume that both the missile and the target are moving only with constant gravitational acceleration, i.e.,

$$\begin{aligned} r_T &= r_{T0} + \left(\frac{1}{2}\right)g(t-t_0)^2, & v_T &= v_{T0} + g(t-t_0) \\ r_M &= r_{M0} + \left(\frac{1}{2}\right)g(t-t_0)^2, & v_M &= v_{M0} + g(t-t_0). \end{aligned} \quad (7)$$

Therefore, the relative position vector  $r$  and the relative velocity vector  $v$  at  $t = t_0$  can be, respectively, expressed as

$$\begin{aligned} r &= r_T - r_M = r_{T0} - r_{M0} = r_0 \\ v &= v_T - v_M = v_{T0} - v_{M0} = v_0 \end{aligned} \quad (8)$$

Thus, zero-effort-miss (ZEM) can be computed as

$$\begin{aligned} \text{ZEM} &= \min_{t \geq t_0} |r_0 + v_0(t-t_0)| \\ &= \sqrt{|r_0|^2 - \frac{(r_0 \cdot v_0)^2}{|v_0|^2}} = |r_0| \sin \theta. \end{aligned} \quad (9)$$

If the convergence of  $v_p$  is fast enough, the relative velocity will soon almost be lined up with LOS (see Fig. 2) and, hence,  $\theta \rightarrow 0$ . Apparently, from (9) one can then conclude that  $\text{ZEM} \rightarrow 0$  as  $t \rightarrow \infty$ , which is our ideal goal.

## III. GUIDANCE SYSTEM DESIGN

The equations of relative motion in terms of the relative position  $r = r_T - r_M$  and the relative velocity  $v = v_T - v_M$  are as follows:

$$\dot{v}(t) = \frac{-F_M}{m} - d_M(t) \text{ and } \dot{r}(t) = v(t) \quad (10)$$

where  $\dot{v}_T = g_M$ ,  $\dot{r}_T = v_T$ , and the translation motion of a missile is defined as (1).

To proceed, we first derive the equation of the relative motion perpendicular to the LOS ( $v_p = v - v_r = v - (v \cdot \hat{r})\hat{r}$ ) as follows:

$$\dot{v}_p(t) = -\bar{a}_{Mp} - d_{Mp} - \left(\frac{1}{|r|}\right)|v_p|^2 \hat{r} - \frac{(v \cdot \hat{r})v_p}{|r|} \quad (11)$$

where  $\bar{a}_{Mp} = F_M/m - ((F_M/m) \cdot \hat{r})\hat{r}$  and  $d_{Mp} = d_M - (d_M \cdot \hat{r})\hat{r}$ .

Referring to (11), an adequate design of  $\bar{a}_{Mp}$  is the following:

$$\bar{a}_{Mp}(t) = \frac{-(v \cdot \hat{r})v_p}{|r|} + k_p v_p + \tau_p \quad (12)$$

where  $k_p = \text{diag}(k_{p1} \quad k_{p2} \quad k_{p3})$  is a positive definite diagonal matrix, and  $\tau_p$  is a switching function of the sliding mode control, which readily yields

$$\dot{v}_p = -k_p v_p - d_{Mp} - \frac{|v_p|^2 \hat{r}}{|r|} - \tau_p. \quad (13)$$

Let  $V_G = (1/2)v_p^T v_p$  be a Lyapunov function candidate, and evaluate the time derivative of  $V_G$  along the trajectories of the system (13) as follows:

