Performance of BFSK FHMA Systems with RTT Side Information over Fading Channels

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***Abstract*— The performance of frequency-hopping multiple-access systems with side information and binary frequency-shift keying signaling is evaluated. The side information is generated by Viterbi's ratio threshold test (RTT). Both synchronous and asynchronous hopping systems over fading channels are examined. The system with RTT side information performs significantly better than that with perfect side information or hard decision. Compared with hard decision, the RTT side information provides more than 20% improvement in capacity for $E_b/N_0 = 10$ dB over Rayleigh channels. Also, we find that the optimum threshold in the RTT scheme should be dynamically adjusted with the number of simultaneous users in the system.**

***Index Terms*— Frequency hopping, multiple access, ratio threshold test.**

I. Introduction

THE PERFORMANCE of frequency-hopping multiple-access (FHMA) systems is severely degraded by multiple-access interference, which is indicated by the probability of signal collision in the same frequency slot [1]. For conventional evaluation, this probability is used to represent the probability of error decision in the system with hard decision or the erasure probability in the system with perfect side information. However, the signal collision may not result in wrong receiving. Therefore, conventional performance evaluation appears to be pessimistic. Moreover, the exaggerative erasure probability predicted by the perfect side information system results in an incorrect conclusion. This exploration was reported by comparing the systems with perfect side information and hard decision over nonfading additive white Gaussian noise (AWGN) and Rayleigh channels [2]. In this paper, we theoretically compare both hopping systems equipped with realistic side information for binary frequency-shift keying (BFSK) signaling over Rician channels. Here the realistic side information is generated by Viterbi's ratio threshold test (RTT), of which the effectiveness has been demonstrated for synchronous hopping systems [3]. Since the time duration of collision from an interference signal, called collision period, is different for both hopping systems, we will reflect it in the analysis.

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II. System Description and Analysis

Under our consideration, each BFSK modulated symbol is hopped to some frequency slot chosen from q available frequency slots according to its respective random hopping pattern. This hopped signal is then transmitted over the common channel, and it may collide with the other signals in the same frequency slot. According to the type of hopping, the probability of collision from a certain interference signal P_h can be expressed as $1/q$ and $(2-1/q)/q$ for synchronous and asynchronous random hopping, respectively [1]. And it can be found that the collision for multiple interference signals would be treated as a binomial trial with probability P_h.

For the receiver with noncoherent detection, the received signal is stripped of frequency hopping and then directed to the envelope detectors for the two BFSK signaling frequencies in band "0" and band "1," respectively, to generate two decision variables. If data "1" and "0" are equally likely transmitted, we can assume that the desired signal (from the user 0) is present in band 0 without loss of generality in the following analysis. If k_i interference signals are present in band i, the decision variables can be expressed as [4], [5]

$$r_{i,k_i} = |Z_i + \delta_{i0}\alpha_0 e^{j\theta_0} + \sum_{m=1}^{k_i} \alpha_{im} x_{im} e^{j\theta_{im}}|, \qquad i = 0, 1 \quad (1)$$

where Z_i is the complex Gaussian noise with zero mean and $E\{Z_i Z_i^*\} \equiv N_0$, δ_{i0} is the Kronecker delta, α_{im} and θ_{im} are the amplitude and phase of the desired or the mth interference signal, and x_{im} is the normalized ratio of hit period to the bit duration from the mth interference signal for band i. x_{im} would be equal to 1 for the synchronous hopping system and would be a random variable uniformly distributed over $[0, 1)$ for the asynchronous hopping system. For the channel under consideration, θ_{im} is uniformly distributed over $[0, 2\pi)$, and $\{\alpha_{im}\}$ are modeled as independently, identically distributed Rician random variables with mean a and variance $2\sigma_f^2$. Here we assume that the same average power is received from each user's signal. Since a^2 and $2\sigma_f^2$ are proportional to the direct and diffused components of the received signal, respectively, it is convenient to define a parameter $\Gamma = a^2/2\sigma_f^2$ to indicate the degree of fading [4]. As a result, $\Gamma = 0$ and $\Gamma = \infty$ represent Rayleigh and nonfading AWGN channels, respectively.

