An Iterative Algorithm for Doppler Spread Estimation in LOS Environments

Po-Ying Chen, Student Member, IEEE, and Hsueh-Jyh Li, Senior Member, IEEE

Abstract—An iterative algorithm is proposed to estimate the maximum Doppler frequency in a line-of-sight (LOS) environment. While the autocorrelation function of a Rayleigh fading signal depends only on the maximum Doppler frequency, that of a Rician fading signal is also related to the K factor and the angle of arrival (AOA) of the LOS component. The performance of conventional Doppler spread estimators based on the Rayleigh fading assumption degrades in LOS environments due to the autocorrelation function mismatch. The proposed estimator estimates the Rician K factor first, then iteratively estimates the Doppler spread and the AOA of the LOS component. The performance degradation due to channel parameter variations is investigated by means of matching the autocorrelation functions. The simulation results indicate that the proposed estimator is not only superior to the conventional autocorrelation-based Doppler spread estimator but also robust to the channel parameter variation in both LOS and non-LOS environments.

Index Terms—Estimation, maximum Doppler frequency, Rician fading.

I. INTRODUCTION

N mobile communication systems, multipath propagation usually gives rise to a fading channel. The Doppler spread is induced when a mobile station is moving in such a multipath environment. It distorts the transmitted signal, and causes difficulties in both channel estimation and synchronization at the receiver. The maximum Doppler frequency is an essential parameter of the channel. Related to the speed of the mobile station, the maximum Doppler frequency provides useful information in adaptive handoff [1] and is also the designed bandwidth of the adaptive channel estimation filter [2]. As the demand on wireless service increases rapidly, adaptive transmission techniques, such as Adaptive Modulation and Coding (AMC) [3], are used to enhance the channel capacity by adaptively changing transmission parameters according to the channel state information (CSI). The adaptation rate should be optimized according to the fading rate of the channel that can be directly deduced from the Doppler spread. With its wide applications, the accurate estimation of Doppler spread is indispensable for the design of a modern mobile communication system.

Manuscript received September 7, 2003; revised April 1, 2004, September 15, 2004, February 17, 2005, and April 6, 2005; accepted May 4, 2005. The associate editor coordinating the review of this paper and approving it for publication was J. Garcia-Frias. This research work was supported by the National Science Council of the Republic of China under the grant number NSC 91-2219-E-002-023 and the MOE program for promoting academic excellence of universities under the grant number 89E-FA06-2-4-7. This paper was presented in part at the IEEE International Conference on Communications (ICC'04), Paris, France.

The authors are with the Graduate Institute of Communication Engineering & Department of Electrical Engineering, National Taiwan University, Taipei 10617, Taiwan, R.O.C. (e-mail: hjli@ew.ee.ntu.edu.tw).

Digital Object Identifier 10.1109/TWC.2006.03466.

Methods to estimate the maximum Doppler frequency have been developed based on the statistics of the fading signals. For noncoherent detection systems, the level-crossing rate (LCR) [4] or the autocovariance of the envelope [1] can be used to estimate the maximum Doppler frequency. In coherent detection systems, where the channel estimation is performed at the receiver, both the autocorrelation and the phase difference of channel estimates provide sufficient statistics to estimate the maximum Doppler frequency [5], [6]. Most of the related works, however, assume the channel is Rayleigh fading. For a Rician fading channel in which the mobile station is moving in an environment with a line-of-sight (LOS) component, the statistics of the LCR and the autocorrelation also depends on the Rician K factor and direction of the LOS component. The performance of the estimator based on the Rayleigh assumption is seriously degraded in such an environment. This paper proposes an estimator that iteratively estimates the maximum Doppler frequency based the autocorrelation function in the LOS environment. Simulation results show that with sufficient iterations, our proposed estimator provides tremendous improvement over the conventional one. The convergence behavior of the algorithm is not addressed in the paper, but is a topic of future research.

The rest of this paper is organized as follows. The Rician fading signal model is described in Section II. In Section III, two K factor estimators are introduced before we propose the iterative Doppler spread estimator. Section IV compares the performance degradation due to channel parameter variations for the conventional and the novel estimators. In Section V, all of the simulation results are presented. Section VI is the conclusion of the paper.