$$\dot{V}_G = -v_p^T k_p v_p - v_p^T (d_{Mp} + \tau_p) \quad (14)$$

$$\text{with } \tau_p = k_1 \text{sgn}(v_p) \quad (15)$$

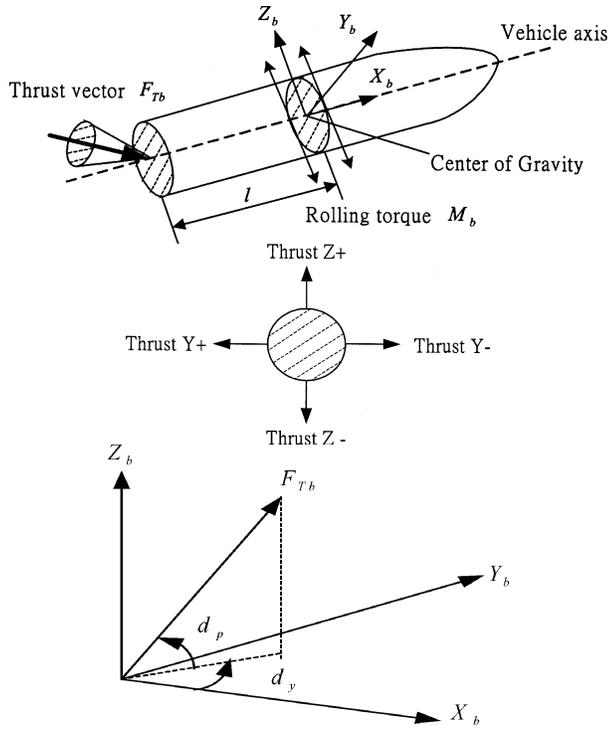


Fig. 1. (a) TVC actuator with single nozzle and rolling torque scheme. (b) Missile's divert control system (bottom view). (c) Two angles of TVC in body coordinate frame.

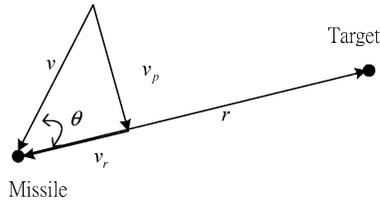


Fig. 2. Relative velocity between missile and target.

where we use the fact  $v_p \cdot \hat{r} = 0$ . It is evident that if we choose  $k_1 = \text{diag}[k_{11} \ k_{12} \ k_{13}]$ ,  $k_{1i} > d_{Mpi}^{\max}$ ,  $i = 1, 2, 3$  and  $d_{Mpi}^{\max} \geq |d_{Mpi}|$ , then (14) becomes

$$\dot{V}_G \leq -\lambda_{\min}(k_p) |v_p|^2 \quad (16)$$

where  $\lambda_{\min}(k_p) (> 0)$  is the minimum eigenvalue of  $k_p$ . Thus, via Lyapunov stability theory, we can conclude that the speed component  $v_p$  will gradually diminish before ZEM phase is reached.

#### IV. AUTOPILOT SYSTEM DESIGN

The attitude of a rigid body may be described in quaternion. Thus, it can be defined as four parameters involving  $n$  and  $\phi$ , i.e.,

$$\begin{aligned} \bar{q} &= [q_1 \ q_2 \ q_3]^T = n \sin\left(\frac{\phi}{2}\right) \\ q_4 &= \cos\left(\frac{\phi}{2}\right). \end{aligned} \quad (17)$$

In theory, it can be verified that the time derivative of a quaternion is as

$$\begin{aligned} \dot{\bar{q}} &= \left(\frac{1}{2}\right) \langle \bar{q} \times \rangle \omega + \left(\frac{1}{2}\right) q_4 \omega \\ \dot{q}_4 &= -\left(\frac{1}{2}\right) \omega \cdot \bar{q}. \end{aligned} \quad (18)$$