The characteristic functions of these two decision variables can be derived as [5]

$$\phi_{i,k_i}(\rho) = e^{-(\sigma^2\rho^2/2)}\phi(\delta_{i0}\rho)\hat{\phi}^{k_i}(\rho) \quad (2)$$

where σ^2 is equal to $N_0/2$, $\phi(y) = e^{-(\sigma_f^2 y^2/2)} J_0(ay)$ is the characteristic function of the desired signal, $J_0(y)$ is the zeroth-order Bessel function, and $\hat{\phi}(\rho)$ is the characteristic function of an interference signal. According to the type of hopping, $\hat{\phi}(\rho)$ can be obtained by averaging over x_{im}

$$\hat{\phi}(\rho) = \begin{cases} e^{-(\sigma_f^2\rho^2/2)} J_0(a\rho), & \text{synchronous hopping} \\ \int_0^1 e^{-(\sigma_f^2 x^2\rho^2/2)} J_0(ax\rho)\, dx, & \text{asynchronous hopping.} \end{cases} \tag{3}$$

For the RTT detection scheme, the receiver employs a threshold t $(0 < t \leq 1)$ to estimate the reliability of the received symbol. If the amplitude ratio of the small one to the large one of two decision variables exceeds this threshold, the symbol will be declared unreliable and therefore erased; otherwise, it will be preserved. For a preserved symbol, a decision error occurs when the amplitude ratio of r_{0,k_0} to r_{1,k_1} is less than t. The conditional probability of error can be determined by [5]

$$P_e(k_0, k_1) = -\int_0^\infty \phi_{0,k_0}(\rho) \frac{d\phi_{1,k_1}(t\rho)}{d\rho}\, d\rho. \tag{4}$$

After simple manipulation, this conditional probability of error can be expressed as

$$P_e(k_0, k_1) = \int_0^\infty \left[\sigma^2 t^2 \rho \hat{\phi}^{k_1}(t\rho) - k_1 X(\rho) \hat{\phi}^{k_1 - 1}(t\rho)\right] \cdot \phi(\rho) \hat{\phi}^{k_0}(\rho) e^{-((t^2+1)\sigma^2\rho^2/2)}\, d\rho \tag{5}$$

where $X(\rho)$ is defined, shown at the bottom of the page.

If K signals appear simultaneously, the average probability of error can be expressed as

$$\begin{aligned} P_e(K) &= \sum_{k=0}^{K-1} \binom{K-1}{k} P_h^k (1-P_h)^{K-1-k} \cdot \sum_{k_0=0}^{k} \binom{k}{k_0} \left(\frac{1}{2}\right)^k P_e(k_0, k-k_0) \\ &= \int_0^\infty \left[\sigma^2 t^2 \rho Y(\rho) - \frac{K-1}{2} P_h X(\rho)\right] \cdot \phi(\rho) Y^{K-2}(\rho) e^{-((t^2+1)\sigma^2\rho^2/2)}\, d\rho \end{aligned} \tag{6}$$

where $Y(\rho) = 1 - P_h + P_h(\hat{\phi}(\rho) + \hat{\phi}(t\rho))/2$.

On the other hand, the symbol is received correctly if the ratio of r_{1,k_1} to r_{0,k_0} is less than t, and the average probability can be deduced similarly

$$P_c(K) = \int_0^\infty \left\{\left[(\sigma^2 + \sigma_f^2) t^2 \rho J_0(at\rho) + at J_1(at\rho)\right] \cdot Y(\rho) - (K-1) P_h J_0(at\rho) X(\rho)/2\right\} \cdot Y^{K-2}(\rho) e^{-(\rho^2/2)[(t^2+1)\sigma^2 + t^2\sigma_f^2]}\, d\rho. \tag{7}$$