II. SIGNAL MODELING

In a scattering environment with a LOS component, the baseband representation of the received flat fading signal, which is composed of a deterministic function and a narrowband Gaussian random process, can be expressed as:

$$x(t) = \sqrt{\frac{K\Omega}{K+1}} e^{j(2\pi f_m \cos \theta_0 t + \phi_0)} + \sqrt{\frac{\Omega}{K+1}} h(t) \qquad (1)$$

where K is the Rician factor defined as the ratio of the power of the LOS component to that of the diffuse component; f_m is the maximum Doppler frequency; θ_0 is the angle of arrival (AOA), that is the angle between the LOS component and the moving direction of the mobile; ϕ_0 is the random phase of the LOS component; h(t) is the narrowband signal composed of scattering components. When the mobile travels with a velocity v, the maximum Doppler frequency is $f_m = vf_c/c$, where f_c is the carrier frequency, and c is the speed of light. For large number of scatterers, h(t) is a Gaussian random process according to the central limit theorem [4]. The total power of the received signal is $\Omega = E\{|x(t)|^2\}$, where $E\{\}$ denotes the operation of taking expectations.

The normalized autocorrelation of the received signal defined as $\mathbb{P}\left(\left(\left(x - y \right) + t \left(y \right) \right) \right)$

$$\rho_x(\tau) = \frac{E\{x(t+\tau)x^*(t)\}}{E\{|x(t)|^2\}}$$
(2)

can be represented as the combination of $\rho_{LOS}(\tau)$ and $\rho_h(\tau)$ as

$$\rho_x(\tau) = \frac{K\rho_{LOS}(\tau) + \rho_h(\tau)}{K+1} \tag{3}$$

where $\rho_{LOS}(\tau)$ and $\rho_h(\tau)$ are the normalized autocorrelation of the LOS component and the diffuse component, respectively. When the well-known Clark-Jakes model is applied to the diffuse component, (3) can be rewritten as

$$\rho_x(\tau) = \frac{K e^{j2\pi f_m \cos\theta_0 \tau} + J_0(2\pi f_m \tau)}{K+1} \tag{4}$$

where $J_0(x)$ is the zeroth order Bessel function of the first kind. Our proposed iterative estimator is mainly based on (4) given an estimate of the K factor.

III. THE ITERATIVE ESTIMATION ALGORITHM

Similar to the conventional Doppler spread estimator, only the autocorrelation of the in-phase or the quadrature-phase component is used (the cross-correlation between them is ignored). Hence the estimator takes the real parts of (2) and (4) for estimating f_m . We first estimate the Rician K factor and the Doppler frequency of the LOS part f_0 , then guess an initial maximum Doppler frequency f_m . With these two values, the information about the AOA of the LOS component is acquired by the relationship of $f_0 = f_m \cos \theta_0$. Next, the estimated AOA and K factor are substituted into (4) and we invert the autocorrelation function to make an estimate of the maximum Doppler frequency. The estimated AOA is then updated with the knowledge of f_0 and the newly-estimated f_m . Thus the iterative estimation procedure is constructed.

In our proposed approach, we introduce two estimators to acquire the Rician K factor information. The first one is based on the moments of the received signal's envelope [7]. It uses the fact that the ratio between the m-th power of the n-th moment and the n-th power of the m-th moment is a function of K only. The Rician K factor is estimated by the inversion of this function. The other one, which was proposed by Tepelenliglu and Abdi [8], is the in- and quadrature-phase (I/Q) components based Rician factor estimator. In this estimator, the power ratio of the LOS part to the diffuse component is calculated for estimating K. Since the estimation procedure is done in frequency domain, the Doppler frequency of the LOS component can also be estimated at the same time.

Before leaving this section, we present the step-by-step procedure of the proposed algorithm as follows.

- **step 1.**Estimate the K factor, \hat{K} , by the moment-based or the I/Q-based estimator; and the Doppler frequency of the LOS component, \hat{f}_0 , by analyzing the Doppler spectrum of the fading signals.
- **step 2.**Set an initial estimate of the maximum Doppler frequency, say $\hat{f}_m^{(0)}$. The initial value can be obtained

by the conventional estimator that estimates f_m by inverting the Bessel function, i.e.,

$$\hat{f}_m^{(0)} = \frac{1}{2\pi m T_s} J_0^{-1}(\hat{\rho})$$
(5)

where T_s is the sampling interval of the fading signal, and $\hat{\rho}$ is the measured correlation coefficient, defined as

$$\hat{\rho} = \frac{\Re\{\frac{1}{N-m}\sum_{n=0}^{N-m-1} x[n]x^*[n+m]\}}{\frac{1}{N}\sum_{n=0}^{N-1} |x[n]|^2} \quad (6)$$

Here mT_s is the correlation lag and it should be kept small or moderate to avoid ambiguity when inverting the autocorrelation function.

