Recursive Clipping and Filtering With Bounded Distortion for PAPR Reduction

Shang-Kang Deng, *Student Member, IEEE*, and Mao-Chao Lin, *Member, IEEE*

***Abstract*—Repeated clipping and filtering (RCF) is a simple method for reducing the peak-to-average power ratio (PAPR) of the signal in the orthogonal frequency-division modulation (OFDM) system. We propose to modify RCF by limiting the distortion on each tone of the OFDM so that both low PAPR and low error can be achieved.**

***Index Terms*—Bounded distortion (BD), clipping and filtering, orthogonal frequency-division modulation (OFDM), peak-to-average power ratio (PAPR).**

I. Introduction

ORTHOGONAL frequency-division modulation (OFDM) is a popular multicarrier modulation technique in modern communication systems. However, a well-known disadvantage of OFDM is the occasional occurrence of high peak-to-average power ratio (PAPR) in the time-domain signal [1], [2]. The simplest PAPR reduction method is to employ clipping in the time domain. The resultant problem is the high out-of-band spectrum. If the out-of-band spectrum is filtered off, it is likely that the reduced PAPR of the clipped signal will regrow [3], [4]. By repeating clipping and filtering several times [5], both low PAPR and low out-of-band spectrum can be achieved. Such a method is called recursive (or repeated) clipping and filtering (RCF). By increasing the number of recursions in RCF, PAPR can be reduced. However, the associated error rate will also be increased, which is due to the increase of in-band distortion (clipping noise) [1], [5].

We modify RCF by bounding the distortion on each tone. The proposed scheme is called RCF with bounded distortion (RCFBD), which includes the technique of active constellation technique (ACE) [6] as a special case. RCFBD can achieve similar PAPR reduction and lower error rates as that of RCF.

The RCFBD itself converges slowly to low PAPR. The smart gradient projection (SGP) [6] algorithm for ACE can achieve convergence to low PAPR within three recursions. However, a direct application of SGP to RCFBD will destroy the constraint of bounded distortion (BD). Thus, we propose a fast RCFBD scheme that employs SGP appropriately so that low PAPR can be achieved within three recursions with the condition of BD intact.

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The authors are with the Graduate Institute of Communication Engineering, National Taiwan University, Taipei 106, Taiwan, R.O.C. (e-mail: d88942008@ntu.edu.tw; mclin@cc.ee.ntu.edu.tw).

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II. RCFBD

Consider an N-tone OFDM system. Let T be the interval of each OFDM symbol and X_k be the complex baseband data carried on the kth tone. The basic operation for the oversampled digital clipping and filtering (OCF) is described as follows.

OCF Algorithm

$\star$ == OCF$((X_0, \cdots, X_{N-1}), L, A(\hat{X}_0, \cdots, \hat{X}_{N-1}))$ ==

1) Complex baseband data $(X_0, X_1, \cdots, X_{N-1})$ are converted to oversampled time-domain signal $(s_L[0], s_L[N], \cdots, s_L[LN-1])$ by zero-padded LN-point inverse discrete Fourier transform (IDFT), where L is the oversampling factor [4], [7], [8], and

$$s_L[n] = s\left(n\frac{T}{LN}\right) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{jk2\pi \frac{n}{LN}}, \quad \text{for } 0 \le n \le LN-1. \tag{1}$$

2) Each $s_L(n)$ is digitally clipped by the soft limiter [2], [4], [7], with output

$$g(y) = \begin{cases} y, & \text{for } \rho \le A \\ Ae^{j\phi}, & \text{for } \rho > A \end{cases} \tag{2}$$

where $y = \rho e^{j\phi}$, $\rho = |y|$ is the input and A is the clipping threshold.

