Optical Orthogonal Codes With Large Crosscorrelation and Their Performance Bound for Asynchronous Optical CDMA Systems

Chi-Shun Weng and Jingshown Wu, *Senior Member, IEEE*

***Abstract*—Optical orthogonal codes (OOCs) are commonly used as signature codes for optical code-division multiple-access (OCDMA) communication systems. Many OOCs have been proposed and investigated. Asynchronous OCDMA systems using conventional OOCs have very limited number of subscribers and a few simultaneous users. Recently, we reported a new code family with large code size by relaxing the crosscorrelation constraint to 2. In this paper by further loosening the crosscorrelation constraint, we adopt the random greedy algorithm to construct a code family which has larger code size and more simultaneous users. We also derive an upper bound of number of simultaneous users for given code length, code weight, and bit error rate. The study shows that it is possible to have codes approaching to this bound.**

***Index Terms*—Maximal system, multiuser interference, optical code-division multiple-access (OCDMA), optical orthogonal code (OOC), perfect difference code, random greedy algorithm.**

I. INTRODUCTION

RECENTLY, there have been many papers that have discussed OOCs for optical code division multiple access (OCDMA) systems [1]–[13]. $(v, w, \lambda_a, \lambda_c)$-OOCs are a family of (0,1) sequences with code length v, code weight w, the maximum value of off-peak autocorrelation λ_a, and the maximum value of crosscorrelation λ_c. For the sake of synchronization and minimizing multiuser interference (MUI), $(v, w, 1)$-OOCs with $\lambda_a = \lambda_c = 1$ are usually adopted as signature codes. In general, the crosscorrelation of any two $(v, w, 1)$-OOCs is either zero or one. Therefore, it is difficult to design a receiver to cancel MUI. The code size, upper bounded by $\lfloor 1/w \lfloor (v-1)/(w-1) \rfloor \rfloor$, of these ideal OOCs is sparse corresponding to the code length. To increase the code size, some code families with nonideal correlation constraint have been reported [10]–[13]. In [10], Chung and Kumar constructed optimal $(p^{2m}-1, p^m+1, 2)$-OOCs, where p is any prime and the family size is $p^m - 2$. In [11], Yang and Fuja investigated $(v, w, 2, 1)$-OOCs and showed that it is impossible to get more than $2(v-1)/(w^2-w)$ codewords whose code size is twice the upper bound of $(v, w, 1)$-OOCs. In [12], Yang also constructed $(v, w, 1, 2)$-OOCs and the code size is w times (for even w) or $w-1$ times (for odd w) the size of the $(v, w, 1)$-OOCs, when w is less than eight. In [13], we have proposed $(v, w, 1, 2)$-OOCs based on $(v, k, 1)$-perfect difference codes, where $v = k^2 - k + 1$, $2 < w < k$, and k is a power of a prime plus one [14]. We have also shown that the performance was improved significantly with larger code weight given a fixed code length.

In [13], we reserved w chips appropriately from the k chips of a $(v, k, 1)$-perfect difference code such that all the codewords fulfilled the crosscorrelation constraint, that is, $\lambda_c = 2$. Because most of the crosscorrelations between any two distinct codes in these family are 0 or 1 (while only very small portion have value of 2), the bit error rate (BER) performance of the systems using the $(v, w, 1, 2)$-OOCs is almost the same as that using the $(v, w, 1)$-OOCs. Moreover, the code size of $(v, w, 1, 2)$-OOCs is upper bounded by $\lfloor k/w \lfloor (k-1)/(w-1) \lfloor (k-2)/(w-2) \rfloor \rfloor \rfloor$ which may be ten times larger than that of ideal $(v, w, 1)$-OOCs. Thus, given a code length and a code size, the code weight of the proposed codes is larger than that of ideal codes. The numerical results showed that the performance of $(v, w, 1, 2)$-OOCs with larger code weight is better than that of ideal codes, because the larger code weight is more robust to interference to a certain extent.

Although the performance of the $(v, w, 1, 2)$-OOCs with larger code weight is better, the code size is reduced sharply as the code weight increases. As a result, it is impossible to increase the code weight of $(v, w, 1, 2)$-OOCs for a given code length and code size further. One feasible way to increase the code size is to relax the crosscorrelation constraint further. In this paper, we investigate $(v, w, 1, \lambda_c)$-OOCs based on the $(v, k, 1)$-perfect difference set with size k, where $2 \leq \lambda_c < w < k$, that is, λ_c is no longer limited to 2. To construct $(v, w, 1, \lambda_c)$-OOCs, it is necessary to choose w-subsets of a $(v, k, 1)$-perfect difference set appropriately, such that any two distinct w-subsets share at most λ_c elements and then each w-subset is corresponding to a code. The problem is the same as how to construct an $m(w, r, k)$ maximal system defined as a family of w-subsets of a k-set such that every r-subset of the k-set is contained in at most one set of the system [15], where $r = (\lambda_c + 1)$ and $r \leq w \leq k$. Trivially, there is an upper bound of the system size (the number of w-subsets in the system) denoted by $\overline{\overline{m}}(w, r, k)$. Exhaustive search is a

Manuscript received May 6, 2002; revised December 6, 2002. This work was supported in part by the National Science Council and Ministry of Education, Taiwan, R.O.C., under Grant NSC89-2215-E-002-012 and Grant 89-E-FA06-2-4.

C.-S. Weng was with the Department of Electrical Engineering, Graduate Institute of Communication Engineering, National Taiwan University, Taipei, Taiwan 10617, R.O.C. He is now with Realtek Semiconductor Corporation, Hsinchu 300, Taiwan, R.O.C.

