Unitary Precoders for ST-OFDM Systems Using Alamouti STBC

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***Abstract*—This paper studies unitary precoders for space-time coded orthogonal frequency division multiplexing (ST-OFDM) systems using Alamouti space-time block codes (STBC). We consider two channel models: 1) the LTI case where the channels are invariant for two consecutive OFDM blocks; 2) the time-varying case where the channels are invariant within one OFDM block but they vary slightly between two OFDM blocks. For the LTI case, we derive the optimal precoder for QPSK modulation. It is shown that the optimal precoder is channel independent and can greatly enhance the performance of the ST-OFDM system. For the time-varying case, the channel variation will cause inter-carrier interference (ICI) in the ST-OFDM system and can result in serious performance degradation. We will show that by properly designing the precoder, we can average the output error variance and hence greatly alleviate the ICI effect. As a result, the bit error rate (BER) performance can be significantly improved.**

***Index Terms*—Space-time block coding, equalization, precoding, OFDM.**

I. Introduction

TRANSMIT diversity has been shown to be an effective method for combating channel fading. Many diversity schemes have been proposed in the past [1]. In particular, the space-time (ST) coding technique based on the orthogonal codes has drawn a lot of attention. It was first proposed by Alamouti [2] for the two antenna case and was generalized by Tarokh *et al.*, [3] to the M antenna case. The advantages of orthogonal codes are that they do not require channel knowledge at the transmitter, and simple linear processing can be employed at the receiver for the symbol recovery. These orthogonal designs guarantee the inter-symbol interference (ISI) free condition when the communication channel is flat fading. For frequency selective channels, orthogonal frequency division multiplexing (OFDM) was combined with the orthogonal code technique to overcome the channel induced ISI [4]. In [4]–[6], the orthogonal code technique was extended to the space and time domains and the result is a space-time coded OFDM (ST-OFDM) system. It has been demonstrated that the ST-OFDM system has a much better performance than the conventional OFDM system.

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It is also known that the performance of a transceiver system can be improved if the transmitter is equipped with a properly designed precoder. The design of the precoders has been studied by many researchers [8]–[17]. In [8], [9], it was shown that a Vandermonde precoding matrix can guarantee multiuser interference elimination as well as symbol recovery. In [10], [11], assuming that the channel information is available at the transmitter, an optimal precoder for multi-input multi-output (MIMO) transmission was derived. Furthermore, the precoder without channel state information was also considered in [12] and [13]. The proposed polynomial ambiguity precoders were employed for blind equalization for MIMO channels. The design of linear precoder for OFDM system with single antenna has been studied by a number of researchers [16]–[20]. Optimal precoders that minimize the bit error rate (BER) or pairwise error probability are derived. It was shown in [16] that the $M \times M$ optimal precoder $\mathbf{T}$ is channel independent and its (j,k)th entry satisfies $|t_{j,k}| = (1/\sqrt{M})$ for all j,k. The DFT matrix and Hadamard matrix are two examples of such matrices. The results in [16] were extended to the case of unitary precoder with memory in [17].

ST-OFDM systems with Hadamard precoders have been studied in [21], [22]. It was shown that the system performance can be improved by adding such precoders at the transmitter. On the other hand, unitarty precoders, such as Hadamard and DFT matrices, have also been studied in [23], [24] for space-frequency coded OFDM (SF-OFDM) system. The space-time block coding technique has also been applied to single-carrier systems with multiple antennas [25]–[27]. Space-time coded cyclic prefixed single carrier (ST-SCCP) systems and zero padded single carrier (ST-SCZP) systems were studied in detail in [25] and [26] respectively. Though it has been numerically demonstrated that these ST coded single carrier systems have a superior performance over ST-OFDM system, their optimality in the BER sense has not been established before.

In this paper, we study the unitarily precoded ST-OFDM (UP-ST-OFDM) systems using Alamouti STBC scheme. Two channel models are considered: (i) the LTI case where the channels are invariant for two consecutive OFDM blocks and (ii) the time-varying case where the channels are invariant within one OFDM block but they vary slightly between two OFDM blocks. We will derive the optimal precoders that minimize the BER for the LTI case. The optimal precoders are channel independent. The results can be viewed as an extension of the results in [16] to the multiple antenna case. We also study the performance of UP-ST-OFDM systems for the time-varying case. This study shows that by properly designing the precoder, the effect of ICI can be greatly mitigated and thus the system performance can be greatly improved.

This paper is organized as follows. In Section II, UP-ST-OFDM systems for the LTI case are studied, and the optimal precoders that minimize the BER are derived. In Section III, the channel model of the time-varying case is first described, and then the performance of UP-ST-OFDM system will be analyzed. Simulation results are given in Section IV. Conclusions are given in Section V. Parts of the results in this paper have been presented in conferences [29], [30].

Notation: Boldfaced letters are used to denote vectors and matrices. The symbols $\mathbf{A}^\dagger$ and $\mathbf{A}^*$ denote the Hermitian and conjugate of $\mathbf{A}$ respectively. $\mathrm{diag}[\mathbf{a}]$ is a diagonal matrix whose diagonal entries are the elements of the vector $\mathbf{a}$. $\mathbf{W}$ is the $M \times M$ DFT matrix with its entries $[\mathbf{W}]_{l,m} = e^{-j2\pi(lm/M)}$. The normalized DFT matrix is defined as $(1/\sqrt{M})\mathbf{W}$. The unit impulse function is denoted by $\delta(k)$ which is equal to one when $k=0$ and zero otherwise. The expectation of a random variable X is denoted by $\mathcal{E}\{X\}$.

II. Precoded ST-OFDM Systems in the LTI Case

The unitarily precoded ST-OFDM (UP-ST-OFDM) system studied in this paper is shown in Fig. 1. The channel impulse response from the ith transmit antenna to the receive antenna is denoted by $h_i(l)$, for $l = 0, \ldots, L$, and $i = 1, 2$. In this section, we assume that the channels are invariant during the transmission of two consecutive OFDM symbols. The transmitted signal vector consisting of modulation symbols is $\mathbf{s}_i = [s_i(0)\ s_i(1)\ \cdots\ s_i(M-1)]^T$, for $i = 1, 2$. The input vectors $\mathbf{s}_i$ are assumed to be zero-mean and have the same covariance matrix:

$$\mathbf{R}_s = \mathcal{E}\{\mathbf{s}_i\mathbf{s}_i^\dagger\} = E_s\mathbf{I}. \tag{1}$$

We assume that the channel noise is AWGN and the modulation symbols and channel noise are uncorrelated. The precoding matrix $\mathbf{T}$ is an $M \times M$ unitary matrix. The vector at the output of the precoding matrix is given by

$$\mathbf{x}_i = \mathbf{T}\mathbf{s}_i, \quad i = 1, 2. \tag{2}$$

In this paper, we assume that the Alamouti code [2] is used to encode the vectors $\mathbf{x}_1$ and $\mathbf{x}_2$. Given two $M \times 1$ vectors $\mathbf{x}_1$ and $\mathbf{x}_2$, the space-time encoder produces two $2M \times 1$ output blocks

$$\text{Block } 1 = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix}, \quad \text{Block } 2 = \begin{bmatrix} -\mathbf{x}_2^* \\ \mathbf{x}_1^* \end{bmatrix}. \tag{3}$$

As shown in Fig. 1, we first take the M-point IDFT of the vectors, $\mathbf{x}_i$ and their conjugates. Then a cyclic prefix (CP) of length L is appended before the transmitter sends the samples through the channels. At the receiver, the CP samples are discarded to remove the inter-block interference and the $M \times M$ DFT operations are applied to the received data. It is known [4] that when the CP length is longer than or equal to the channel order, the two consecutive DFT output vectors are respectively given by

$$\mathbf{r}_1 = \mathbf{\Lambda}_1\mathbf{x}_1 + \mathbf{\Lambda}_2\mathbf{x}_2 + \mathbf{q}_1,$$
$$\mathbf{r}_2 = -\mathbf{\Lambda}_1\mathbf{x}_2^* + \mathbf{\Lambda}_2\mathbf{x}_1^* + \mathbf{q}_2, \tag{4}$$

where the noise vectors $\mathbf{q}_1$ and $\mathbf{q}_2$ are uncorrelated zero-mean circularly-symmetric complex Gaussian random vectors with