step 3. Iteratively estimate the maximum Doppler frequency and the AOA of the LOS component. Let M be the number of iterations, for i = 1, 2, ..., M

$$\hat{\theta}_0^{(i)} = \cos^{-1}(\hat{f}_0 / \hat{f}_m^{(i-1)}) \tag{7}$$

Construct the function

$$F^{(i)}(x) = \frac{\hat{K}\cos(x\cos\hat{\theta}_0^{(i)}) + J_0(x)}{\hat{K} + 1}$$
(8)

then

$$\hat{f}_m^{(i)} = \frac{1}{2\pi m T_s} F^{(i)^{-1}}(\hat{\rho}) \tag{9}$$

The function $F^{(i)}(x)$ is given by the real part of (4). The variable x and the maximum Doppler frequency f_m are related by $x/2\pi = f_m\tau$. Because of the difficulty to find a closed-form expression, the inverse function $F^{(i)^{-1}}(x)$ can be computed by a look-up table. For the simulations in Section V, a look-up table with 351 entries is used. It records the values calculated by (8), with the variable x varying from 0 to 3.5. The table is updated once a new AOA $\hat{\theta}_0^{(i)}$ is estimated.

IV. PERFORMANCE ANALYSIS

In this section, we investigate the impact of parameter variations on the performance of the maximum Doppler frequency estimator based on the autocorrelation function. The in-phase part of the autocorrelation functions of the Rician fading signal, i.e. the real part of (4), is plotted in Fig. 1 for different channel parameters. The unit of the *x*-axis is in radian.

In Fig. 1(a), for a fixed AOA of the LOS component at $\theta_0 = \pi/3$, the autocorrelation function is plotted for different K factors for comparison. It deviates more from the Bessel function as the K factor increases. Hence the degradation of the conventional estimator becomes severer with the increase of the K factor.

In Fig. 1(b), for a fixed K factor of 3 dB, the AOA of θ_0 is varied from 0 to $\pi/2$. The correlation coefficient increases from 0 to $\pi/2$. There exists an AOA of θ_0 between 0 and $\pi/2$ such that the corresponding autocorrelation function is close to the Bessel function. This θ_0 is solved by the equation

$$J_0(2\pi f_m \tau) = \frac{K \cos(2\pi f_m \cos \theta_0 \tau) + J_0(2\pi f_m \tau)}{K+1}$$
(10)

This value changes with the channel parameters, including the K factor and the value $2\pi f_m \tau$. With this AOA, the conventional Doppler spread estimator performs well since the actual correlation coefficient is close to that calculated from the Rayleigh fading signal under the same f_m .

At the end of this section, the effect of the noise corruption is investigated. Assume $\tilde{\rho}$ and ρ are the correlation coefficients with and without noise corruption, it is easy to derive the relation

$$\tilde{\rho} = \frac{E\{(x(t) + n(t))(x(t - \tau) + n(t - \tau))^*\}}{E\{|x(t) + n(t)|^2\}}$$
$$= \frac{E\{x(t)x(t - \tau)^*\}}{E\{|x(t)|^2\} + E\{|n(t)|^2\}}$$
$$= \frac{\rho}{1 + 1/\gamma}$$
(11)

where γ is the signal-to-noise ratio (SNR). When K = 3 dB and $\theta_0 = \pi/3$, Fig. 1(c) plots the autocorrelation functions corrupted by additive noise under different SNRs. We can predict that even when both K and θ_0 can be estimated perfectly, noise degrades the proposed estimator seriously. Note that each autocorrelation function has an intersection point with the Bessel function. If the measured correlation coefficient locates at one of the intersection points, the conventional Doppler spread estimator can provide a perfect estimate. Given $x = 2\pi f_m \tau$, the SNR γ corresponding to the intersection point is found by solving the following equation

$$J_0(2\pi f_m \tau) = \frac{1}{1+1/\gamma} \frac{K \cos(2\pi f_m \cos\theta_0 \tau) + J_0(2\pi f_m \tau)}{K+1}$$
(12)

Although the conventional maximum Doppler frequency estimator can perform well under certain channel parameters in Rician fading channels, the estimation degrades seriously even when these parameters change slightly. On the other hand, our proposed estimator can adaptively change the kernel function of (8) according to the channel parameters instead of relying solely on the Bessel function. Therefore, it performs consistently better than the conventional estimator.