To demonstrate the robustness of the controller, we allow (2) and (18) to be subjected to (bounded) input disturbances  $d$  and the uncertain quantities  $\Delta \dot{J}$  and  $\Delta J$ , where  $J = J_0 + \Delta J$ ,  $\dot{J} = \dot{J}_0 + \Delta \dot{J}$ . Let the torque input be proposed as

$$\begin{aligned} T_b &= -k_a S_a + \dot{J}_0 \omega - \left(\frac{1}{2}\right) \dot{J}_0 S_a - \left(\frac{1}{2}\right) J_0 P \langle \bar{q}_e \times \rangle \omega_e \\ &\quad + q_{e4} \omega_e + \omega \times (J_0 \omega) + J_0 \dot{\omega}_d + \tau \end{aligned} \quad (19)$$

where  $\tau = [\tau_1 \ \tau_2 \ \tau_3]^T$ ,  $\tau_i = -k_i(q, \omega, q_d, \dot{q}_d, \ddot{q}_d) \cdot \text{sgn}(S_{ai})$ ,  $i = 1, 2, 3$ , whereas  $\text{sgn}(S_{ai})$  is a sign function, and  $S_a = [S_{a1} \ S_{a2} \ S_{a3}]^T$  is a sliding surface variable defined as  $S_a = p \bar{q}_e + \omega_e$ , with  $P = \text{diag}[P_1 \ P_2 \ P_3]$ . If the inequality condition shown below can be guaranteed

$$K_i(q, \omega, q_d, \dot{q}_d, \ddot{q}_d) > \delta_i^{\max}(q, \omega, q_d, \dot{q}_d, \ddot{q}_d) \geq |\delta_i| \quad (20)$$

where

$$\begin{aligned} \delta &= -\Delta \dot{J} \omega - \omega \times (\Delta J \omega) + d - \Delta J \dot{\omega}_d + \Delta J P \left(\frac{1}{2}\right) \\ &\quad \langle \bar{q}_e \times \rangle \omega_e + q_{e4} \omega_e + \left(\frac{1}{2}\right) \Delta \dot{J} S_a \end{aligned} \quad (21)$$

whose bounding function  $\delta$  is obviously a function of  $q$ ,  $\omega$ ,  $q_d$ ,  $\dot{q}_d$  and  $\ddot{q}_d$ . Here, let the Lyapunov function candidate be set as  $V_s = (1/2) S_a^T J S_a$ .

From the sliding-mode theory, once the reaching condition is satisfied, the system is eventually forced to stay on the sliding manifold, i.e.,  $S_a = P \bar{q}_e + \omega_e = 0$ , where  $\omega_e = \omega - \omega_d$ . It has been shown that [2] the system origin  $(\bar{q}_e, \omega_e) = (0_{3 \times 1}, 0_{3 \times 1})$  is indeed exponentially stable. Taking the first-order derivative of  $V_s$ , we have

$$\begin{aligned} \dot{V}_s &= S_a^T \left[ -\dot{J} \omega - \omega \times (J \omega) + T_b + d - J \dot{\omega}_d + J P \left(\frac{1}{2}\right) \right. \\ &\quad \left. \langle \bar{q}_e \times \rangle \omega_e + q_{e4} \omega_e + \left(\frac{1}{2}\right) \dot{J} S_a \right]. \end{aligned} \quad (22)$$

Therefore, it is evident that (22) becomes

$$\dot{V}_s \leq -\sigma_{\min}(k_a) |S_a|^2 < 0 \quad (23)$$

for  $S_a \neq 0$ , where  $\sigma_{\min}(k_a)$  is the minimum eigenvalue of  $k_a$ , a positive definite diagonal matrix. As a result, the exponential stability and robustness of the autopilot system can be achieved.

#### V. INTEGRATED STABILITY ANALYSIS

To verify the stability of the overall system, we define the Lyapunov function candidate of the overall system as  $V = V_s +$

$V_G$ . The first-order time derivative of the Lyapunov function can be derived as

$$\begin{aligned} \dot{V} = & S_a^T \left[ -\dot{J}\omega - \omega \times (J\omega) + T_b + d - J\dot{\omega}_d \right. \\ & + \left( \frac{1}{2} \right) JP \left( \langle \bar{q}_e \times \rangle \omega_e + q_e 4\omega_e \right) + \left( \frac{1}{2} \right) \dot{J}S_a \left. \right] \\ & + v_p^T \left( -k_p v_p - d_{Mp} - \left( \frac{1}{|r|} \right) |v_p|^2 \hat{r} - \tau_p \right) \quad (24) \end{aligned}$$

referring to (14) and (22).