3) Then, $(g(s_L[0]), g(s_L[1]), \cdots, g(s_L[LN-1]))$ are converted to $(\hat{X}_0, \hat{X}_1, \cdots, \hat{X}_{N-1}, \cdots, \hat{X}_{LN-1})$ by using the LN-point DFT

$$\hat{X}_k = \frac{\sqrt{N}}{LN} \sum_{n=0}^{LN-1} g\left(s_L[n]\right) e^{-j2\pi \frac{nk}{LN}}, \quad \text{for } 0 \le n \le LN-1. \tag{3}$$

4) The filtering operation removes the out-of-band components and obtains $(\hat{X}_0, \cdots, \hat{X}_{N-1}, 0, \cdots, 0)$. The time-domain output of the OCF is

$$\hat{s}_L[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \hat{X}_k e^{jk2\pi \frac{n}{LN}}, \qquad \text{for } 0 \le n \le LN-1. \tag{4}$$

$\star$ ============================

The filtering operation will lead to peak power regrowth, i.e., $\max_{0 \le n < LN} |\hat{s}_L[n]|$ will be greater than $|A|$ again. By repeating OCF several times, we can reduce the likelihood of peak power regrowth. Such a procedure is called RCF [5]. More specifically, if the number of recursions is J, the scheme is denoted RCF-J. At the jth recursion, the operation

OCF$((\hat{X}_0^{(j-1)}, \cdots, \hat{X}_{N-1}^{(j-1)}), L, \mathrm{A}, (\hat{X}_0^{(j)}, \cdots, \hat{X}_{N-1}^{(j)}))$ applies the clipping and filtering operations on the frequency-domain signal $(\hat{X}_0^{(j-1)}, \cdots, \hat{X}_{N-1}^{(j-1)})$ to obtain the output which is the frequency-domain signal $(\hat{X}_0^{(j)}, \cdots, \hat{X}_{N-1}^{(j)})$, where $(\hat{X}_0^{(0)}, \cdots, \hat{X}_{N-1}^{(0)}) = (X_0, \cdots, X_{N-1})$. The improvement of further PAPR reduction is usually very small for J beyond 3 in RCF.

Each operation of OCF will increase the distortion of the baseband data. That means $|\hat{X}_k^{(j)} - X_k|$ is likely to be significant, and hence, the error rate will be large. Note that some form of distortion will not increase the error rate. For example, in the binary phase-shift keying (BPSK) constellation with signal points $\{-\sqrt{E_b}, \sqrt{E_b}\}$, if $X_k = \sqrt{E_b}$ and $\hat{X}_k^{(j)} > \sqrt{E_b}$, the likelihood of erroneous detection will not be increased (in case the increase of average power is not taken into account). To reduce the error rate, we would like to bound the distortion unless the distortion will not increase the likelihood of erroneous detection.

Consider the baseband data at the kth tone, for which the constellation Ω_k contains M_k signal points, $V_{k,1}, V_{k,2}, \cdots, V_{k,M_k}$, $k = 0, 1, \cdots, N-1$. For simplicity, we describe here only the condition that $\Omega_k = \Omega$, $M_k = M$, and $V_{k,i} = V_i$ for all k. Suppose the Ω is the square quadrature amplitude modulation (QAM). Let $E = (\sum_{i=1}^{M} |V_i|^2/M)$, and let

$$\gamma = \frac{(\sqrt{M}-2)\sqrt{E}}{\sqrt{2(M-1)/3}} \tag{5}$$

which is the threshold to judge whether a signal point V_i is on the boundary of Ω or not. For the 4-QAM constellation, $\gamma = 0$. For the 16-QAM constellation, $\gamma = (2\sqrt{E}/\sqrt{10})$. For the 64-QAM constellation, $\gamma = (6\sqrt{E}/\sqrt{42})$. Let $X = (a, b)$ be the original data signal point in Ω. Let $\hat{X} = (\hat{a}, \hat{b})$ be a distorted version of X. Let δ be the bound to constrain the distortion at either the real part or the imaginary part. We use the BD algorithm given in the following to move $\hat{X}$ to a point $\tilde{X} = (\tilde{a}, \tilde{b})$ such that the distortion between $\tilde{X}$ and X is bounded unless the distortion will not increase the likelihood of erroneous detection.