J. Wu is with the Department of Electrical Engineering, Graduate Institute of Communication Engineering, National Taiwan University, Taipei, Taiwan 10617 R.O.C. (e-mail: wujsh@cc.ee.ntu.edu.tw).

Digital Object Identifier 10.1109/JLT.2003.809547

way to construct the largest size of such system. However, it is infeasible due to its complexity. Fortunately, the *random greedy algorithm* can construct asymptotically good maximal systems [16], [17]. In this paper, we adopt this algorithm to construct a suboptimal code family of $(v, w, 1, \lambda_c)$-OOCs. The crosscorrelation between two codes is 0 or 1 if they are not aligned with each other. When two codes are aligned (the probability is only $1/v$), the value of crosscorrelation is between 0 and λ_c. As a result, the crosscorrelation property of $(v, w, 1, \lambda_c)$-OOCs is only slightly different from that of $(v, w, 1)$-OOCs. Therefore, it is reasonable to anticipate that the performances of $(v, w, 1, \lambda_c)$-OOCs and $(v, w, 1)$-OOCs are similar to each other. To simplify the BER performance analysis, we assume that chips of two codes are synchronous among users. We also take the presumption that the crosscorrelation is λ_c (which is the worst situation of interference) when two distinct codes are aligned with each other and the interfered λ_c chips are randomly distributed among w chips. Based on these assumptions, an upper bound of the BER can be derived according to the principle of inclusion and exclusion. The numerical results show that the performance is improved significantly with the increase of w when the code weight is not large. However, when the code weight is larger than a certain value, the performance gets worse as the code weight increases. This is because the larger code weight increases the interfering probability which offsets the robustness. The results also show that it is possible to approach the upper bound of the number of simultaneous users given a code length and a code size.

The remainder of this paper is organized as follows. In Section II, we describe and construct the $(v, w, 1, \lambda_c)$-OOCs based on perfect difference sets and the random greedy algorithm. In Section III, we analyze the BER performance of the systems in conjunction with $(v, w, 1, \lambda_c)$-OOCs and double hard-limiters [18]. The numerical results are given in Section IV. We conclude in Section V.

II. $(v, w, 1, \lambda_c)$-OOCs Based on Perfect Difference Sets

In this section, we describe the formulation of a code family of $(v, w, 1, \lambda_c)$-OOCs based on perfect difference sets.

Let W be the v-set of the integers $0, 1, \ldots, v-1$ modulo v. A set $D = \{d_1, d_2, \ldots, d_k\}$ is a k-subset of W. For every $a \neq 0 \pmod v$, there is exactly one ordered pair (d_i, d_j), $i \neq j$, such that

$$d_i - d_j \equiv a \pmod v. \tag{1}$$

A set D satisfying these requirements is called a $(v, k, 1)$-perfect difference set. The existence of the $(q^2+q+1, q+1, 1)$-perfect difference set, where q is a power of a prime, has been proved and constructed by Singer [19]. We can construct a perfect difference code $C = \{c_0, c_1, \ldots, c_i, \ldots, c_{(v-1)}\}$ based on the perfect difference set D with the rule

$$c_i = \begin{cases} 1, & \text{if } i \in D \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

The code weight and code length are k and v, respectively, where $v = k^2 - k + 1$ and $k = q + 1$. The off-peak auto correlation of such code is always equal to one. This property is useful to construct a code family with crosscorrelation equal to one by cyclically shifting such code $(v-1)$ times for synchronous OCDMA [14]. However, for asynchronous OCDMA, we have to modify the $(v, k, 1)$-perfect difference code [13]. A code family of $(v, w, 1, \lambda_c)$-OOCs is formed by reserving some w chips from the k chips of a $(v, k, 1)$-perfect difference code such that all the new codes fulfill the crosscorrelation constraint. That is, the maximum crosscorrelation between any two codes is not larger than λ_c. The problem is the same as how to construct an $m(w, r, k)$ maximal system defined as a family of w-subsets of a k-set such that every r-tuple of the k-set is contained in at most one set of the system [15], where $r = (\lambda_c + 1)$ and $r \leq w \leq k$. It is well known that the system size, denoted by $\overline{\overline{m}}(w, r, k)$, is upper bounded by

$$\overline{\overline{m}}(w, r, k) \leq \binom{k}{r}\binom{w}{r}^{-1} \tag{3}$$

or more tightly [20]

$$\overline{\overline{m}}(w, r, k) \leq \left\lfloor \frac{k}{w} \left\lfloor \frac{k-1}{w-1} \left\lfloor \cdots \left\lfloor \frac{k-r+2}{w-r+2} \left\lfloor \frac{k-r+1}{w-r+1} \right\rfloor \right\rfloor \cdots \right\rfloor \right\rfloor \right\rfloor. \tag{4}$$

The density of the system is defined as

$$d(w, r, k) = \overline{\overline{m}}(w, r, k) \cdot \binom{w}{r}\binom{k}{r}^{-1}. \tag{5}$$

Trivially, $d(w, r, k) \leq 1$ holds. If $d(w, r, k) = 1$, the system is also called *Steiner system* $S(w, r, k)$ in which every r-tuple of the k-set is contained in *exactly* one w-set of the system. To find all parameters (w, r, k) for $S(w, r, k)$ is a long-standing unsolved problem. There are an infinite number of known Steiner systems with $r = 2$ and 3 and a finite number of Steiner systems with $r = 4$ and 5. Moreover, no Steiner systems with $r \geq 6$ are known [21].