Now, we are ready to state the following theorem which will provide conditions under which the proposed overall sliding mode guidance and autopilot system controlled by TVC, DCS, and rolling moment guarantee the stability of the entire system and the target-reaching objective is achieved.

*Theorem 1:* Let the equation of relative translational motion perpendicular to LOS and the relative rotational motion be described as in (2), (11), and (18), the sliding mode guidance law be proposed as in (12), the torque input of the autopilot be given as in (19), and the guidance law be switched to ZEM phase when  $|r| \leq r_{\min}$ . If  $v$  is such that  $v(t_0) \cdot \hat{r}(t_0) < 0$ , where  $t_0$  is the starting time, and  $v$  is bounded away from zero, then the integrated overall guidance and autopilot systems will drive the missile to enter the ZEM phase eventually so that the bounded target interception of the integrated system can be achieved.

*Sketch Proof of Theorem 1:* From (24), the expression of  $\dot{V}$  can be simplified as

$$\begin{aligned} \dot{V} = & -v_p^T k_p v_p - v_p^T (d_{Mp} + \tau_p) - S_a^T k_a S_a \\ & - \sum_{i=1}^3 |S_{ai}| [k_i - \delta_i \text{sgn}(S_{ai})]. \quad (25) \end{aligned}$$

Let the desired acceleration  $\bar{a}_{Mp}$  be defined in (12) perpendicular to LOS for the sake of guidance, and let the torque input be shown in (19) for attitude tracking, where both can be absolutely computed. Based on the methodology in the aforementioned,  $\dot{V}$  can be re-expressed as

$$\dot{V} \leq -\sigma_{\min}(k_p) |v_p|^2 - \sigma_{\min}(k_a) |S_a|^2 \quad (26)$$

which means that  $-\dot{V}$  is positive definite and, hence,  $S_a \rightarrow 0$ ,  $v_p \rightarrow 0$  as  $t \rightarrow \infty$  via use of Lyapunov stability theory before entering ZEM phase. Due to the derivation of ZEM phase, the minimum distance between the missile and the target will be less than the prespecified value  $r_{\min}$  during ZEM phase, and the target will be destroyed by choosing a smaller  $r_{\min}$  and triggering the missile's explosion when the closest distance between the missile and the target is within the indicated effective interception range. Finally, to show that Theorem 1 is satisfied, we need to show that  $v \cdot \hat{r} < 0$  before entering ZEM phase, which has been proved in [1]. Therefore, the target-tracking objective during the flight before entering ZEM phase can be completed as derived by the aforementioned proof of theorem 1, but after the ZEM phase, the principal goal of bounded target interception as claimed by the aforementioned theorem can be achieved. ■

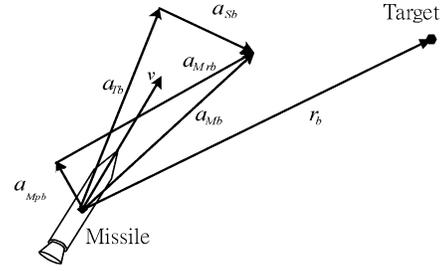


Fig. 3. Relative relation diagram in body coordinate.