BD Algorithm

$\star$ ======== BD$(X; \hat{X}, \delta, \gamma, \tilde{X})$ =========

1) Compute $\Delta x = \hat{a} - a, \Delta y = \hat{b} - b$.
2) Set $\tilde{a} = \hat{a}$ for any of the following conditions:
   1) $|\Delta x| \leq \delta$; 2) $a > \gamma$ and $\Delta x > 0$; 3) $a < -\gamma$ and $\Delta x < 0$.
   Otherwise, set $\tilde{a} = a + \mathrm{sign}(\Delta x)\delta$.
3) Set $\tilde{b} = \hat{b}$ for any of the following conditions:
   1) $|\Delta y| \leq \delta$; 2) $b > \gamma$ and $\Delta y > 0$; 3) $b < -\gamma$ and $\Delta y < 0$.
   Otherwise, set $\tilde{b} = b + \mathrm{sign}(\Delta y)\delta$.

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The regions of all the possible $\tilde{X}$ points obtained from the BD algorithm for the 16-QAM constellation are shown in Fig. 1.

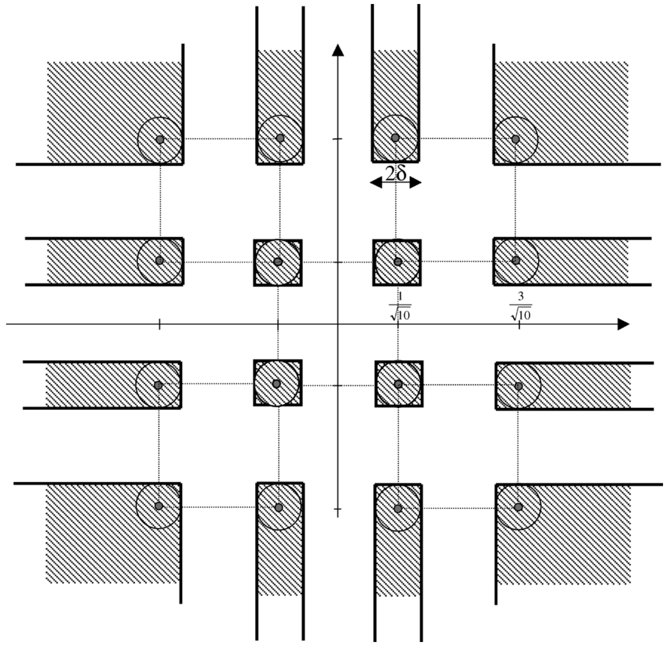

Fig. 1. Regions of BD control for 16-QAM.

If we repeat the combined operation of OCF and BD several times, we have a scheme called RCFBD. If the number of recursions in RCFBD is J, then it is denoted RCFBD-J. For the jth recursion $1 \leq j \leq J$, the operation includes OCF$((X_0^{(j)}, \cdots, X_{N-1}^{(j)}), L, A, (\hat{X}_0^{(j)}, \cdots, \hat{X}_{N-1}^{(j)}))$ and BD$(X_k; \hat{X}_k^{(j)}, \delta, \gamma, \tilde{X}_k^{(j)})$ for $k = 0, 1, \cdots, N-1$, where $(X_0^{(j)}, \cdots, X_{N-1}^{(j)}) = (\tilde{X}_0^{(j-1)}, \cdots, \tilde{X}_{N-1}^{(j-1)})$ and $(\tilde{X}_0^{(0)}, \cdots, \tilde{X}_{N-1}^{(0)}) = (X_0, \cdots, X_{N-1})$. Although BD control can limit the distortion, it also limits the capability of PAPR reduction. Usually, RCFBD-8 will have PAPR reduction similar to RCF-3. RCF is a special case of RCFBD with $\delta = \infty$. On the other hand, the active constellation extension (ACE) proposed in [6] can be regarded as a special case of RCFBD with $\delta = 0$, which only employs the distortion that will not increase the erroneous detection to reduce PAPR.

For some applications, there is an advantage by varying the parameters A and δ in different recursions. Moreover, the signal constellation Ω and the parameter δ can vary for different tones. For example, we may incorporate the reserved tone technique [7] into RCFBD so that arbitrary distortion is allowed for each reserved tone and BD is allowed for each data tone.

III. Fast Convergence for RCFBD

The operation of RCFBD described in the last section has the problem of slow convergence in searching for the best PAPR reduction. In [6], the SGP algorithm is applied to ACE, called ACE-SGP, to speed up the convergence to low PAPR. The ACE-SGP algorithm estimates a proper step size μ which magnifies the clipped and bounded signal in the operation of OCF and BD (with $\delta = 0$) to accelerate the rate of convergence.