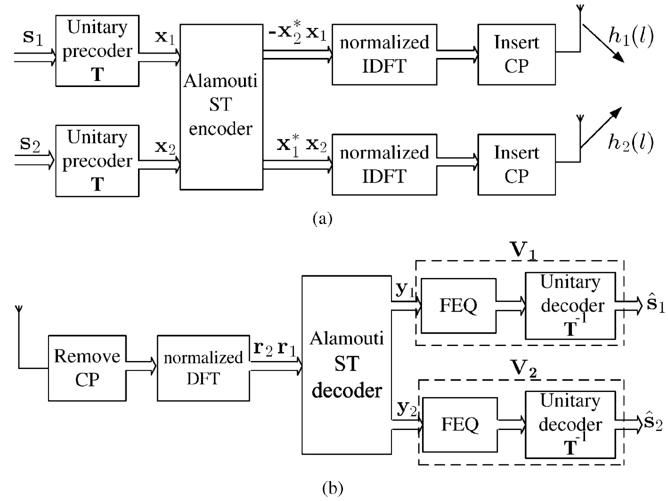

Fig. 1. Unitarily precoded ST-OFDM systems. (a) Transmitter. (b) Receiver.

the covariance matrix $\mathbf{R}_{q_i} = N_0\mathbf{I}$. The matrix $\mathbf{\Lambda}_i$ is the diagonal matrix

$$\mathbf{\Lambda}_i = \mathrm{diag}[H_i(0) \quad H_i(1) \quad \cdots \quad H_i(M-1)] \tag{5}$$

where $H_i(k) = \sum_{l=0}^{L} h_i(l)e^{-j2\pi kl/M}$ are the kth subchannel gains of the ith transmit antenna. Given $\mathbf{r}_1$ and $\mathbf{r}_2$, the space-time decoder produces two vectors $\mathbf{y}_1$ and $\mathbf{y}_2$ as

$$\mathbf{y}_1 = \mathbf{\Lambda}_1^*\mathbf{r}_1 + \mathbf{\Lambda}_2\mathbf{r}_2^*,$$
$$\mathbf{y}_2 = \mathbf{\Lambda}_2^*\mathbf{r}_1 - \mathbf{\Lambda}_1\mathbf{r}_2^*. \tag{6}$$

Define the diagonal matrix

$$\mathbf{D} = \mathrm{diag}[d_0 \quad d_1 \quad \cdots \quad d_{M-1}] = \mathbf{\Lambda}_1^*\mathbf{\Lambda}_1 + \mathbf{\Lambda}_2^*\mathbf{\Lambda}_2 \tag{7}$$

where $d_k = |H_1(k)|^2 + |H_2(k)|^2$. Then from (4) and (7), we obtain

$$\mathbf{y}_i = \mathbf{D}\mathbf{x}_i + \boldsymbol{\nu}_i = \mathbf{D}\mathbf{T}\mathbf{s}_i + \boldsymbol{\nu}_i \tag{8}$$

where $\boldsymbol{\nu}_i$ is the noise vector. It can be shown that $\boldsymbol{\nu}_1$ and $\boldsymbol{\nu}_2$ are zero-mean complex Gaussian random vectors and their covariance matrices are

$$\mathbf{R}_{\nu_i} = N_0\mathbf{D}. \tag{9}$$

From (8), we see that the system from $\mathbf{x}_i$ to $\mathbf{y}_i$ can be viewed as a set of parallel subchannels with kth subchannel gain given by

$$d_j = |H_1(j)|^2 + |H_2(j)|^2. \tag{10}$$

Given $\mathbf{y}_i$, we can employ either a zero-forcing (ZF) or an minimum mean squared error (MMSE) receiver for the recovery of the transmitted vectors $\mathbf{s}_i$.

A. ZF Receivers

As both the diagonal matrix $\mathbf{D}$ and the unitary matrix $\mathbf{T}$ are $M \times M$, from (8) it is clear that the ZF receiver is given by

$$\hat{\mathbf{s}}_i = \mathbf{T}^\dagger\mathbf{D}^{-1}\mathbf{y}_i = \mathbf{s}_i + \mathbf{T}^\dagger\mathbf{D}^{-1}\boldsymbol{\nu}_i. \tag{11}$$

In this case, the frequency domain equalizer (FEQ) in Fig. 1 is the diagonal matrix $\mathbf{D}^{-1}$. Define the output error of the kth subchannel as $e_i(k) = \hat{s}_i(k) - s_i(k)$, for $i = 1,2$ and $k = 0,1,\ldots,M-1$. The variance of $e_i(k)$ can be obtained from (11) and it is given by

$$\sigma_{i,T}^2(k) = \sum_{j=0}^{M-1} N_0 |t_{j,k}|^2 d_j^{-1}. \quad (12)$$

Notice that these noise variances are independent of i. The two input signals $s_1(k)$ and $s_2(k)$ are corrupted by noises with the same variance. Therefore, in the following discussions we will drop the index i. The SNR of the kth subchannel is given by

$$\eta_T(k) = \frac{\gamma}{\sum_{j=0}^{M-1} |t_{j,k}|^2 d_j^{-1}} \quad (13)$$

where $\gamma = E_s/N_0$ and d_j is the jth diagonal entry of $\mathbf{D}$. Moreover, the total output noise variance of the UP-ST-OFDM system is

$$\sigma_{\text{total}}^2 = \sum_{k=0}^{M-1} \sigma_{i,T}^2(k) = N_0 \sum_{j=0}^{M-1} d_j^{-1} \quad (14)$$

where we have used the fact that $\sum_{k=0}^{M-1} |t_{j,k}|^2 = 1$ to simplify the expression.

When the precoding matrix $\mathbf{T}$ is chosen to be the identity matrix $\mathbf{I}$, the system in Fig. 1 reduces to the conventional ST-OFDM system [4], and the noise variance and the SNR of the kth subchannel are respectively given by

$$\sigma_{\text{ofdm}}^2(k) = N_0 d_k^{-1} \quad (15)$$
$$\eta_{\text{ofdm}}(k) = \gamma d_k. \quad (16)$$

From (14) and (15), it is clear that the UP-ST-OFDM system has the same total output noise variance as the conventional ST-OFDM system. On the other hand, when the precoding matrix $\mathbf{T}$ is chosen as the normalized DFT matrix $(1/\sqrt{M})\mathbf{W}$, the system becomes an ST-SCCP system [25]. Substituting the fact that $|t_{l,m}|^2 = |(1/\sqrt{M})e^{-j2\pi(lm)/(M)}|^2 = (1/M)$ into (12) and (13), the noise variance and SNR are respectively given by

$$\sigma_{\text{sccp}}^2(k) = \frac{N_0}{M} \sum_{j=0}^{M-1} d_j^{-1} = \frac{1}{M}\sigma_{\text{total}}^2 \quad (17)$$
$$\eta_{\text{sccp}}(k) = \frac{\gamma}{\frac{1}{M}\sum_{j=0}^{M-1} d_j^{-1}} \equiv \eta_{\text{sccp}}. \quad (18)$$

Note that both the output noise variances and SNRs of the ST-SCCP system are independent of the frequency index k. All the subchannels have the same noise variances and they are equal to the average of those in the ST-OFDM system. Hence, all the subchannels of the ST-SCCP system have the same SNR η_{sccp}. From the above discussions, we see that the UP-ST-OFDM system, ST-OFDM system and ST-SCCP system have the same total output noise variance. The unitary precoder $\mathbf{T}$ does not change the total output noise variance, and it simply redistributes the subchannel variances. When $\mathbf{T} = (1/\sqrt{M})\mathbf{W}$, it averages all the subchannel variances.