Accordingly, the desired overall acceleration  $a_{Mpb}$  perpendicular to the LOS can be derived due to the result in Section III, which together with  $a_{Mrb}$  in the  $r$  direction leads to the desired acceleration  $a_{Mb}$  (see Fig. 3) of the missile, namely

$$a_{Mb} = a_{Tb} + a_{Sb} = a_{Mpb} + a_{Mrb}.$$

Hence, the resulting acceleration  $a_{Mb}$  of the missile due to TVC and DCS together will lie on the plane  $a_{Mpb} - r_b$ . We note the following two facts: 1) projection of the desired resulting acceleration  $a_{Mb}$  onto the axis of  $a_{Mpb}$  is simply  $|a_{Mpb}|$  and 2) projection of  $a_{Mb}$  onto the axis perpendicular to the plane  $a_{Mpb} - r_b$  will be identically zero. Then, we can derive the following constraint equations of  $a_{Sb}$  in the body coordinate frame as:

$$\begin{aligned} a_{Sb} \cdot \hat{a}_{Mpb} &= -a_{Tb} \cdot \hat{a}_{Mpb} + |a_{Mpb}| \\ a_{Sb} \cdot \hat{a}_{\perp b} &= -a_{Tb} \cdot \hat{a}_{\perp b} \\ \text{where } \hat{a}_{\perp b} &= \frac{a_{Mpb} \times \hat{r}_b}{|a_{Mpb} \times \hat{r}_b|}, \hat{r}_b = B_b^T(q)\hat{r} \\ \hat{a}_{Mpb} &= B_b^T(q)\hat{a}_{Mp}, \hat{a}_{Mp} = \frac{a_{Mp}}{|a_{Mp}|}. \end{aligned}$$

By Cramer's rule, the acceleration  $a_{Sb}$  [see Fig. 1(b)] generated by the divert control system, denoted as  $a_{Sb} = [0 \ a_{Sby} \ a_{Sbz}]^T$ , can be derived as

$$\begin{aligned} a_{Sby} &= \frac{\Delta_y}{\Delta}, a_{Sbz} = \frac{\Delta_z}{\Delta} \\ \text{where } \Delta &= \hat{a}_{Mpby}\hat{a}_{\perp bz} - \hat{a}_{Mpbz}\hat{a}_{\perp by} \\ \Delta_y &= (-a_{Tb} \cdot \hat{a}_{Mpb} + |a_{Mpb}|)\hat{a}_{\perp bz} + \hat{a}_{Mpbz}a_{Tb} \cdot \hat{a}_{\perp b} \\ \Delta_z &= -\hat{a}_{Mpby}a_{Tb} \cdot \hat{a}_{\perp b} - (-a_{Tb} \cdot \hat{a}_{Mpb} + |a_{Mpb}|)\hat{a}_{\perp by} \\ \hat{a}_{Mpb} &= [\hat{a}_{Mpbx} \ \hat{a}_{Mpb y} \ \hat{a}_{Mpbz}]^T \\ \hat{a}_{\perp b} &= [\hat{a}_{\perp bx} \ \hat{a}_{\perp by} \ \hat{a}_{\perp bz}]^T. \end{aligned}$$

*Remark 1:* To avoid the singularity for computing the  $a_{Sb}$ , we propose one possible solution to modify the force from the divert control system when the singularity condition " $\Delta = 0$ " occurs as follows:

$$\begin{aligned} a_{Sby} &= \frac{(-a_{Tb} \cdot \hat{a}_{Mpb} + |a_{Mpb}|)}{\hat{a}_{Mpby}}, \text{ and } a_{Sbz} = 0 \\ &\text{if } \hat{a}_{Mpby} \neq 0 \\ \hat{a}_{Sby} &= 0, \text{ and } a_{Sbz} = \frac{(-a_{Tb} \cdot \hat{a}_{Mpb} + |a_{Mpb}|)}{\hat{a}_{Mpbz}} \end{aligned}$$

if  $\hat{a}_{Mpy} = 0$ .