According to the idea of ACE-SGP, we may try RCFBD-SGP for which the operation in the jth recursion $1 \leq j \leq J$ includes

$\mathrm{OCF}((X_0^{(j-1)},\cdots,X_{N-1}^{(j-1)}),L,\mathrm{A},(\hat{X}_0^{(j)},\cdots,\hat{X}_{N-1}^{(j)}))$ and $\mathrm{BD}(X_k;\hat{X}_k^{(j)},\delta,\gamma,\tilde{X}_k^{(j)})$ for $k=0,1,\cdots,N-1$ and

$$X_k^{(j)} = X_k^{(j-1)} + \mu_j\left(\tilde{X}_k^{(j)} - X_k^{(j-1)}\right), \quad \text{for } 0\leq k\leq N-1 \tag{6}$$

where $((X_0^{(0)},\cdots,X_{N-1}^{(0)}) = (X_0,\cdots,X_{N-1})$. The step size μ is computed according to [6, eq. (22)], and is not described here due to space limitation. For ACE-SGP, equivalently, RCFBD-SGP with $\delta = 0$, the output at each iteration $X_k^{(j)}$ always falls in the desired regions, i.e., follows the constraint of distortion bound. As to RCFBD-SGP with $\delta > 0$, $X_k^{(j)}$ may not follow the constraint of distortion bound for two reasons. The first is that the step size μ_j may magnify the distortion. The second is that the distortion in $X_k^{(j-1)}$ will be added to the distortion in $\mu_j(\tilde{X}_k^{(j)} - X_k^{(j-1)})$. In order that the final output $X_k^{(J)}$ be bounded as desired, we need to vary δ in the BD operation in each iteration, i.e., replacing δ by δ_j and avoiding the SGP operation in the last iteration. We find that by using $\delta_j = 0$ for $1 \leq j \leq J-1$ and $\delta_J = \delta > 0$ is a simple and effective way of constructing RCFBD-SGP, which is denoted ACESGP-(J-1)+OCFBD.

IV. Performance Evaluation

We consider two clipping operations. The first, called preclip, is used in the OCF operations of RCFBD. The second, called PA-clip (power amplifier clip), is used to simulate the nonlinearity of the power amplifier, which is also modelled as a soft limiter. There are two corresponding oversampling factors, i.e., L and L_a. We use $L = 2$ in the preclip process and $L_a = 8$ to approximate the analog behavior of the signal and the power amplifier [6], [7].

The performance, including the complementary cumulative distribution function (CCDF) of PAPR, out-of-band power spectral density (PSD) and bit-error rate (BER) for RCF, and ACESGP-2+OCFBD is verified by simulation. We use constellation of unit symbol energy, i.e., $E_k = 1$ for each tone. The long-term average power of the OFDM signals after preclip is denoted P_{av}, which is assumed to be 1 in case there is no PAPR reduction. The value of P_{av} after PAPR reduction will vary for different schemes. ACESGP-3, which is similar to ACESGP-2+OCFBD with $\delta = 0$, will have P_{av} greater than 1, since the extension of boundary signal points in the operation of ACE will increase the symbol energy. By increasing δ above zero, P_{av} of ACESGP-2+OCFBD will gradually decline. For RCF-J, which can be considered as RCFBD-J with infinitely large δ, will have P_{av} smaller than 1, while larger J will yield smaller P_{av}. The average power at the output of power amplifier is denoted P_{AV}. The phenomenon of constellation shrinkage pointed out in [2], [9] also occurs to RCF. Constellation shrinkage comes from both the clipping of RCF and PA-clip, and can be estimated by the average output power. By considering constellation shrinkage at the receiver, the BER for RCF can be reduced as compared with that without considering constellation shrinkage. However, the constellation shrinkage model is not suitable for RCFBD, because BD breaks the isotropic shrinking phenomenon in the constellation. Hence, we do not consider constellation shrinkage for RCFBD in our simulation.