One can obtain from (13) that the SNRs of the UP-ST-OFDM system satisfy

$$\gamma \cdot d_{\min} \le \eta_T(k) \le \gamma \cdot d_{\max} \quad (19)$$

where $d_{\min} = \min_j d_j$ and $d_{\max} = \max_j d_j$ are respectively the smallest and largest subchannel gains in (10). Note that $\gamma d_{\min}$ and $\gamma d_{\max}$ are respectively the SNRs of the worst and the best subchannels in the ST-OFDM system. For any unitary precoders, the subchannel SNRs of the precoded ST-OFDM system are lower bounded and upper bounded by the worst and best subchannels of the ST-OFDM system respectively:

$$\min_j \eta_{\text{ofdm}}(j) \le \eta_T(k) \le \max_j \eta_{\text{ofdm}}(j). \quad (20)$$

In the following, we will study the BER performances of the ST-OFDM system, the ST-SCCP system and the UP-ST-OFDM system.

BER Analysis: Assume that the input vectors $\mathbf{s}_1$ and $\mathbf{s}_2$ consist of QPSK symbols. Then BER of the UP-ST-OFDM system is given by

$$\text{BER}_T = \frac{1}{M}\sum_{k=0}^{M-1} Q(\sqrt{\eta_T(k)}) \quad (21)$$

where the Q function is defined as $Q(x) = (1/\sqrt{2\pi})\int_x^\infty e^{-t^2} dt$. For convenience of discussion, we define

$$f(x) = Q\left(\frac{1}{\sqrt{x}}\right), \text{for } x \ge 0. \quad (22)$$

Using $f(x)$, the BER of the ST-OFDM, UP-ST-OFDM and ST-SCCP systems are respectively given by

$$\begin{aligned}\text{BER}_{\text{ofdm}} &= \frac{1}{M}\sum_{k=0}^{M-1} f\left(\eta_{\text{ofdm}}^{-1}(k)\right)\\ &= \frac{1}{M}\sum_{k=0}^{M-1} f\left(\frac{1}{\gamma d_k}\right) \quad (23)\\ \text{BER}_T &= \frac{1}{M}\sum_{k=0}^{M-1} f\left(\eta_T^{-1}(k)\right)\\ &= \frac{1}{M}\sum_{k=0}^{M-1} f\left(\frac{1}{\gamma}\sum_{j=0}^{M-1}|t_{j,k}|^2 d_j^{-1}\right) \quad (24)\\ \text{BER}_{\text{sccp}} &= f\left(\eta_{\text{sccp}}^{-1}(k)\right) = f\left(\frac{1}{M\gamma}\sum_{j=0}^{M-1} d_j^{-1}\right). \quad (25)\end{aligned}$$

It is shown in [16] that $f(x)$ is a monotonic increasing function and it is convex when $0 \le x \le 1/3$ and concave when $x \ge 1/3$. That is, the BER of the kth subchannel of ST-OFDM system is in the convex region if $\eta_{\text{ofdm}}(k) \ge 3$ and it is in the concave region if $\eta_{\text{ofdm}}(k) \le 3$. Let us define two SNR regions

$$\mathcal{R}_{\text{low}} = \left\{\gamma \,|\, \gamma \le 3d_{\max}^{-1}\right\}, \mathcal{R}_{\text{high}} = \left\{\gamma \,|\, \gamma \ge 3d_{\min}^{-1}\right\}. \quad (26)$$

When $\gamma \in \mathcal{R}_{\text{low}}$, all the subchannels of the ST-OFDM system have SNRs of γd_k, which are ≤ 3; all the subchannels are in the concave region. From (20), we can conclude that all the subchannels of the precoded system will fall in the concave region when $\gamma \in \mathcal{R}_{\text{low}}$. On the other hand, when $\gamma \in \mathcal{R}_{\text{high}}$, all the subchannels of ST-OFDM system are in the convex region, so are those of the precoded system. By exploiting the convexity and concavity of $f(x)$, we are able to establish the following theorem.

Theorem 1: Consider the UP-ST-OFDM system in Fig. 1. Suppose that the modulation symbols are QPSK and the zero-forcing receiver in (11) is employed. Then for any unitary precoder $\mathbf{T}$, we have

$$\mathrm{BER}_{\mathrm{ofdm}} \leq \mathrm{BER}_T \leq \mathrm{BER}_{\mathrm{sccp}}, \quad \text{for } \gamma \in \mathcal{R}_{\mathrm{low}} \quad (27)$$

$$\mathrm{BER}_{\mathrm{ofdm}} \geq \mathrm{BER}_T \geq \mathrm{BER}_{\mathrm{sccp}}, \quad \text{for } \gamma \in \mathcal{R}_{\mathrm{high}}. \quad (28)$$

Furthermore, $\mathrm{BER}_T = \mathrm{BER}_{\mathrm{sccp}}$ if and only if the unitary precoder $\mathbf{T}$ satisfies

$$|t_{i,j}| = \frac{1}{\sqrt{M}}, \quad \text{for all } i, j,$$

and $\mathrm{BER}_T = \mathrm{BER}_{\mathrm{ofdm}}$ if and only if $\mathbf{T} = \mathbf{I}$.

Proof: When $\gamma \in \mathcal{R}_{\mathrm{low}}$, all the subchannels of ST-OFDM system fall into the concave region. So are all the subchannels of the unitarily precoded system. By using the concavity of $f(x)$, the BER of unitarily precoded system in (24) satisfies the following inequality

$$\frac{1}{M}\sum_{k=0}^{M-1}\sum_{j=0}^{M-1}|t_{j,k}|^2 f\left(\frac{d_j^{-1}}{\gamma}\right) \leq \mathrm{BER}_T \leq f\left(\frac{1}{M\gamma}\sum_{k=0}^{M-1}\sum_{j=0}^{M-1}|t_{j,k}|^2 d_j^{-1}\right). \quad (29)$$

Using the fact that $\sum_{k=0}^{M-1}|t_{j,k}|^2 = 1$, we can simplify the above equation as

$$\frac{1}{M}\sum_{j=0}^{M-1} f\left(\frac{d_j^{-1}}{\gamma}\right) \leq \mathrm{BER}_T \leq f\left(\frac{1}{M}\sum_{j=0}^{M-1}\frac{d_j^{-1}}{\gamma}\right). \quad (30)$$

From (23) and (25), we see that the lower and upper bounds of BER_T are $\mathrm{BER}_{\mathrm{ofdm}}$ and $\mathrm{BER}_{\mathrm{sccp}}$ respectively. This proves the inequalities in (27). Moreover the lower bound is achieved if and only if the unitary precoder $\mathbf{T}$ is chosen as the identity matrix $\mathbf{I}$. And the upper bound is achieved if and only if the unitary precoder $\mathbf{T}$ satisfies $|t_{i,j}| = (1/\sqrt{M})$.

Using the convexity of $f(x)$, one can apply a similar procedure to prove that when $\gamma \in \mathcal{R}_{\mathrm{high}}$, the inequalities in (28) hold. ■

The above theorem can be viewed as a generalization of the result in [16] to the multiple antenna case. From Theorem 1, we see that the design of $\mathbf{T}$ depends on the SNR regions when a zero-forcing receiver is employed. In the low SNR region $\mathcal{R}_{\mathrm{low}}$, the optimal precoder is $\mathbf{T} = \mathbf{I}$ and the system becomes the conventional ST-OFDM system. In other words, the conventional ST-OFDM system will outperform any UP-ST-OFDM system in $\mathcal{R}_{\mathrm{low}}$. In the high SNR region $\mathcal{R}_{\mathrm{high}}$, the ST-SCCP system has the best BER performance. To achieve this optimal BER, the precoder can be chosen as any matrix satisfying $|t_{i,j}| = (1/\sqrt{M})$. Two well-known examples are the normalized DFT matrix $(1/\sqrt{M})\mathbf{W}$ and the normalized Hadamard matrix. Notice that both of these systems have low computational complexity. When $\mathbf{T} = (1/\sqrt{M})\mathbf{W}$, it can be combined with the normalized IDFT matrix at the transmitter (Fig. 1(a)) and the result is an ST-SCCP system. When $\mathbf{T}$ is the normalized Hadamard matrix, no multiplication is needed for its computation.