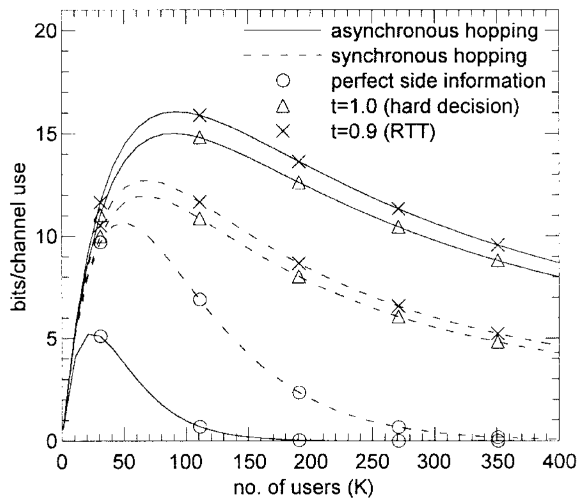

Fig. 1. Capacity of the BFSK FHMA system with RTT side information: $E_b/N_0 = 10$ dB, $q = 50$, and $\Gamma = 0$. t: threshold.

Finally, the average probability of erasure can be obtained by $P_x(K) = 1 - P_c(K) - P_e(K)$. Note that $P_x(K)$ is equal to zero for $t = 1$, and the decision would be degenerated to the conventional hard decision.

With the binary erasure channel model [6], the total capacity, in terms of bits per channel use, can be computed by summing the individual channel capacity as

$$C(K) = K\left\{P_c(K) \log_2 P_c(K) + P_e(K) \log_2 P_e(K) + [1 - P_x(K)] \log_2 \frac{2}{1 - P_x(K)}\right\}. \tag{8}$$

III. Numerical Results and Discussion

Fig. 1 shows the capacity of the various systems for $E_b/N_0 = 10$ dB and $q = 50$ over Rayleigh fading channels, where $E_b = a^2 + 2\sigma_f^2$ is the average bit energy. For the systems with perfect side information derived from [2], the asynchronous system performs poorer than the synchronous system. On the contrary, the asynchronous system achieves better performance than the synchronous system for realistic RTT side information, as shown in Fig. 1. Moreover, the capacity of the system with RTT side information surpasses that with perfect side information, regardless of the type of hopping. These results confirm that exaggerative erasure derived from the perfect side information assumption, especially for the asynchronous hopping systems, cannot be used to correctly deduce the performance of the system implemented with realistic side information. When the direct signal power (large Γ) or the average signal power (large E_b/N_0) increases, as shown in Figs. 2 and 3, respectively, the behavior of system performance is similar to that shown

$$X(\rho) = \frac{d\hat{\phi}(t\rho)}{d\rho} = \begin{cases} -\left[\sigma_f^2 t^2 \rho J_0(at\rho) + at J_1(at\rho)\right] e^{-(\sigma_f^2 t^2 \rho^2/2)}, & \text{synchronous hopping} \\ -\int_0^1 \left[\sigma_f^2 t^2 x^2 \rho J_0(atx\rho) + atx J_1(atx\rho)\right] e^{-(\sigma_f^2 t^2 x^2 \rho^2/2)}\, dx, & \text{asynchronous hopping} \end{cases}$$

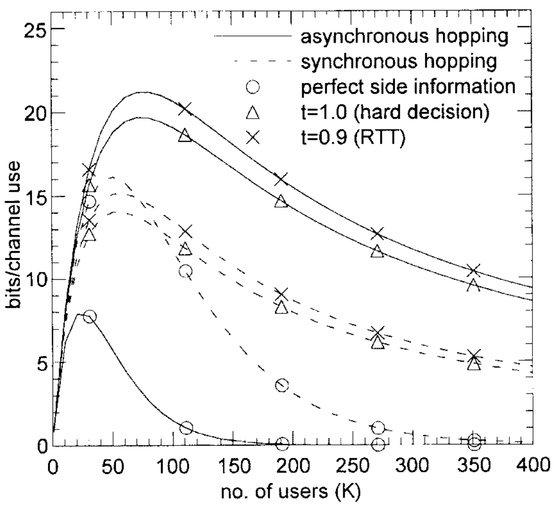

Fig. 2. Capacity of the BFSK FHMA system with RTT side information: $E_b/N_0 = 10$ dB, $q = 50$, and $\Gamma = 10$. t: threshold.