The determination of the maximal value of $\overline{\overline{m}}(w, r, k)$ is still an unsolved problem [20]. Fortunately, some useful results have been reported. In 1963, Erdös and Hanani conjectured that for every r and w, $r < w$ [15]

$$\lim_{k \to \infty} d(w, r, k) = 1. \tag{6}$$

They proved (6) for $r = 2$ and every w and for $r = 3$ and $w = p + 1$, where p is a prime power. Eventually, Rödl proved this conjecture in 1985 [22].

One way to construct a maximum size of an $m(w, r, k)$ maximal system is to compare all collections of w-tuples of the k-set and choose one which forms an $m(w, r, k)$ maximal system with largest size. The number of possible collections is $O(2^{\binom{k}{w}})$, which is too complex to apply exhaustive search. Fortunately, it was proved that the random greedy algorithm can almost surely construct asymptotically good maximal systems [16], [17]. That is, the density $d(w, r, k)$ tends to 1 as k approaches to infinity.

In this paper, we adopt the random greedy algorithm to construct a family of $(v, w, 1, \lambda_c)$-OOCs as follows.

1. Construct a $(v, k, 1)$-perfect difference set D with k elements according to [19], where $k = q + 1$, $v = q^2 + q + 1$, and q is a power of a prime.
2. Let M denote an $m(w, \lambda_c + 1, k)$ maximal system which is empty initially.
3. Construct a complete list of candidate w-subsets of D.

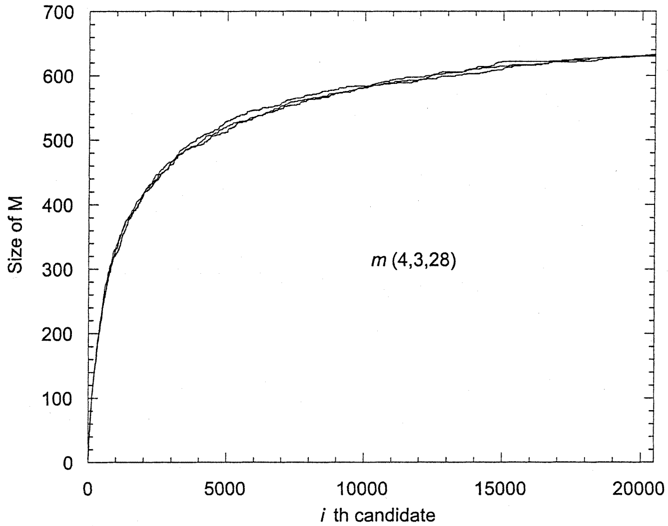

Fig. 1. Three searching results of random greedy algorithm for $m(4, 3, 28)$.

4. Pick one w-subset randomly from the list and eliminate it from the list. If the w-subset shares at most λ_c elements with all selected w-tuples in M, then include it in M, otherwise, discard it.

5. Repeat Step 4 until there are no more candidates in the list.

6. The lth subset, E^l, in M corresponds to a code C^l according to (2). All the codes C^l form a family of $(v, w, 1, \lambda_c)$-OOCs.

The crosscorrelation between any two distinct codes can be expressed as

$$\Phi = \begin{cases} 0 \text{ or } 1, & \text{two codes not aligned} \\ 0 \sim \lambda_c, & \text{two codes aligned.} \end{cases} \tag{7}$$

Since this algorithm involves randomness, the density of the system may be far from 1 with small probability. On the other hand, the density may be close to 1. Roughly speaking, the density is somewhere in between, but not very far from 1 [16], [17]. In Fig. 1, we apply this algorithm three times to form three distinct maximal systems of $m(4, 3, 28)$ with system sizes 634, 630, and 632, respectively. The total number of candidates is 20 475. Fig. 1 shows that the size of M grows quickly in the early stage of iterative due to the small size of M. However, when the size is getting larger it grows slowly because most of the candidates are discarded. This fact is helpful because we can construct the most part of codes during early searching stage especially when the number of candidates is too large to search through. Note that the optimal maximal system of $m(4, 3, 28)$ is also a Steiner system of $S(4, 3, 28)$ whose size is equal to 819. In other words, the density $d(4, 3, 28)$ is about 0.77, which is not far from 1.

III. Performance Analysis

We analyze the performance of the systems using double hard-limiters with consideration of shot noise, thermal noise, APD bulk, and surface leakage currents. We adopt $(v, w, 1, \lambda_c)$-codes as the signature codes. The receiver structure is shown in Fig. 2 [18]. To simplify the analysis, we assume that chips are synchronous among users because it is the worst case and results in an upper bound [3].

The average photon arrival rate λ per pulse is given by

$$\lambda = \frac{\eta P_W}{hf} \tag{8}$$

where η is the APD quantum efficiency, P_W is the received signal power per pulse, h is the Planck's constant, and f is the optical frequency. There are only two states after the second hard-limiter, denoted by S_1 and S_0, respectively. The state S_1 means that the average photon arrival rate is equal to λ. The other state S_0 is that the photon arrival rate is zero (this occurs only when the desired data bit is zero and the MUI is removed completely by the two hard-limiters). For states S_i, $i \in \{0, 1\}$, the probability density function of the output Y_i after the photo detector can be expressed as [23]

$$P_{Y_i}(y_i) = \frac{1}{\sqrt{2\pi\sigma_i^2}}\, e^{-(y_i-\mu_i)^2/2\sigma_i^2} \tag{9}$$