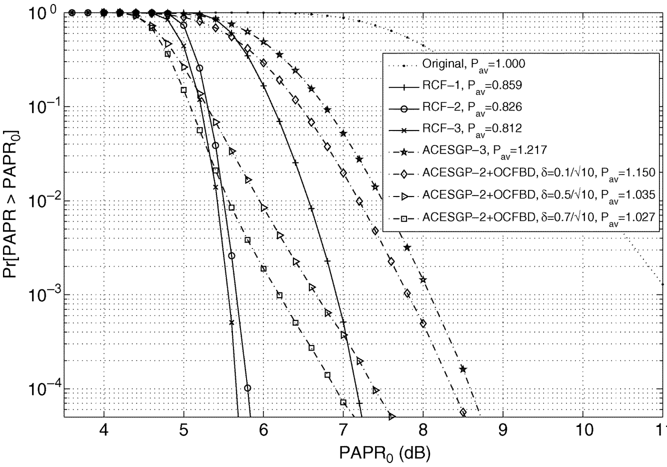

Fig. 2. CCDF of PAPR for 16-QAM/128-tone OFDM system.

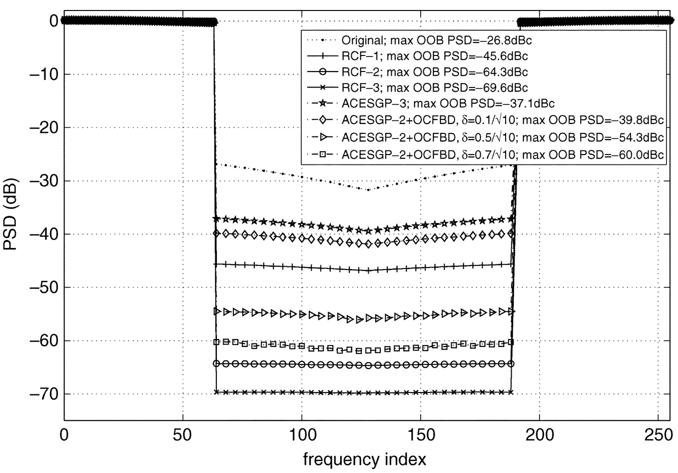

Fig. 3. PSD under 5-dB IBO for 16-QAM/128-tone OFDM system.

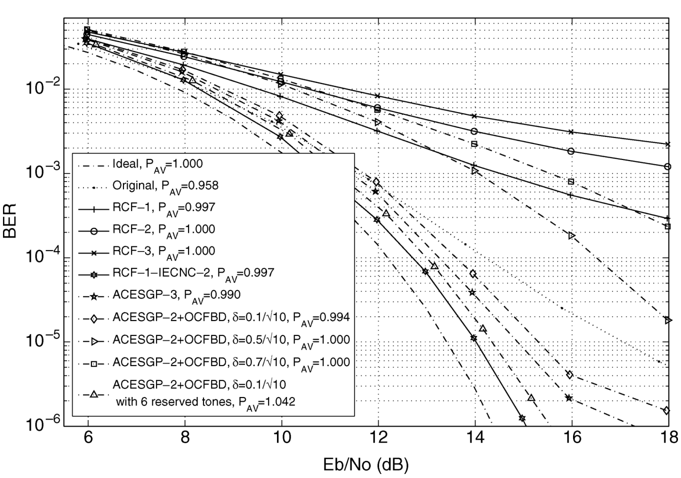

Fig. 4. BER under 5-dB IBO for 16-QAM/128-tone OFDM system.

The simulation results of CCDF of PAPR, out-of-band PSD, and BER are shown in Figs. 2–4, respectively. In the figures, the curve denoted "Original" represents the OFDM system which does not use any PAPR reduction technique but will encounter

PA-clip distortion; the curve denoted "Ideal" stands for the OFDM with ideal power amplifier so that there is no PAPR reduction and no PA-clip distortion. The signal constellation is 16-QAM. The preclip is set to be 3 dB ($A = 1.413$). The PA clip is based on 5-dB input power backoff (IBO). From Figs. 2 and 3, we observe that the PAPR reduction capability of RCF-2 is close to that of RCF-3, and is much better than that of RCF-1. We also see that a larger distortion bound results in better PAPR reduction and lower out-of-band PSD. From Fig. 4, the BER curves show that a smaller δ will provides better BER performance. This complies with the expectation that the smaller distortion bound will yield better BER performance. Simulations for OFDM based on quaternary (Q)PSK and 64-QAM have also been implemented, which are not shown here. The trend of performance variation with the distortion bound δ is similar to that of 16-QAM-OFDM.