From (17), we see the output noise variance of the ST-SCCP system is proportional to the average of d_j^{-1}, where $d_j = |H_1(j)|^2 + |H_2(j)|^2$. When both the transmission channels have a small gain at the same frequency, the value of η_{sccp} will be small. This can happen when the two channels $h_1(l)$ and $h_2(l)$ are highly correlated. For uncorrelated channels, it is very unlikely that both $|H_1(j)|$ and $|H_2(j)|$ will be small for the same j. To cope with this channel null problem, one can employ an MMSE receiver.

B. MMSE Receivers

Consider the receiver in Fig. 1. Let the decoding matrix be $\mathbf{V}_i$ so the output $\hat{\mathbf{s}}_i = \mathbf{V}_i\mathbf{y}_i$. Define the error $\mathbf{e}_i = \hat{\mathbf{s}}_i - \mathbf{s}_i$. The MMSE decoding matrix is the matrix $\mathbf{V}_i$ that minimizes $\mathcal{E}\{\|\mathbf{e}_i\|^2\}$. Given $\mathbf{y}_i$, we want to derive the MMSE decoding matrix. From the orthogonality principle, we know that the best $\mathbf{V}_i$ is such that $\mathcal{E}\{\mathbf{e}_i\mathbf{y}_i^\dagger\} = 0$. By using the channel and signal models given at the beginning of Section II, one can show that the MMSE decoding matrix is given by

$$\mathbf{V}_i = \mathbf{T}^\dagger(\mathbf{D} + \gamma^{-1}\mathbf{I})^{-1}. \quad (31)$$

Like the ZF receivers, the MMSE receivers for the transmitted signal $\mathbf{s}_1$ and $\mathbf{s}_2$ are the same and we will drop the index i in the discussion below. Notice that $(\mathbf{D} + \gamma^{-1}\mathbf{I})^{-1}$ is a diagonal matrix with diagonal entries

$$[(\mathbf{D} + \gamma^{-1}\mathbf{I})^{-1}]_{kk} = \frac{1}{d_k + \gamma^{-1}}. \quad (32)$$

The MMSE receiver is also given by a cascade of a diagonal FEQ followed by the unitary matrix $\mathbf{T}^\dagger$. Thus its implementation complexity is the same as the ZF receiver. When the receiver is MMSE, the output vector is

$$\hat{\mathbf{s}}_i = \mathbf{T}^\dagger(\mathbf{D} + \gamma^{-1}\mathbf{I})^{-1}\mathbf{D}\mathbf{T}\mathbf{s}_i + \mathbf{T}^\dagger(\mathbf{D} + \gamma^{-1}\mathbf{I})^{-1}\boldsymbol{\nu}_i. \quad (33)$$

It is not difficult to verify that the product $\mathbf{T}^\dagger(\mathbf{D} + \gamma^{-1}\mathbf{I})^{-1}\mathbf{D}\mathbf{T} \neq \mathbf{I}$ and the system is not zero-forcing. Thus the output error $\hat{\mathbf{s}}_i - \mathbf{s}_i$ contains both inter-carrier interference (ICI) and channel noise. After some algebraic simplifications, one can compute the error variance $\sigma_T^2(k)$ and the signal-to-interference-noise ratio (SINR) $\rho_T(k)$ of the UP-ST-OFDM system as

$$\sigma_T^2(k) = \sum_{l=0}^{M-1}\frac{E_s}{\gamma d_l + 1}|t_{l,k}|^2 \quad (34)$$

$$\rho_T(k) = \frac{\sum_{l=0}^{M-1}\frac{d_l}{d_l+\gamma^{-1}}|t_{l,k}|^2}{\sum_{l=0}^{M-1}\frac{\gamma^{-1}}{d_l+\gamma^{-1}}|t_{l,k}|^2}. \quad (35)$$

By setting $\mathbf{T} = \mathbf{I}$ and $\mathbf{T} = (1/\sqrt{M})\mathbf{W}$, we can obtain the the error variances and the SINRs for the ST-OFDM system and ST-SCCP system respectively as

$$\sigma_{\mathrm{ofdm}}^2(k) = \frac{E_s}{\gamma d_k + 1}, \quad (36)$$

$$\rho_{\mathrm{ofdm}}(k) = \eta_{\mathrm{ofdm}}(k) = \gamma d_k \quad (37)$$

and

$$\sigma^2_{\rm sccp}(k) = \frac{1}{M}\sum_{l=0}^{M-1}\frac{E_s}{\gamma d_l + 1} = \frac{1}{M}\sum_{l=0}^{M-1}\sigma^2_{\rm ofdm}(l) \quad (38)$$

$$\rho_{\rm sccp}(k) = \frac{\sum_{l=0}^{M-1}\frac{d_l}{d_l+\gamma^{-1}}}{\sum_{l=0}^{M-1}\frac{\gamma^{-1}}{d_l+\gamma^{-1}}} \equiv \rho_{\rm sccp}. \quad (39)$$

The SINR $\rho_{\rm ofdm}(k)$ of the ST-OFDM system with an MMSE receiver is the same as the SNR $\eta_{\rm ofdm}(k)$ of the ST-OFDM system with a ZF receiver. This implies that for the ST-OFDM system, both ZF and MMSE receivers have the same BER performance. From (38), we find that the error variance of the ST-SCCP system is also independent of k and also equal to the average of that of the ST-OFDM system. The SINR of the ST-SCCP is also independent of k; all its outputs have the same SINR. One can see from (34), (36) and (38) that the UP-ST-OFDM system, the ST-OFDM system and the ST-SCCP system have the same total output error variance. The unitary precoder $\mathbf{T}$ does not change the total output variance but simply redistributes the error variance to different subchannels. When $\mathbf{T} = (1/\sqrt{M})\mathbf{W}$, it averages all the subchannel error variances. As we will see below, this averaging effect in fact gives the smallest BER.

BER Analysis: Since the system is no longer ICI-free, the errors consist of both the noise term and the ICI term from other subchannels. However, it was verified in [28] that when the number of subchannels is large, the Gaussian tail renders a very good approximation of the BER and the SINR is a good measure of system performance. In other words, BER of the kth subchannel can be very well approximated by $Q(\sqrt{\rho_T(k)})$ for QPSK modulation symbols. If we define the function $h(x) = Q(\sqrt{x^{-1}-1})$, then the BERs of the three systems are respectively given by

$$\mathrm{BER}_{\rm ofdm,mmse} = \frac{1}{M}\sum_{k=0}^{M-1} h\left(\frac{1}{1+\rho_{\rm ofdm}(k)}\right) \quad (40)$$

$$\mathrm{BER}_{T,\rm mmse} = \frac{1}{M}\sum_{k=0}^{M-1} h\left(\frac{1}{1+\rho_T(k)}\right) \quad (41)$$

$$\mathrm{BER}_{\rm sccp,mmse} = h\left(\frac{1}{1+\rho_{\rm sccp}}\right). \quad (42)$$

It is shown in [16] that the function $h(x)$ is convex for all $0 \le x \le 1$. Using this fact, we can prove the theorem below.

Theorem 2: For any unitary precoder $\mathbf{T}$, BER of UP-ST-OFDM systems satisfies

$$\mathrm{BER}_{\rm sccp,mmse} \le \mathrm{BER}_{T,\rm mmse} \le \mathrm{BER}_{\rm ofdm,mmse}. \quad (43)$$

Furthermore, $\mathrm{BER}_{T,\rm mmse} = \mathrm{BER}_{\rm sccp,mmse}$ if and only if $|t_{i,j}| = (1/\sqrt{M})$ for all $0 \le i,j \le M-1$, and $\mathrm{BER}_{T,\rm mmse} = \mathrm{BER}_{\rm ofdm,mmse}$ if and only if $\mathbf{T} = \mathbf{I}$.