where the mean μ_i can be expressed as

$$\mu_i = GT_c(i\lambda + I_b/e) + T_c I_s/e \tag{10}$$

where G is the average APD gain, T_c is the chip duration, e is the electron charge, I_b/e is the contribution of the APD bulk leakage current to the APD output, I_s is the APD surface leakage current, and the variance σ_i^2 can be written as

$$\sigma_i^2 = G^2 F_e T_c(i\lambda + I_b/e) + T_c I_s/e + \sigma_{\text{th}}^2 \tag{11}$$

where F_e is the excess noise factor given by

$$F_e = k_{\text{eff}} G + (2 - 1/G)(1 - k_{\text{eff}}) \tag{12}$$

where k_{eff} is the APD effective ionization ratio and σ_{th}^2 is the variance of thermal noise expressed as

$$\sigma_{\text{th}}^2 = 2k_B T_r T_c/(e^2 R_L) \tag{13}$$

where k_B is Boltzmann's constant, T_r is the receiver noise temperature, and R_L is the receiver load resistance.

After the photo detector, the signal is fed into the on-off keying (OOK) decoder. If the output Y_i is larger than a constant threshold θ, we declare that the output data bit b_o is one, otherwise, zero. To minimize the error probability, we set the suboptimal value of the constant threshold θ to be

$$\theta = \frac{\mu_0\sigma_1 + \mu_1\sigma_0}{\sigma_1 + \sigma_0}. \tag{14}$$

The probability that the state S_1 (or S_0) is decoded incorrectly as $b_o = 0$ (or $b_o = 1$) can be expressed as

$$\Pr(b_o = 0|S_1) = \Pr(b_o = 1|S_0) = \frac{1}{2}\,\text{erfc}\left(\frac{\mu_1 - \theta}{\sqrt{2\sigma_1^2}}\right) \tag{15}$$

where $\text{erfc}(\cdot)$ stands for the complementary error function, defined as

$$\text{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^{\infty} \exp(-u^2)\, du. \tag{16}$$

The probability that the state S_1 (or S_0) is decoded correctly to be $b_o = 0$ (or $b_o = 1$) can be expressed as

$$\Pr(b_o = 1|S_1) = 1 - \Pr(b_o = 0|S_1) \tag{17}$$

and

$$\Pr(b_o = 0|S_0) = 1 - \Pr(b_o = 1|S_0) \tag{18}$$

respectively.

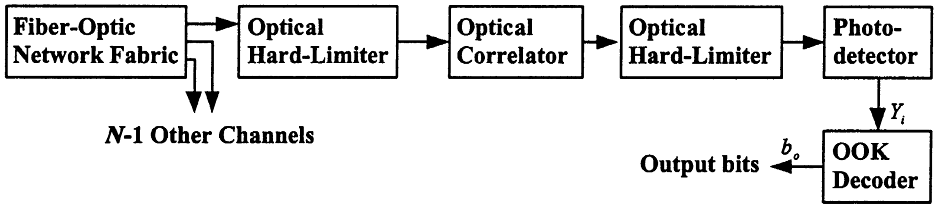

Fig. 2. Receiver structure of OCDMA systems with double hard-limiters.

A. Upper Bound of Performance

Without loss of generality, we consider the user U^1 assigned Code $C^1 = \{c_0^1, c_1^1, \ldots, c_{v-1}^1\}$ (based on Subset E^1) is the desired user and the desired data bit is b. The user U^2 assigned Code $C^2 = \{c_0^2, c_1^2, \ldots, c_{v-1}^2\}$ (based on Subset E^2) represents any other user. If their relative cyclic shift is j, $j \in \{0, 1, \ldots, v-1\}$, the crosscorrelation can be expressed as

$$I_j = \sum_{i=0}^{v-1} c_i^1 c_{i\oplus j}^2 \tag{19}$$

where $\oplus$ denotes the addition modulo v. The value of I_j is given by

$$I_j = |E^1 \cap E_j^2| \tag{20}$$

where E_j^2 is a set formed by adding j to each element of E^2 and $|\cdot|$ represents the size of a set. When $j = 0$, the value of I_j is not larger than λ_c. On the other hand, the value of I_j is not larger than 1 when $j \neq 0$.

To simplify the analysis and to derive the upper bound of performance, we assume that the crosscorrelation is always equal to λ_c when two distinct codes are aligned with each other (that is, $j = 0$). We also assume that the interfered λ_c chips are randomly distributed among w chips. Let p_1 and p_c denote the probabilities that I_j is 1 and λ_c, respectively. Because the value of I_j is λ_c only when $j = 0$, the value of p_c is $1/v$ and then $p_1 = (w^2 - \lambda_c)/v$. We have the expected value of I_j as [4]

$$E(I_j) = p_1 + \lambda_c p_c = \frac{w^2}{v}. \tag{21}$$

The probabilities that U^2 contributes 1 and λ_c pulse positions are given by

$$q_1 = \Pr(I_j = 1) = p_1/2 \tag{22}$$

and

$$q_c = \Pr(I_j = \lambda_c) = p_c/2 \tag{23}$$

respectively, where the factor 1/2 means the equiprobable 0 and 1 symbols.

If the desired data bit b is 1, the state after the second hard-limiter must be S_1, that is

$$\Pr(S_1|b = 1) = 1. \tag{24}$$

On the other hand

$$\Pr(S_0|b = 1) = 1 - \Pr(S_1|b = 1) = 0. \tag{25}$$

When the desired data bit b is 0, the state after the second hard-limiter should be S_0 if the two hard-limiters can completely eliminate MUI. However, when each of the w chips in the desired code is interfered by at least one user, the hard-limiters cannot remove MUI entirely. As a result, the state will be S_1. In the following, we derive the probability $\Pr(S_1|b = 0)$ using the principle of inclusion and exclusion.