## VI. SIMULATION

To validate the proposed sliding-mode guidance and autopilot of the missile system, we provide a realistic computer simulation in this section. We assume the target is launched from somewhere 600 km far away. The missile has a sampling period of 10 ms. The bandwidth of the TVC is 20 Hz and the two angular displacements are both limited to  $5^\circ$ . Here, we consider the missile's variation of the moment of inertia. Thus, the inertia matrix and the rate of its variation including the nominal part  $J_0$ ,  $\dot{J}_0$  and the uncertain part  $\Delta J$ ,  $\Delta \dot{J}$  used here are as follows:

$$J = J_0 + \Delta J (\text{kg} \cdot \text{m}^2)$$

$$\dot{J} = \dot{J}_0 + \Delta \dot{J} (\text{kg} \cdot \text{m}^2)$$

where

$$J_0 = \begin{bmatrix} 100 & 10 & 300 \\ 10 & 3000 & 300 \\ 300 & 300 & 3000 \end{bmatrix}$$

$$\Delta J = \begin{bmatrix} 10 & 10 & 300 \\ 10 & 300 & 300 \\ 300 & 300 & 300 \end{bmatrix}$$

$$\dot{J} = \begin{bmatrix} -0.4 & -0.1 & -0.2 \\ -0.1 & -12 & -0.2 \\ -0.2 & -0.2 & -12 \end{bmatrix}$$

$$\Delta \dot{J} = \begin{bmatrix} -0.04 & -0.01 & -0.02 \\ -0.01 & -1.2 & -0.02 \\ -0.02 & -0.02 & -1.2 \end{bmatrix}$$

Apparently, all the components of the inertia matrix and its variation depend on the mass and size of the missile [11], the specific impulse and fuel mass fraction of the propellant [12]. The attitude's initial conditions of the missile is set as  $q = [0 \ -0.707 \ 0 \ 0.707]^T$ , i.e., vertical onto the launch pad, the missile's initial angular velocity is as  $\omega(0) = [0 \ 0 \ 0]^T$ , and the variation of missile's mass is as  $\dot{m} = -4$  (kg/s) for the initial mass  $m = 1000$  (kg) and the specific impulse  $I_{sp} = 250$  (s). Furthermore, we also consider the aerodynamic force and wind gusts exerted on the missile by including the term  $d_i(t) = \sin(t) + 10(u(t-20) - u(t-21))$  (Nt-m) for the rotation motion as described in (2) and  $d_{Mi}(t) = \sin(t) + 10(u(t-20) - u(t-21))(m/s^2)$  for the translation motion as described in (6), for  $i = 1, 2, 3$ , where  $u(t)$  is the step function. Besides that, we also check the force which is produced by the divert control system equipped at the center of gravity.

In numerical simulation, the feasibility of the presented approach is satisfactorily demonstrated by the results of simulation scenario shown in Fig. 4. The total simulation time of intercepting phase is 109.46 s, and the ZEM phase starts when the distance between the missile and the target is less than 100 m and lasts for only about 0.02 s. Finally, the shortest distance at the intercepting point is less than 1 m. The effect of the intercepting missile can be shown as in Fig. 4(a) and (b) since the lower velocity missile intercepts the ballistic missile with higher velocity after entering its reentry phase. The final velocity of the intercepting missile is 1230.6 m/s, which is almost one-third of