Proof: Substituting (35) into the expression of $\mathrm{BER}_{T,\rm mmse}$ in (41), we get

$$\mathrm{BER}_{T,\rm mmse} = \frac{1}{M}\sum_{k=0}^{M-1} h\left(\sum_{l=0}^{M-1}\frac{\gamma^{-1}|t_{l,k}|^2}{d_l+\gamma^{-1}}\right). \quad (44)$$

Using the convexity of $h(x)$, we can obtain the following inequality:

$$h\left(\frac{1}{M}\sum_{k=0}^{M-1}\sum_{l=0}^{M-1}\frac{\gamma^{-1}|t_{l,k}|^2}{d_l+\gamma^{-1}}\right) \le \mathrm{BER}_{T,\rm mmse} \le \frac{1}{M}\sum_{k=0}^{M-1}\sum_{l=0}^{M-1}|t_{l,k}|^2 h\left(\frac{\gamma^{-1}}{d_l+\gamma^{-1}}\right). \quad (45)$$

Using the fact that $\sum_{k=0}^{M-1}|t_{k,l}|^2 = 1$, we can simplify the above equation as

$$h\left(\frac{1}{M}\sum_{l=0}^{M-1}\frac{\gamma^{-1}}{d_l+\gamma^{-1}}\right) \le \mathrm{BER}_{T,\rm mmse} \le \frac{1}{M}\sum_{l=0}^{M-1} h\left(\frac{\gamma^{-1}}{d_l+\gamma^{-1}}\right). \quad (46)$$

Notice that from (37) and (39), the upper and lower bounds of the above equation are simply $\mathrm{BER}_{\rm ofdm,mmse}$ and $\mathrm{BER}_{\rm sccp,mmse}$ respectively. This proves the inequalities in (43). Moreover the lower bound is achieved if and only if the unitary precoder $\mathbf{T}$ satisfies $|t_{j,k}| = (1/\sqrt{M})$. And the upper bound is achieved if and only if $\mathbf{T} = \mathbf{I}$. ■

Theorem 2 can be viewed as a generalization of Theorem 2 in [16] to the two antenna case. From this theorem, we can conclude that for the case of the MMSE receivers, no matter what the SNR γ is, the UP-ST-OFDM systems always outperform the ST-OFDM systems. And the optimal unitary precoder also satisfies $|t_{i,j}| = (1/\sqrt{M})$. In this case, the BER is the smallest and it is equal to $\mathrm{BER}_{\rm sccp,mmse}$.

III. Precoded ST-OFDM Systems for the Time-Varying Cases

In previous section, we assume that the channels are invariant during the transmissions of two consecutive OFDM blocks. Based on this assumption, the ICI-free condition can be obtained by using a zero-forcing receiver. However, in many applications, channels may encounter a small variation during the transmission of two OFDM blocks. The channel for the second block can be slightly different from the channel for the first block. In this section, we will study the performance of UP-ST-OFDM system under this time-varying environment.

Channel Model: We assume that the channels remain constant during the transmission of one OFDM block but they vary slightly between two OFDM blocks.[1] Let $h_i(l)$ and $\tilde{h}_i(l)$ denote the channels for the first and second OFDM block respectively. These channels satisfy

$$\tilde{h}_i(l) = h_i(l) + \phi_i(l) \quad (47)$$

where $\phi_i(l)$ is modeled as a complex random variable. It is assumed that the receiver knows $h_i(l)$ but not the variation $\phi_i(l)$. We assume that the channel variations satisfy

$$\mathcal{E}\{\phi_i(k)\} = 0, \mathcal{E}\{\phi_i(l)\phi_j^*(k)\} = \sigma^2_{\phi_i}(l)\delta(i-j)\delta(l-k). \quad (48)$$

[1]It is more realistic to assume that the channels also vary within one OFDM block. For the sake of tractability, we will adopt this simplified model.

Let $H_i(k)$, $\tilde{H}_i(k)$ and $\Phi_i(k)$ be respectively the M-point DFT coefficients of $h_i(l)$, $\tilde{h}_i(l)$ and $\phi_i(l)$. Then, they are related by $\tilde{H}_i(k) = H_i(k) + \Phi_i(k)$, for $k = 0, \ldots, M-1$. Moreover, the variations $\Phi_i(k)$ satisfy

$$\mathcal{E}\{\Phi_i(k)\} = 0, \quad \text{and} \quad \mathcal{E}\{|\Phi_i(k)|^2\} = \sum_{l=0}^{L} \sigma^2_{\phi_i}(l) \tag{49}$$

for $k = 0, 1, \ldots, M-1$. Under the above channel model, we analyze the performance of the UP-ST-OFDM system in Fig. 1. As the channels vary between two successive blocks, the expression for $\mathbf{r}_2$ in (4) is no longer valid. For the time-varying case, the two consecutive received vectors at the output of the DFT matrix become

$$\begin{aligned}\mathbf{r}_1 &= \mathbf{\Lambda}_1\mathbf{x}_1 + \mathbf{\Lambda}_2\mathbf{x}_2 + \mathbf{q}_1\\ \mathbf{r}_2 &= -\tilde{\mathbf{\Lambda}}_1\mathbf{x}_2^* + \tilde{\mathbf{\Lambda}}_2\mathbf{x}_1^* + \mathbf{q}_2\end{aligned} \tag{50}$$

where $\tilde{\mathbf{\Lambda}}_i$ is a diagonal matrix given by

$$\tilde{\mathbf{\Lambda}}_i = \mathrm{diag}[\tilde{H}_i(0) \quad \tilde{H}_i(1) \quad \cdots \tilde{H}_i(M-1)]. \tag{51}$$

As the channel variations are not known, only $h_i(l)$ are employed at the space time decoder to obtain the vectors $\mathbf{y}_1$ and $\mathbf{y}_2$ (Fig. 1):

$$\begin{bmatrix}\mathbf{y}_1\\ \mathbf{y}_2\end{bmatrix} = \begin{bmatrix}\mathbf{\Lambda}_1^* & \mathbf{\Lambda}_2\\ \mathbf{\Lambda}_2^* & -\mathbf{\Lambda}_1\end{bmatrix}\begin{bmatrix}\mathbf{r}_1\\ \mathbf{r}_2^*\end{bmatrix}. \tag{52}$$

Using (50), we can rewrite the above equation as

$$\begin{bmatrix}\mathbf{y}_1\\ \mathbf{y}_2\end{bmatrix} = \begin{bmatrix}\mathbf{U}_{11} & \mathbf{U}_{12}\\ \mathbf{U}_{21} & \mathbf{U}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{bmatrix} + \begin{bmatrix}\boldsymbol{\nu}_1\\ \boldsymbol{\nu}_2\end{bmatrix} \tag{53}$$

where $\mathbf{U}_{ij}$ are $M \times M$ diagonal matrices. Their kth diagonal entries are respectively given by

$$\begin{aligned}[\mathbf{U}_{11}]_{kk} &= d_k + H_2(k)\Phi_2^*(k)\\ [\mathbf{U}_{12}]_{kk} &= -H_2(k)\Phi_1^*(k)\\ [\mathbf{U}_{22}]_{kk} &= d_k + H_1(k)\Phi_1^*(k)\\ [\mathbf{U}_{21}]_{kk} &= -H_1(k)\Phi_2^*(k).\end{aligned} \tag{54}$$

Therefore, both $\mathbf{y}_1$ and $\mathbf{y}_2$ contain contributions from both $\mathbf{x}_1$ and $\mathbf{x}_2$. As $\Phi_i(k)$ are unknown, it is impossible to recover $\mathbf{x}_1$ and $\mathbf{x}_2$ perfectly even when there is no channel noise. In the following, we will introduce two methods for the symbol recovery. For simplicity, we will only describe the recovery of $\mathbf{s}_1$ below as the recovery of $\mathbf{s}_2$ is identical.