For any i chips of w mark chips of a desired code, the probability that the i chips are not interfered by one other user is

$$F_i = 1 - q_1 - q_c + q_1 \cdot \frac{w-i}{w} + q_c \cdot \frac{\binom{w-i}{\lambda_c}}{\binom{w}{\lambda_c}}. \tag{26}$$

Given the number of simultaneous users N, the probability that the i chips are not interference by the other $N-1$ users is $(F_i)^{N-1}$ if all the N users are independent to each other. According to the principle of inclusion and exclusion, the probability $\Pr(S_1|b = 0)$ can be expressed as

$$\Pr(S_1|b = 0, N) = \sum_{i=0}^{w} (-1)^i \binom{w}{i} (F_i)^{N-1}. \tag{27}$$

On the other hand

$$\begin{aligned}\Pr(S_0|b = 0, N) &= 1 - \Pr(S_1|b = 0, N)\\ &= \sum_{i=1}^{w} (-1)^{i+1} \binom{w}{i} (F_i)^{N-1}.\end{aligned} \tag{28}$$

Therefore, the bit error probability can be written as

$$\begin{aligned}P_e =& \Pr(b_o = 0|S_1)\Pr(S_1|b = 1)\Pr(b = 1)\\ &+ \Pr(b_o = 0|S_0)\Pr(S_0|b = 1)\Pr(b = 1)\\ &+ \Pr(b_o = 1|S_1)\Pr(S_1|b = 0, N)\Pr(b = 0)\\ &+ \Pr(b_o = 1|S_0)\Pr(S_0|b = 0, N)\Pr(b = 0)\\ =& \frac{1}{2}\operatorname{erfc}\left(\frac{\mu_1 - \theta}{\sqrt{2\sigma_1^2}}\right) + \frac{1}{2}\Pr(S_1|b = 0, N)\\ &\cdot \left(1 - \operatorname{erfc}\left(\frac{\mu_1 - \theta}{\sqrt{2\sigma_1^2}}\right)\right).\end{aligned} \tag{29}$$

If $\operatorname{erfc}\left((\mu_1 - \theta)/\sqrt{2\sigma_1^2}\right) \ll 1$, (29) can be approximated as

$$P_e \cong \frac{1}{2}\operatorname{erfc}\left(\frac{\mu_1 - \theta}{\sqrt{2\sigma_1^2}}\right) + \frac{1}{2}\Pr(S_1|b = 0, N). \tag{30}$$

The first and second terms in (30) represent the noise power and interference contributions, respectively.

TABLE I
LINK PARAMETERS

Name	Symbol	Value
Light wavelength		$1.3\mu m$
APD Quantum efficiency	η	0.6
APD gain	G	100
APD effective ionization ratio	k_{eff}	0.02
APD bulk leakage current	I_b	0.1nA
APD surface leakage current	I_s	10nA
chip duration	T_c	0.1ns
bit rate	$\frac{1}{T_b} = \frac{1}{vT_c}$	
Receiver noise temperature	T_r	300k
Receiver load resistor	R_L	1030Ω

B. Lower Bound of Performance

Because we assume that the crosscorrelation is always equal to λ_c when two distinct codes are aligned with each other, the bit error probability is upper bounded by (30). On the other hand, we will derive the lower bound of performance for given codes and noise level.

Because a family of $(v, w, 1, \lambda_c)$-OOCs are formed by choosing w chips from k chips of a $(v, k, 1)$-perfect difference code, any two $(v, w, 1, \lambda_c)$-OOCs must at least share $\max(2w - k, 0)$ chips of the k mark chips. Thus, the crosscorrelation is larger than or equal to $\max(2w - k, 0)$, if two distinct codes are aligned with each other. To derive the lower bound of the BER, we assume that the crosscorrelation between any two codes is equal to $\max(2w - k, 0)$, if the two codes are aligned. From (26) and (27), we have

$$P_e > \frac{1}{2}\operatorname{erfc}\left(\frac{\mu_1 - \theta}{\sqrt{2\sigma_1^2}}\right) + \frac{1}{2}\sum_{i=0}^{w}(-1)^i\binom{w}{i} \cdot \left(1 - q_1 - q_c + q_1 \cdot \frac{w-i}{w} + q_c \cdot \frac{\binom{w-i}{\lambda_c}}{\binom{w}{\lambda_c}}\right)^{N-1} \quad (31)$$

where $\lambda_c = \max(2w - k, 0)$.

IV. NUMERICAL RESULTS

In this section, we present the numerical results of the systems with $(v, w, 1, \lambda_c)$-OOCs. The parameters used are given in Table I.