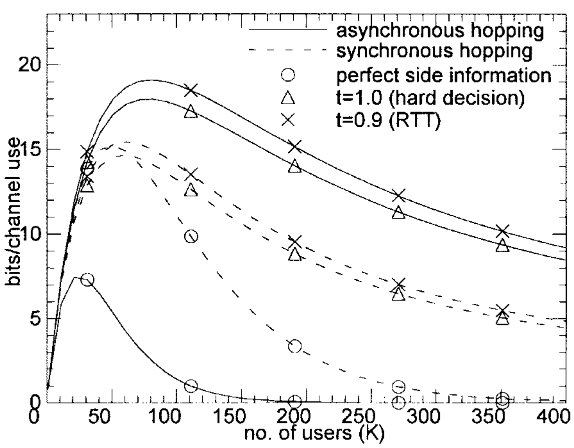

Fig. 3. Capacity of the BFSK FHMA system with RTT side information: $E_b/N_0 = 16$ dB, $q = 50$, and $\Gamma = 0$. t: threshold.

in Fig. 1. However, for the synchronous hopping system, the achievable maximum capacity with RTT side information may be lower than that with perfect side information for higher Γ or E_b/N_0. This degradation lies in the fact that the hit symbol is more unreliable in the system with RTT side information for higher Γ or E_b/N_0. Such degradation can be mitigated with lower threshold to generate more erasure information. Since the useful information decreases as the erasure increases, capacity increasing results from reduction of error decisions. However, the excessive erasure (extremely low threshold) may not be offset by reduction of error decisions, which leads to capacity decreasing. Therefore, there exists an optimum threshold to maximize the capacity. Through numerical search, the optimum threshold and capacity for the system with $E_b/N_0 = 10$ dB and $q = 50$ over Rayleigh channels are presented in Fig. 4. Besides varying with the number of users, the optimum threshold for the asynchronous hopping system is higher than that for the synchronous hopping system, due to less interference produced by the partial collision for the same number of users in the asynchronous hopping system.

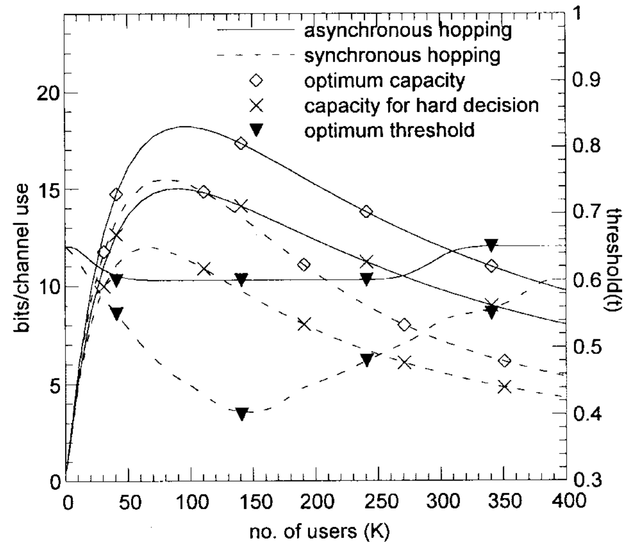

Fig. 4. Optimum capacity and threshold of the BFSK FHMA system with RTT side information: $E_b/N_0 = 10$ dB, $q = 50$, and $\Gamma = 0$.

Evidently, the RTT system with optimum threshold provides more than 20% improvement in achievable maximum capacity regardless of the type of hopping, when compared with the hard decision system. Certainly, this optimum capacity would surpass the capacity of the system with perfect side information.

IV. Conclusion

The capacity of the BFSK FHMA system with RTT side information over fading channels is derived. Contrary to the envisaged perfect side information to produce excessive erasure, the RTT scheme is feasible and can generate appropriate erasure information to improve system capacity regardless of the type of hopping. To maximize the improvement, the threshold of RTT should be adjusted with the number of simultaneous users in the system. With numerical search, the optimum threshold is illustrated. This achievable maximum capacity can be improved over 20% of that obtained by hard decision.

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