A. Method 1

In the first method, we assume that the ZF receiver for time-invariant case in Section II-A is used for symbol recovery. That means the output vector is

$$\hat{\mathbf{s}}_1 = \mathbf{T}^\dagger\mathbf{D}^{-1}\mathbf{y}_1 \tag{55}$$

where $\mathbf{D}$ is a diagonal matrix given by (7). From (53), we get

$$\hat{\mathbf{s}}_1 = \mathbf{T}^\dagger\mathbf{D}^{-1}\mathbf{U}_{11}\mathbf{T}\mathbf{s}_1 + \mathbf{T}^\dagger\mathbf{D}^{-1}\mathbf{U}_{12}\mathbf{T}\mathbf{s}_2 + \mathbf{T}^\dagger\mathbf{D}^{-1}\boldsymbol{\nu}_1. \tag{56}$$

Define the error vector as $\mathbf{e} = \hat{\mathbf{s}}_1 - \mathbf{s}_1$. It is shown in the Appendix that the variance of the kth component of $\mathbf{e}$ is given by

$$\sigma_T^2(k) = E_s\sum_{j=0}^{M-1}|H_2(j)|^2 d_j^{-2}|t_{j,k}|^2\psi(j) + N_0\sum_{j=0}^{M-1}|t_{j,k}|^2 d_j^{-1} \tag{57}$$

where

$$\psi(j) = |\Phi_1(j)|^2 + |\Phi_2(j)|^2. \tag{58}$$

Notice that the output error variances consist of two parts. The first term at the right hand size of (57) is the error variance due to the ICI whereas the second term is the error variance due to the channel noise. When the channels do not vary for two consecutive OFDM blocks, we have $\phi_i(j) = 0$. Then the ICI term becomes zero and it reduces to the LTI case. Comparing (57) and (12), we find that the error variances in the time-varying case are always larger than those in the LTI case due to the ICI effect.

For the ST-OFDM and ST-SCCP systems, the corresponding error variances can be obtained by letting $\mathbf{T} = \mathbf{I}$ and $\mathbf{T} = (1/\sqrt{M})\mathbf{W}$ respectively, and they are given by

$$\sigma^2_{\mathrm{ofdm}}(k) = d_k^{-2}(E_s\psi(k)|H_2(k)|^2 + N_0 d_k) \tag{59}$$

$$\sigma^2_{\mathrm{sccp}}(k) = \frac{1}{M}\sum_{j=0}^{M-1} d_j^{-2}(E_s\psi(j)|H_2(j)|^2 + N_0 d_j). \tag{60}$$

Notice from the above expressions that for both ST-OFDM and ST-SCCP systems, the output error variances also consist of an ICI term and a noise term. For the ST-OFDM case, the ICI term depends on both the kth subchannel gain and the variation $\phi_i(k)$. On the other hand, for the ST-SCCP system, both the ICI term and the noise term are independent of k. All the subchannels have the same subchannel variance σ^2_{sccp} and they are equal to the average of $\sigma^2_{\mathrm{ofdm}}(k)$. As we will see in the numerical simulations in Section IV, by averaging these subchannel error variances, the system performance can be significantly improved.

B. Method 2: MMSE Receiver

In the second method, we consider the MMSE receiver. In the following, we will only derive the MMSE receiver for $\mathbf{s}_1$ because the derivation of that for $\mathbf{s}_2$ is very similar. Given $\mathbf{y}_1$, we want to find an MMSE receiver for $\mathbf{s}_1$. Supposing that the receiving matrix is $\mathbf{V}_1$, then the MMSE receiver output is given by $\hat{\mathbf{s}}_1 = \mathbf{V}_1\mathbf{y}_1$. Define the error vector as $\mathbf{e} = \hat{\mathbf{s}}_1 - \mathbf{s}_1$, and we want to find $\mathbf{V}_1$ such that $\mathcal{E}\left\{\|\mathbf{e}\|^2\right\}$ is minimized. Notice that $\mathbf{e}$ depends on three random variables, s_i, ν_i, and $\Phi_i(k)$. The statistical expectation is taken with respect to all three random variables. By using the orthogonality principle $\mathcal{E}\{\mathbf{e}\mathbf{y}_1^\dagger\} = 0$, we have $\mathcal{E}\{\hat{\mathbf{s}}_1\mathbf{y}_1^\dagger\} = \mathcal{E}\{\mathbf{s}_1\mathbf{y}_1^\dagger\}$. Using (1), (9), and (49), it is not difficult to show that

$$\mathcal{E}\{\mathbf{s}_1\mathbf{y}_1^\dagger\} = E_s\mathbf{T}^\dagger\mathbf{D} \tag{61}$$

and

$$\mathcal{E}\{\hat{\mathbf{s}}_1\mathbf{y}_1^\dagger\} = \mathbf{V}_1\tilde{\mathbf{D}} \tag{62}$$

where $\tilde{\mathbf{D}}$ is a diagonal matrix with its kth entry given by

$$[\tilde{\mathbf{D}}]_{kk} = E_s(d_k^2 + \alpha^2 |H_2(k)|^2) + N_0 d_k \tag{63}$$

where $\alpha^2 = \sum_{l=0}^{L}(\sigma_{\phi_1}^2(l) + \sigma_{\phi_2}^2(l))$. From these expressions, we obtain the MMSE receiving matrix as[2]

$$\mathbf{V}_1 = \mathbf{T}^\dagger \mathbf{Q} \tag{64}$$

where $\mathbf{Q}$ is a diagonal matrix with

$$[\mathbf{Q}]_{kk} = d_k \theta_k \tag{65}$$

where

$$\theta_k = (d_k^2 + \alpha^2 |H_2(k)|^2 + \gamma^{-1} d_k)^{-1}. \tag{66}$$

From (64), we see that the receiver continues to have the same structure as that in Fig. 1. The FEQ in this case becomes the diagonal matrix $\mathbf{Q}$. Moreover, it can be shown that when α is zero, the MMSE receiving matrix of the time-varying case reduces to that of the LTI case. Applying the decoding matrix in (64) to (53), we get the output vector

$$\hat{\mathbf{s}}_1 = \mathbf{T}^\dagger \mathbf{Q} \mathbf{U}_{11} \mathbf{T} \mathbf{s}_1 + \mathbf{T}^\dagger \mathbf{Q} \mathbf{U}_{12} \mathbf{T} \mathbf{s}_2 + \mathbf{T}^\dagger \mathbf{Q} \boldsymbol{\nu}_1. \tag{67}$$

It is shown in the Appendix that the variance of the kth subchannel error $e(k) = \hat{s}_1(k) - s_1(k)$ is given by

$$\sigma_T^2(k) = E_s \sum_{j=0}^{M-1} |\lambda_j - 1|^2 |t_{j,k}|^2 + N_0 \sum_{j=0}^{M-1} \theta_j^2 |t_{j,k}|^2 d_j^3 + E_s \sum_{j=0}^{M-1} d_j^2 |H_2(j)|^2 |\Phi_1(j)|^2 \theta_j^2 |t_{j,k}|^2 \tag{68}$$

where $\lambda_j = d_j^2 \theta_j + d_j H_2(j) \Phi_2^*(j) \theta_j$.

Like the results of Method 1 in (57), the error variance of the kth subchannel for the MMSE receiver also consists of two parts. The first and third terms at the right hand side of (68) are the error variances due to the ICI whereas the second term is the error variance due to the channel noise. When the channels do not vary for two consecutive OFDM blocks, we have $\phi_i(k) = 0$. In this case, the ICI term becomes zero and it reduces to the LTI case. Comparing (68) and (34), we find that the error variance in the time-varying case is always larger than that in the LTI case.

If $\mathbf{T} = \mathbf{I}$, we get the error variance of the ST-OFDM system as

$$\sigma_{\text{ofdm}}^2(k) = E_s d_k^2 |H_2(k)|^2 \psi(k) \theta_k^2 + N_0 \theta_k^2 d_k^3. \tag{69}$$

If we choose the precoding matrix $\mathbf{T}$ as the normalized DFT matrix $(1/\sqrt{M})\mathbf{W}$, we can get the error variance of the ST-SCCP system as

$$\sigma_{\text{sccp}}^2(k) = \frac{E_s}{M} \sum_{j=0}^{M-1} |\lambda_j - 1|^2 + \frac{N_0}{M} \sum_{j=0}^{M-1} \theta_j^2 d_j^3 + \frac{E_s}{M} \sum_{j=0}^{M-1} d_j^2 |H_2(j)|^2 |\Phi_1(j)|^2 \theta_j^2 = \sigma_{\text{sccp}}^2. \tag{70}$$

[2]Given $\mathbf{y}_2$, the MMSE receiving matrix $\mathbf{V}_2$ for $\mathbf{s}_2$ is very similar to the matrix $\mathbf{V}_1$ in (64) except that $|H_2(k)|^2$ in $[\mathbf{Q}]_{kk}$ is replaced by $|H_1(k)|^2$.