The bit error probabilities versus code weight with or without double optical hard-limiters are given in Figs. 3 and 4. Note that to simplify the calculation of the BER performance without optical hard-limiter, we assume that the maximum crosscorrelation between any two codes is one and we only take the interference contribution into consideration. These assumptions result in the lower bound of BER performance. Consider the systems with double optical hard-limiters. Part of the BER induced by the noise power is 2×10^{-11} under $P_W = 0.5\ \mu$W. In Fig. 3, the

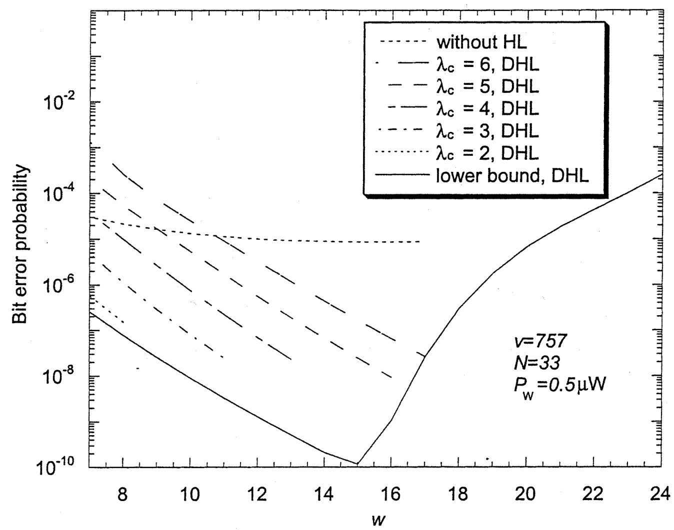

Fig. 3. Bit error probabilities versus the code weight under $v = 757, N = 33$, and $P_W = 0.5\ \mu$W.

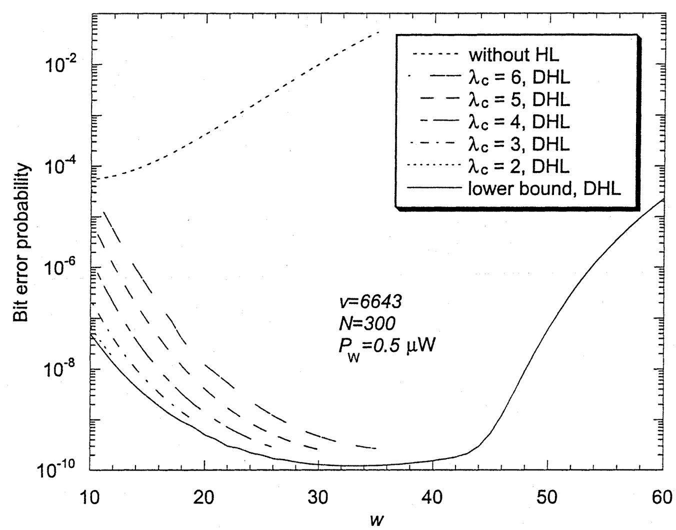

Fig. 4. Bit error probabilities versus the code weight under $v = 6643, N = 300$, and $P_W = 0.5\ \mu$W.

performance of the systems with double optical hard-limiters is improved as the code weight increases. This is because the larger code weight is more robust to interference. The performance of codes with larger λ_c is worse than that of codes with smaller λ_c. However, codes with larger λ_c have larger code size. Moreover, when $w > 15$, the lower bound of the BER increases sharply due to $\max(2w - k, 0)$ increasing with w. As a result, the lowest BER of the lower bound is about 10^{-10} under $v = 757$ and $N = 33$. In other words, given $v = 757$, the maximum number of simultaneous users is about 33, no matter what value of w. This figure also shows that the performance of the system without optical hard-limiter is not improved significantly with the increasing of code weight. The phenomenon is due to the larger the code weight, the larger the interfering probability between any two codes. Therefore, the advantage of a large code weight is diluted. Comparing the performances between systems with or without double optical hard-limiters, we find that the system with double optical hard-limiters outperforms that without any

TABLE II
UPPER BOUND OF $\overline{\overline{m}}(w, \lambda_c + 1, 82)$ VERSUS w AND λ_c UNDER BER $\leq 10^{-9}$, $N = 300$ AND $v = 6643$

w	$\lambda_c = 3$	4	5	6
19	349			
20				
21		1120		
22		916		
23		745		
24		478	2439	
25		373	1492	
26			1154	
27			962	3261
28			755	2723
29			653	2052
30			563	1716
31				1256
32				622
33				546
34				484
35				388

optical hard-limiters for the most part. In fact, under suitable signal power, the performance of the system with double optical hard-limiters outperforms that without optical hard-limiter because the former could remove some interference patterns. Fig. 4 shows the performance of $(6643, w, 1, \lambda_c)$-OOCs. Fig. 4 has similar characteristics with respect to Fig. 3 except that the performance of the system without optical hard-limiter is getting worse with the increasing code weight. The reason is similar to that in Fig. 3. However, the advantage of larger code weight cannot offset the increasing of interfering probability. On the other hand, because the interference increases due to large code weight for the system with double optical hard-limiters, there is an optimal value of code weight for a given code length and number of simultaneous users. Fig. 4 shows that the best performance is when the code weight is around 30. In such a case, the bit error probability of $(6643, 30, 1, 5)$-OOCs or $(6643, 30, 1, 6)$-OOCs is about 4.0×10^{-10} which is very close to the lower bound. Consequently, the maximum number of simultaneous users is around 300 given $v = 6643$. Fig. 4 also shows that the larger λ_c results the worse performance, especially when code weight is less than 10. However, when the code weight is larger than 20, the phenomenon is not obvious, because a large code weight dilutes the effect of interference. The upper bound of $\overline{\overline{m}}(w, \lambda_c + 1, 82)$ versus the code weight under BER $\leq 10^{-9}$, $N = 300$, and $v = 6643$ is listed in Table II. It shows that the code family of $(6643, 27, 1, 6)$-OOCs has the largest upper bound of code size.