In [10], the error rate of RCF is reduced by iteratively estimating and cancelling the clipped noise (IECNC) at the receiver. To reduce both PAPR and error rate, RCFBD (including ACE) works on the transmitter side by limiting the distortion of the transmitted signal, while IECNC works on the receiver side by iteratively repeating the operations the same as those conducted in the transmitter, and comparing the results with the detected data to estimate the clipped noise (distortion), which are then deducted from the received signal for improved detection. The BER curve of RCF-1 using IECNC with two iterations, denoted RCF-1-IECNC-2, is given in Fig. 4, from which we see that RCF with IECNC provides better error rates than RCFBD. As a rough comparison, RCFBD-SGP and RCF with IECNC have similar overall (transmitter plus receiver) complexity. However, most complexity of RCFBD lies in the transmitter side and most complexity of RCF with IECNC lies in the receiver side. Hence, in the wireless communications, the transmitter using RCFBD and the receiver using IECNC are suitable to be implemented in the base station, and the transmitter using RCF and the RCFBD receiver (i.e, the receiver in the original form) are suitable to be implemented in the mobile station. RCF with IECNC can achieve better PAPR reduction and error rates as compared with RCFBD. A drawback of RCF with IECNC is that the receiver needs the knowledge of threshold of the preclip and the threshold of the power amplifier at the transmitter side. If the power amplifier can not be modeled by a soft limiter, then the situation will be more complicated. RCFBD has an additional advantage. As indicated in Section II, in case there are reserved tones which are not used for data transmission, RCFBD can use them without additional complexity. Suppose that there are six reserved tones numbered 61, 62, ..., 66 out of the 128 tones numbered 0 (dc tone), 1, 2, ..., 126, 127. Through the help of six reserved tones, the maximum out-of-band PSD for ACESGP-2+OCFBD with δ equal to $0.1/\sqrt{10}$, $0.5/\sqrt{10}$, and $0.7/\sqrt{10}$, respectively, are -42.3, -62.3, and -69.0 dB, respectively, which are close to those obtained by RCF-1, RCF-2, and RCF-3, respectively. The BER curve of ACESGP-2+OCFBD plus six reserved tones with δ equal to $0.1/\sqrt{10}$ is also given in Fig. 4, which is improved as compared with the case of no reserved tones. In case there are more reserved tones, better error rates and PAPR reduction can be achieved for RCFBD, but not for RCF with IECNC.

V. CONCLUSION

We have proposed a modified RCF by bounding the distortion after each recursion of clipping and filtering to simultaneously reduce the PAPR and error rate of OFDM. Also, an efficient method that employs the SGP algorithm is proposed to reduce the number of recursions in RCFBD. A comparison of the proposed method with that of RCF using IECNC is given.

Note: After the initial submission of our work, we find that in [11], a generalization of ACESGP with constrained region as shown in Fig. 1 is considered. However, the SGP operation in [11] causes the aggregation of distortion. Henceforth, the approach in [11] does not guarantee the resulting distortion to be bounded.

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Shang-Kang Deng (S'04) was born in Chiayi, Taiwan, R.O.C., in 1968. He received the B.S. and M.S. degrees in electrical engineering in 1990 and 1992, respectively, from National Taiwan University (NTU), Taipei, Taiwan, R.O.C., where he is currently working toward the Ph.D. degree in the Graduate Institute of Communication Engineering.

His research interests include coding theory and orthogonal frequency-division modulation.

Mao-Chao Lin (S'86–M'86) was born in Taipei, Taiwan, R.O.C., on December 24, 1954. He received the Bachelor's and Master's degrees, both in electrical engineering, from National Taiwan University (NTU), Taipei, Taiwan, R.O.C., in 1977 and 1979, respectively, and the Ph.D. degree in electrical engineering from University of Hawaii at Manoa, in 1986.

From 1979 to 1982, he was an Assistant Scientist with the Chung-Shan Institute of Science and Technology, Lung-Tan, Taiwan, R.O.C. He is currently a Professor in the Department of Electrical Engineering, NTU. His research interests are in the area of coding theory and its applications.