For both ST-OFDM and ST-SCCP systems, the output variances also consist of the ICI term and one noise term. In the ST-OFDM case, we see that the ICI term depends on both the kth subchannel gain and the variation $\phi_i(k)$. For the ST-SCCP system, both the ICI terms and the noise term are independent of k. The precoder $\mathbf{T} = (1/\sqrt{M})\mathbf{W}$ averages not only the noise variance but also the error variance due to ICI. All the subchannels have the same subchannel variance σ_{sccp}^2 and they are equal to the average of $\sigma_{\text{ofdm}}^2(k)$. Simulation results in Section IV show that by averaging these subchannel error variances, the performance of the ST-SCCP system is much better than that of the ST-OFDM system.

IV. Simulation Results

In this section, we carry out Monte-Carlo experiments to verify the performance of UP-ST-OFDM systems. The modulation symbols are QPSK. The channel taps $h_i(l)$ are i.i.d. complex Gaussian random variables. The two channels $h_1(l)$ and $h_2(l)$ are uncorrelated. The channel order is $L = 31$. The variance of the channel taps is normalized by $\sum_{l=0}^{L} \mathcal{E}\{|h_i(l)|^2\} = 1$. A total of 1000 pairs of random channels are generated for the time-invariant case while a total of 2000 pairs of random channels are generated for the time-varying case. Noise is modeled as circularly symmetric complex Gaussian random variable. The CP length is $L = 31$. The size of FFT matrix is $M = 512$. We consider the BER performance for both cases with and without a convolutional code. The convolutional code is the rate $(1/2)$ code with generator (53,75) [31]. The convolutional code is applied over one OFDM symbol.

The performance of UP-ST-OFDM systems for the LTI case is shown in Fig. 2. It is seen from Fig. 2(a) that at a BER of 10^{-4}, the ST-OFDM system outperforms the OFDM system with one transmit and one receive antenna by as much as 13 dB. The ST-SCCP system with a ZF receiver is better than the ST-OFDM system by 7 dB. The use of an MMSE receiver can further improve the performance of ST-SCCP system. Notice from Fig. 2(a) that when the SNR is low ($\leq$6 dB), the ST-OFDM system is better than the ST-SCCP system with a ZF receiver. On the other hand, the ST-SCCP system with an MMSE receiver is always better than the ST-OFDM system. These results confirm the claims in Theorem 1 and 2. When the rate $(1/2)$ convolutional code is applied, from Fig. 2(b) we can make a similar conclusion about the performances of these systems. From Fig. 2(b), we find that with a convolutional code, the gain with the MMSE receiver over the ZF receiver is more obvious than that without a convolutional code.

For the purpose of comparison, we also show the results for correlated channels. We assume that the correlation of $h_1(l)$ and $h_2(l)$ is 0.9. In Fig. 3(a), it is found that at a BER of 10^{-4}, the gain of ST-OFDM systems over OFDM systems is reduced to 11 dB, which is smaller than the uncorrelated case in Fig. 2(a). Comparing the performance of ST-OFDM and ST-SCCP in Figs. 2(a) and 3(a), we can see that the gain using unitary precoders is still about 7 dB. The BER of the systems employing a convolutional code is shown in Fig. 3(b) and we can observer similar results.

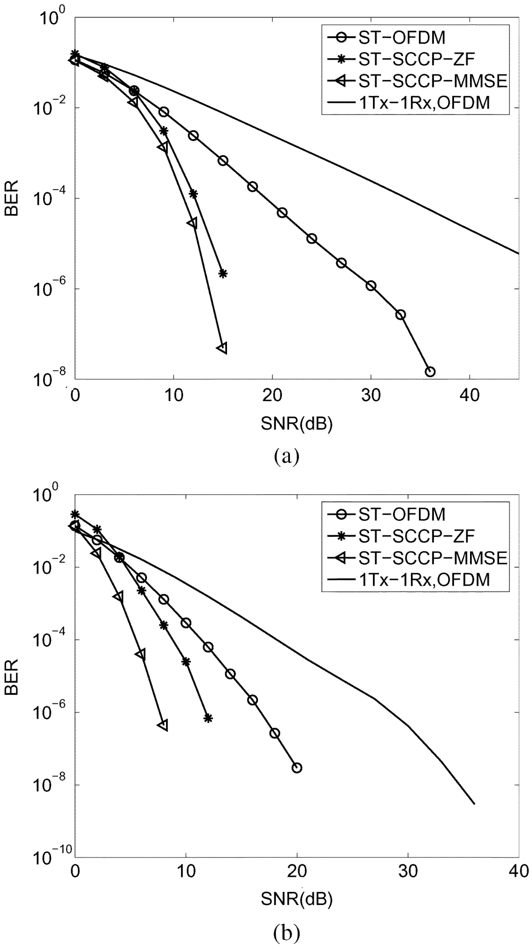

Fig. 2. Performance of UP-ST-OFDM system in time-invariant channels: (a) without a convolutional code, and (b) with a convolutional code.

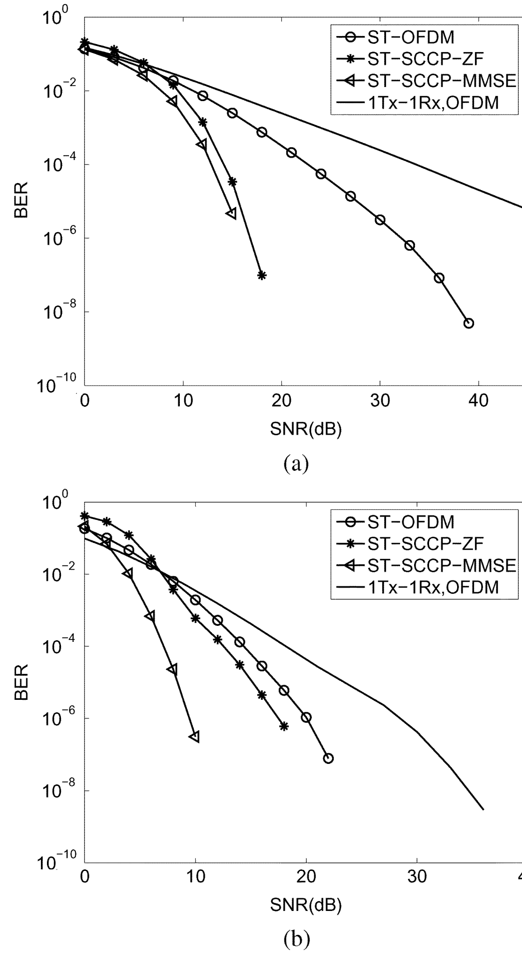

Fig. 3. Performance of UP-ST-OFDM system in correlated channels: (a) without a convolutional code, and (b) with a convolutional code.

Next, we show the simulation results of the precoded ST-OFDM system in the time-varying case channels. We assume that the variance of the channel variation is $\sigma^2_{\phi_i}(l) = (1/3200)$, for all $0 \leq l \leq 31$. From Fig. 4, it is seen that the performance of the conventional ST-OFDM system suffers an error flooring due to the ICI effect. When SNR ≥ 30 dB, OFDM with one transmit and one receive antenna is better than the ST-OFDM system. The performance of the ST-SCCP system with a ZF receiver is about 14 dB better than the ST-OFDM system. Moreover, the error flooring does not occur for the BER range of interest. The use of an MMSE receiver can further improve the performance of ST-SCCP system. And the ST-SCCP system with an MMSE receiver is always better than the ST-OFDM system. Similarly, from Fig. 4(a) when the SNR ≤ 6 dB, the ST-OFDM system is better than the ST-SCCP system with a ZF receiver. When the convolutional code is applied, the BER performance is shown in Fig. 4(b). The same results as in Fig. 4(a) can be observed when the convolutional code is used. The gain with an MMSE receiver with a convolutional code is more obvious than that without a convolutional code.