Fig. 5 shows the bit error probabilities versus the code weight under $v = 6643$, $N = 300$, $\lambda_c = 6$, and three different values of the average power per bit. Considering the value of the average power per bit is equal to 1 nW, the BER performance is improved with the increasing of the code weight under $w < 19$ because the interference contribution in (30) is the dominant term. On the other hand, the BER performance is getting worse under $w > 19$ because the larger values of code weight reduce the signal power per pulse. As a result, the noise power contribution in (30) is getting larger and dominates the bit error probability. Similarly, when the value of the average power per bit is equal to 2 nW, the lowest BER is about 7.0×10^{-10}. However, because it has larger average power per bit, the effect of the noise power is not obvious until $w > 28$. When the value of the average power per bit is equal to 3 nW, the curve is almost the same as the corresponding one in Fig. 4. This is because the average power per bit is large enough that the noise power contribution no longer dominates the bit error probabilities.

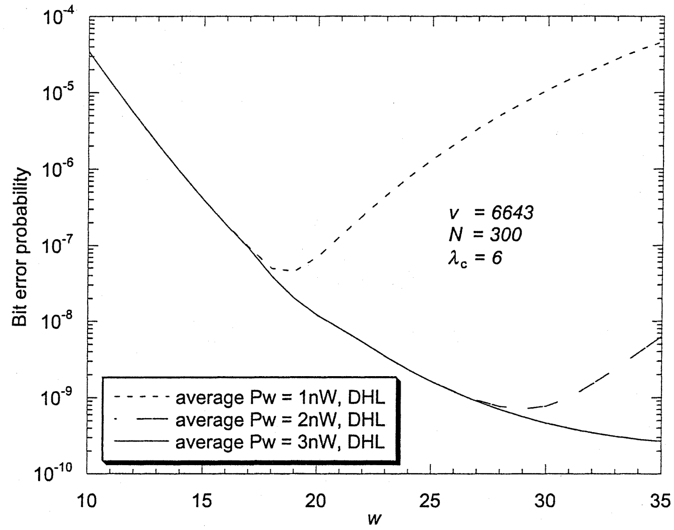

Fig. 5. Bit error probabilities versus the code weight under $v = 6643$, $N = 300$, $\lambda_c = 6$, and three different values of the average received power per bit.

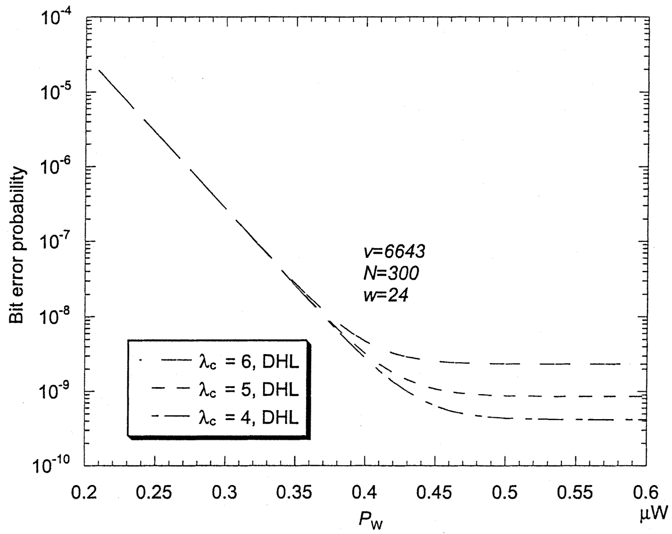

Fig. 6. Bit error probabilities versus P_W under $v = 6643$, $N = 300$, and $w = 24$.

Fig. 6 shows the bit error probabilities versus the received power per pulse under $v = 6643$, $N = 300$, and $w = 24$. The three curves illustrate that there exists an error floor which is actually equal to the interference contribution in (30).

The bit error probabilities versus number of simultaneous users are presented in Figs. 7 and 8. For the systems with double optical hard-limiters, these figures indicate that the performances of codes with larger crosscorrelation are worse than those of codes with smaller one. However, when the number of simultaneous users is large, the phenomenon is not

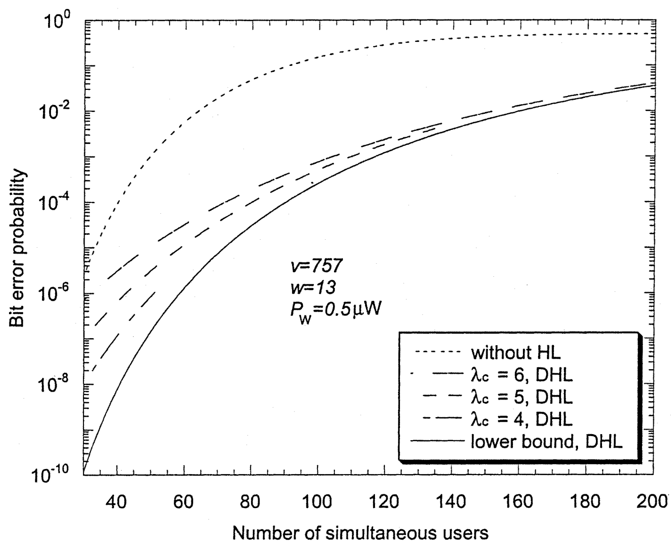

Fig. 7. Bit error probabilities versus the number of simultaneous users under $v = 757$, $w = 13$, and $P_W = 0.5$ μW.