V. SIMULATIONS

In this section, each numerical result is simulated independently 500 times to yield the mean value and the corresponding root mean-square-error (RMSE), which is the square root of the sum of the variance and the square of the bias of estimates. We also generalized the work done in [9] to derive the Cramér-Rao bounds (CRB) of the maximum Doppler frequency for comparison. The scattering part of the fading signal is generated by Jakes-like generator [10] with 21 scattering points to ensure the Rayleigh statistics. In most of these simulations, the maximum Doppler frequency is set as 150 Hz and the AOA of the LOS component is $\theta_0 = \pi/3$. We choose the channel sampling rate as 1.5 kHz, and a sequence of data with length 256 is generated in each simulation, this corresponds to an overall simulation period of 170.67 ms. The mobile station is assumed to remain at the same velocity with the same scattering points during each simulation period. The correlation lag is set as $\tau = 3T_s$.

Fig. 2 shows the convergence rates of our iterative algorithm under different K factors estimated by the I/Q-based method







Fig. 1. Autocorrelation functions of Rician fading signals under different channel parameters. (a) Without noise, $\theta_0 = \pi/3$, varying the Rician K factor; (b) Without noise, K = 3 dB, varying the AOA of the LOS component; (c) With noise, K = 3 dB, $\theta_0 = \pi/3$, varying SNR.



Fig. 2. (a) The mean and (b) the RMSE of Doppler spread estimates versus the number of iterations under different K factors, $\theta_0 = \pi/3$, $f_m = 150$ Hz, and SNR = 40 dB.

with average SNR of $\gamma = 40$ dB. The number of iterations increases for large K factors. Here the initial maximum Doppler frequency, which corresponds to the zeroth iteration in Fig. 2, is obtained by the conventional Doppler spread estimator. As the K factor increases, the channel deviates more from the Rayleigh assumption, so this algorithm takes more iterations to obtain a convergent estimate. When K is equal to -9 dB, the estimation is biased regardless of the number of iterations and the RMSE is slightly larger. This is because the LOS component in spectral domain is difficult to detect accurately for small K, and the estimation of the K factor is degraded.

In order to evaluate the performance of the novel and the conventional estimators, they are compared in different channel environments. Here the two K factor estimators mentioned in Section III are both applied in the proposed algorithm. The second and forth moments of the envelope of the received signal are used in the moment-based K factor estimator. Fig. 3 compares the performance of these estimators for different



Fig. 3. The RMSE of Doppler spread estimates versus K factors under $\theta_0 = \pi/3$, $f_m = 150$ Hz, and SNR = 40 dB; with 60 iterations.

K. It also shows the RMSE obtained by our estimator when the actual K factor and the AOA of the LOS component are both known to the receiver. All of the RMSEs for the iterative approach are computed after 60 iterations to ensure a meaningful statistics. The result indicates that if both the K factor and the AOA θ_0 can be estimated perfectly, our estimator can approach CRB for large K factors, since the variance of the measured correlation coefficients is reduced when the power of the deterministic part of the received signal increases. For low K factors, because the I/Q-based K factor estimator provides better estimates than the moment-based method, the novel iterative estimation algorithm equipped with the I/Q-based K factor estimator performs better than that with the moment-based K factor estimator. However, because the channel is close to a Rayleigh channel for small K factors, the conventional estimator has smaller RMSE than the proposed one no matter how the K factor is estimated. For channels with moderate or large K factors, the novel estimator has tremendous improvement over the traditional approach.

In the sequel, the simulation results are compared in the Rician channel with K = 3 dB, and the numerical results are obtained after 20 iterations. Fig. 4 shows the effect of additive noise upon the estimators. The performance of the iterative algorithm is much better than the conventional estimator for those SNRs larger than 7 dB. However, the conventional estimator provides smaller RMSE in low SNR region. The reason is that as indicated in the previous section, at certain SNR, the correlation coefficient corrupted by noise is very close to the value obtained by the Bessel function with the same correlation lag. The corresponding SNR can be solved by (12) to be $\gamma \approx 1.66$ dB. When the SNR is in the neighborhood of 1.66 dB, the conventional estimator provides excellent estimation. Its performance degrades a lot at other SNRs.

Fig. 5 shows that the RMSE obtained by the proposed estimator does not vary much with respect to θ_0 . However, the RMSE curve of the conventional estimator forms a V-shape with the minimum RMSE for AOAs around 0.8 radian. This was expected, since for high SNR the conventional estimator



Fig. 4. The RMSE of Doppler spread estimates versus SNRs under K = 3 dB, $\theta_0 = \pi/3$, $f_m = 150$ Hz; with 20 iterations.