V. Conclusion

In this paper, we studied the UP-ST-OFDM systems. For the LTI case, the optimal precoder for ST-OFDM system with a ZF receiver is SNR dependent. For a moderate high SNR region, the optimal precoder satisfies $|t_{j,k}| = 1/\sqrt{M}$. For an MMSE receiver, the optimal precoder always satisfies $|t_{j,k}| = 1/\sqrt{M}$. For the time-varying case, we show that the output error variance of all subchannels can be averaged if the precoder satisfies $|t_{j,k}| = (1/\sqrt{M})$. Numerical experiment shows that by averaging the error variances, the BER performance can be improved substantially.

It is possible to extend the results to the case of more transmit antennas using other orthogonal codes [3]. In this case, the input vectors $\mathbf{s}_i$ (more than 2) are passed through a unitary precoder before the application of orthogonal codes. The derivations are similar to the case of Alamouti code.

Appendix A
The Derivation of $\sigma^2_T(k)$ for Method 1

From (56), we can write the error vector as

$$\mathbf{e} = \mathbf{T}^\dagger(\mathbf{D}^{-1}\mathbf{U}_{11} - \mathbf{I})\mathbf{T}\mathbf{s}_1 + \mathbf{T}^\dagger\mathbf{D}^{-1}\mathbf{U}_{12}\mathbf{T}\mathbf{s}_2 + \mathbf{T}^\dagger\mathbf{D}^{-1}\boldsymbol{\nu}_1. \tag{71}$$

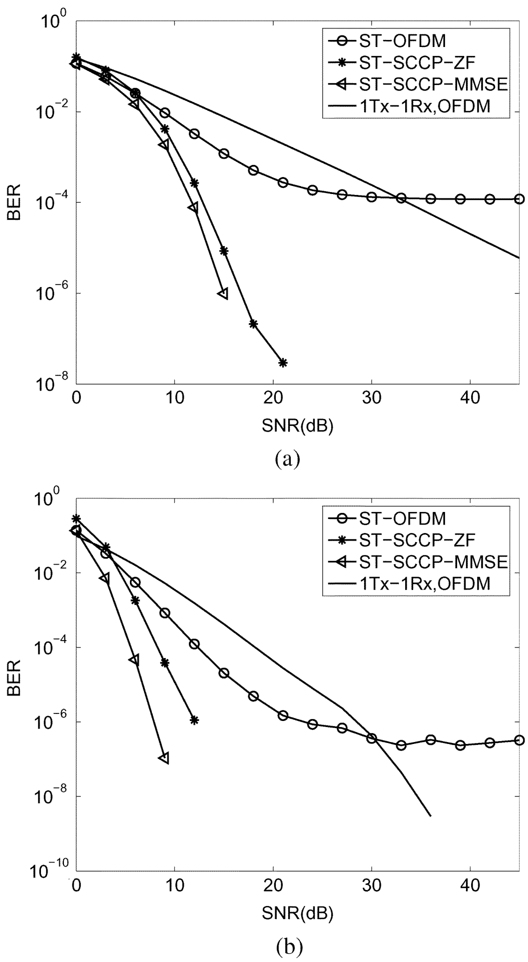

Fig. 4. Performance of UP-ST-OFDM system in time-varying channels: (a) without a convolutional code, and (b) with a convolutional code.

From the above expression, we can get the kth component of the error vector as

$$e(k) = \underbrace{\sum_{j=0}^{M-1}\sum_{l=0}^{M-1} H_2(j)\Phi_2^*(j)d_j^{-1}t_{j,k}^*t_{j,l}s_1(l)}_{\mathrm{ICI}_1} \underbrace{- \sum_{j=0}^{M-1}\sum_{l=0}^{M-1} H_2(j)\Phi_1^*(j)d_j^{-1}t_{j,k}^*t_{j,l}s_2(l)}_{\mathrm{ICI}_2} + \underbrace{\sum_{j=0}^{M-1} t_{j,k}^* d_j^{-1}\nu_1(j)}_{N_1}. \tag{72}$$

Since the channel deviations $\Phi_1(j)$ and $\Phi_2(j)$ are not known, ICI_1 and ICI_2 represent the interferences from $\mathbf{s}_1$ and $\mathbf{s}_2$ respectively. N_1 is the channel noise with variance

$$\mathcal{E}\{|N_1|^2\} = \sum_{j=0}^{M-1} N_0 d_j^{-1}|t_{j,k}|^2. \tag{73}$$

Based on the assumption that $\boldsymbol{\nu}_1$, $\mathbf{s}_1$ and $\mathbf{s}_2$ are uncorrelated, the error variance of $e(k)$ can be computed as

$$\sigma_T^2(k) = \mathcal{E}\{|\mathrm{ICI}_1|^2\} + \mathcal{E}\{|\mathrm{ICI}_2|^2\} + \mathcal{E}\{|N_1|^2\}. \tag{74}$$

Using the fact that $\mathcal{E}\{\mathbf{s}_1\mathbf{s}_1^\dagger\} = E_s\mathbf{I}$ and $\mathbf{T}^\dagger\mathbf{T} = \mathbf{I}$, we have

$$\mathcal{E}\{|\mathrm{ICI}_1|^2\} = E_s \sum_{j=0}^{M-1} |H_2(j)|^2|\Phi_2(j)|^2 d_j^{-2}|t_{j,k}|^2. \tag{75}$$

Similar results can be observed for the variance of ICI_2

$$\mathcal{E}\{|\mathrm{ICI}_2|^2\} = E_s \sum_{j=0}^{M-1} |H_2(j)|^2|\Phi_1(j)|^2 d_j^{-2}|t_{j,k}|^2. \tag{76}$$

Combining the above results, we get

$$\sigma_T^2(k) = E_s \sum_{j=0}^{M-1} |H_2(j)|^2 d_j^{-2}|t_{j,k}|^2\psi(j) + N_0 \sum_{j=0}^{M-1} |t_{j,k}|^2 d_j^{-1} \tag{77}$$

where $\psi(j) = |\Phi_1(j)|^2 + |\Phi_2(j)|^2$.

APPENDIX B
THE DERIVATION OF $\sigma_T^2(k)$ FOR THE MMSE RECEIVER

From (67), the error vector $\mathbf{e} = \hat{\mathbf{s}}_1 - \mathbf{s}_1$ is given by

$$\mathbf{e} = \mathbf{T}^\dagger(\mathbf{Q}\mathbf{U}_{11} - \mathbf{I})\mathbf{T}\mathbf{s}_1 + \mathbf{T}^\dagger\mathbf{Q}\mathbf{U}_{12}\mathbf{T}\mathbf{s}_2 + \mathbf{T}^\dagger\mathbf{Q}\boldsymbol{\nu}_1. \tag{78}$$

The kth component of $\mathbf{e}$ is given by

$$\begin{aligned} e(k) = &\sum_{j=0}^{M-1}\sum_{l=0}^{M-1}(\lambda_j - 1)t_{j,k}^*t_{j,l}s_1(l) \\ &- \sum_{j=0}^{M-1}\sum_{l=0}^{M-1} d_jH_2(j)\Phi_1^*(j)\theta_j t_{j,k}^*t_{j,l}s_2(l) \\ &+ \sum_{j=0}^{M-1} d_j\theta_j t_{j,k}^*\nu_1(j). \end{aligned} \tag{79}$$

where $\lambda_j = d_j^2\theta_j + d_jH_2(j)\Phi_2^*(j)\theta_j$, and $\theta_j = (d_k^2 + \alpha^2|H_2(k)|^2 + \gamma^{-1}d_k)^{-1}$. Applying the same procedure as in Method 1, the error variance of $e(k)$ can be computed as

$$\begin{aligned} \sigma_T^2(k) = &E_s \sum_{j=0}^{M-1} |\lambda_j - 1|^2 + N_0 \sum_{j=0}^{M-1} d_j^3\theta_j^2|t_{j,k}|^2 \\ &+ E_s \sum_{j=0}^{M-1} d_j^2|H_2(j)|^2|\Phi_1(j)|^2\theta_j^2|t_{j,k}|^2. \end{aligned} \tag{80}$$

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