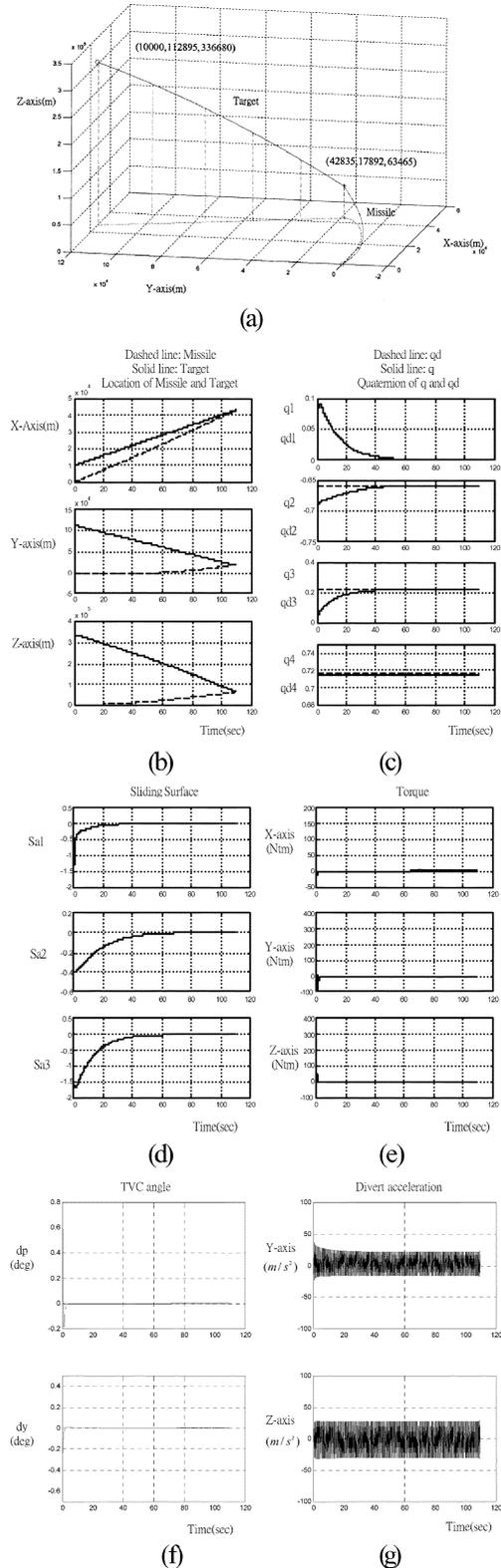


Fig. 4. Results of simulation scenario.

the velocity, 3168.6 m/s, of the target at the final time. On the other hand, the attitude tracking can be verified by the results of Fig. 4(c) and (d), which demonstrate successful tracking effects in terms of the quaternion angle and the sliding surface variable, respectively, for the rotation motion. From Fig. 4(e) and (f),

which show that a relatively larger value of control torque input occurs at the start during the entire flight phase of the missile, one can conclude that economical power consumption should be enough to complete the attitude tracking almost throughout the intercepting period. Finally, the plausible divert acceleration of DCS, less than  $3g$ , where  $g$  is the gravitational acceleration, is shown in Fig. 4(g). In order to strengthen the applicability of the DCS actuator, we adopt a low-pass filter with bandwidth 5 Hz to limit the response performances of the actuator, and the desired force of the intercepting missile for the translation motion remain realistic enough to achieve the satisfactory ballistic missile interception.

## VII. CONCLUSION

The overall process of intercepting a ballistic missile generally includes two parts: midcourse and terminal phases. In this brief, we focus on the overall phase composed of the above two phases of the interception, which is a period of time lasting until the ballistic missile is destroyed by the intercepting missile. Considering the properties of TVC, DCS, and nonideal conditions during the interception phase, we employed the controller incorporating variable structure based nonlinear missile guidance and autopilot systems, which can robustly adjust not only the missile attitude but also the translational displacement even under the conditions of model uncertainty and disturbances, such as variation of missile's inertia, influence of aerodynamic force and unpredictable wind gusts. We respectively proved the stability of the individual guidance, autopilot and integrated systems via Lyapunov stability theory. Finally, after switch to ZEM phase, a bounded target interception can be achieved. A numerical simulation has been conducted to verify the feasibility of the integrated sliding-mode guidance and the autopilot systems with 5-DOF inputs.

TABLE I  
SIMULATION SCENARIO

Target:

	X	Y	Z
Initial Position (m)	10000	112895	336680
Initial Velocity (m/sec)	300	-868	-1960

Missile:

	X	Y	Z
Initial Position (m)	0	0	0
Initial Velocity (m/sec)	0	0	100

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