Fig. 5. The RMSE of Doppler spread estimates versus AOAs of the LOS component under K=3 dB, $f_m=150$ Hz, and SNR = 40 dB; with 20 iterations.

has good performance at the θ_0 solved by (10), which is found to be $\theta_0 \approx 0.827$ radian and agrees with the numerical result in Fig. 5. As the AOA approaches $\pi/2$, the slightly increase of the RMSE, which is calculated by the proposed algorithm with known parameters, results from the variation of the measured correlation coefficient. From Fig. 1(b), it has been shown that the slope of the autocorrelation function decreases as the AOA increases from 0 to $\pi/2$. With smooth autocorrelation function, even small variations of the measured correlation coefficients result in varying values after the inverse of the function. This leads to the increase of the variance of the maximum Doppler frequency estimates. This effect is not so apparent for the estimator with estimated AOA when we perform the iterative algorithm with the I/Q-based K factor estimator, since, as we substitute the estimated AOA in the lookup table iteratively, the function of (8) is also adapting. This adaptation makes the estimates less variant when channel





Fig. 6. (a) The mean and (b) the RMSE of Doppler spread estimates versus Doppler spreads under K = 3 dB, $\theta_0 = \pi/3$, and SNR = 40 dB; with 20 iterations.

parameters are changed. Notice that since the estimated K factors obtained by the moment-based method have larger bias and variance, the adaptation property of the iterative algorithm becomes a non-issue. Therefore, the performance of the novel estimator using the moment-based method also degrades when the AOA approaches $\pi/2$.

Before leaving this section, we investigate the performance of the proposed estimator for different maximum Doppler frequencies. Fig. 6 shows that the proposed estimator is unbiased after convergence, while the bias of the conventional estimator increases with the Doppler spread. The main contribution of the RMSE for low f_m comes from the inaccuracy on the estimation of K factors due to the short data length. A tremendous improvement is found for the novel iterative estimator over the conventional approach, and the RMSE is generally smaller than 15 Hz.

VI. CONCLUSION

A Doppler spread estimator is proposed to estimate the maximum Doppler frequency in an environment where a LOS exists with a given or an estimated K factor. Based on the behavior of the autocorrelation functions, we investigate the impact of channel parameter variations on the proposed and the conventional estimator. The simulation results show that the proposed estimator based on the assumption of Rician fading channel yields smaller RMSE than the conventional Rayleigh channel-based Doppler spread estimator. With moderate to large SNRs, the iterative approach is more robust to channel variations. Even when the channel is close to Rayleigh fading, i.e. for small K factors, the proposed algorithm can still perform well. For moderate K factors, a small number of iterations is sufficient for the convergence of the RMSE.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for their constructive and helpful comments and suggestions.

REFERENCES

 J. M. Holtzman and A. Sampath, "Adaptive averaging methodology for handoffs in cellular systems," *IEEE Trans. Veh. Technol.*, vol. 44, pp. 56–66, Feb. 1995.

- [2] M. Sakamoto, J. Huoponen, and I. Niva, "Adaptive channel estimation with velocity estimator for W-CDMA receiver," in *Proc. IEEE Vehicular Technology Conference (VTC'2000)*, pp. 2024–2028.
- [3] S. Catreux, V. Erceg, D. Gesbert, and R. W. Heath, Jr., "Adaptive modulation and MIMO coding for broadband wireless data networks," *IEEE Commun. Mag.*, vol. 40, pp. 108–115, June 2002.
- [4] G. L. Stuber, *Principles of Mobile Communications*. Boston, MA: Kluwer, 1996.
- [5] M. Sakamoto and Y. Akaiwa, "Time correlation based maximum doppler frequency estimator for W-CDMA receiver," in *Proc. IEEE Vehicular Technology Conference (VTC'2001)*, pp. 2026–2029.
- [6] K.-J. Han and E.-K. Hong, "A channel estimation algorithm for mobile communication systems in a fading environment," *IEICE Trans. Commun.*, vol. E85-B, pp. 682–685, March 2002.
- [7] L. J. Greenstein, D. G. Michelson, and V. Erceg, "Moment-method estimation of the Ricean K-factor," *IEEE Commun. Lett.*, vol. 3, no. 6, pp. 175–176, June 1999.
- [8] C. Tepelenliglu and A. Abdi, "Estimation of the Rice factor from the I/Q components," in *Proc. of the Conference on Information Systems* and Sciences 2002, pp. 423–428.
- [9] C. Tepelenliglu, A. Abdi, and G. B. Giannakis, "The Ricean K factor: estimation and performance analysis," *IEEE Trans. Wireless Commun.*, vol. 2, pp. 799–810, July 2003.
- [10] J. K. Cavers, *Mobile Channel Characteristics*. Boston, MA: Kluwer, 2000.