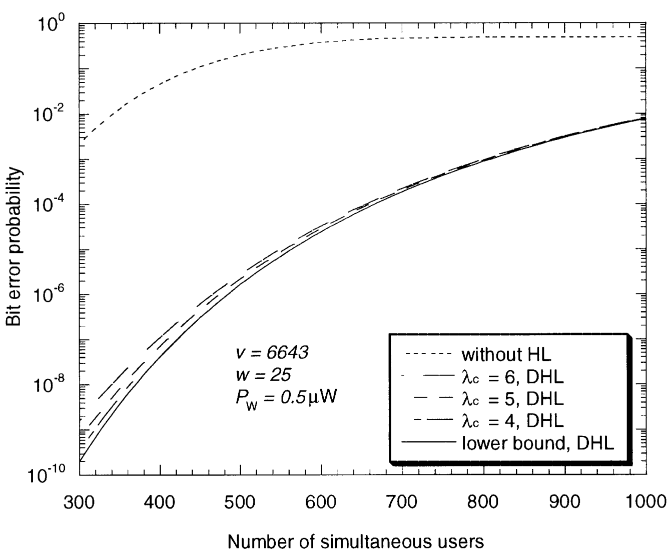

Fig. 8. Bit error probabilities versus the number of simultaneous users under $v = 6643$, $w = 25$, and $P_W = 0.5$ μW.

obvious due to q_1 dominating the BER. These figures also show that the performances of the systems with double optical hard-limiters outperform those of the systems without any optical hard-limiters.

V. Conclusion

One feasible way to improve the performances of OCDMA systems is to increase the code weight with the penalty of decreasing the code size when the code weight is less than a certain value. In this paper, we further relax the crosscorrelation constraint to get larger code size and derive the upper and lower bounds of performances. Because the optimal construction of a maximal system using exhaustive search is infeasible, we adopt the random greedy algorithm to construct $(v, w, 1, \lambda_c)$-OOCs. It is proved that this algorithm almost surely can construct asymptotically good maximal systems. An example of constructing $m(4, 3, 28)$ maximal systems shows that the system density is around 0.77, which is not far from 1. Moreover, most of the codes can be formed at early constructed stage. This fact is very helpful especially when the number of candidates is too large to search through. Because most of the crosscorrelations between any two codes are 0 or 1 (while only a very small portion have value of λ_c), the numerical results show that the larger value of crosscorrelation does not decrease the performance significantly, especially when the code weight or number of simultaneous users is large. For the systems with double optical hard-limiters, the results also imply that there is an optimal value of code weight for a given code length. We demonstrate that it is possible to approach an upper bound of the number of simultaneous users for a given code length and code size. Meanwhile, the code size is maintained on a satisfactory level.

References

[1] F. R. K. Chung, J. A. Salehi, and V. K. Wei, "Optical orthogonal codes: Design, analysis, and applications," *IEEE Trans. Inform. Theory*, vol. 35, pp. 595–604, May 1989.
[2] J. A. Salehi, "Code division multiple-access techniques in optical fiber networks-Part I: Fundamental principles," *IEEE Trans. Commun.*, vol. 37, pp. 824–833, Aug. 1989.
[3] J. A. Salehi and C. A. Brackett, "Code division multiple-access techniques in optical fiber networks-Part II: Systems performance analysis," *IEEE Trans. Commun.*, vol. 37, pp. 834–842, Aug. 1989.
[4] M. Azizoglu, J. A. Salehi, and Y. Li, "Optical CDMA via temporal codes," *IEEE Trans. Commun.*, vol. 40, pp. 1162–1170, July 1992.
[5] R. Fuji-Hara and Y. Miao, "Optical orthogonal codes: Their bounds and new optimal constructions," *IEEE Trans. Inform. Theory*, vol. 46, pp. 2396–2406, Nov. 2000.
[6] G.-C. Yang, "Variable-weight optical orthogonal codes for CDMA networks with multiple performance requirements," *IEEE Trans. Commun.*, vol. 44, pp. 47–55, Jan. 1996.
[7] S. Bitan and T. Etzion, "Constructions for optimal constant weight cyclically permutable codes and difference families," *IEEE Trans. Inform. Theory*, vol. 41, pp. 77–87, Jan. 1995.
[8] J.-G. Zhang, "Design of a special family of optical CDMA address codes for fully asynchronous data communications," *IEEE Trans. Commun.*, vol. 47, pp. 967–973, July 1999.
[9] S. V. Marić, M. D. Hahm, and E. L. Titlebaum, "Construction and performance analysis of a new family of optical orthogonal codes for CDMA fiber-optic networks," *IEEE Trans. Commun.*, vol. 43, pp. 485–489, Feb. 1995.
[10] H. Chung and P. V. Kumar, "Optical orthogonal codes—New bounds and an optimal construction," *IEEE Trans. Inform. Theory*, vol. 36, pp. 866–873, July 1990.
[11] G.-C. Yang and T. E. Fuja, "Optical orthogonal codes with unequal auto- and cross-correlation constraints," *IEEE Trans. Inform. Theory*, vol. 41, pp. 96–106, Jan. 1995.
[12] G.-C. Yang, "Some new families of optical orthogonal codes for code-division multiple-access fiber-optic networks," *IEE Proc. Commun.*, vol. 142, pp. 363–368, Dec. 1995.
[13] C.-S. Weng and J. Wu, "Optical orthogonal codes with nonideal cross-correlation," *J. Lightwave Technol.*, vol. 19, pp. 1856–1863, Dec. 2001.
[14] ——, "Perfect difference codes for synchronous fiber-optic CDMA communication systems," *J. Lightwave Technol.*, vol. 19, pp. 186–194, Feb. 2001.
[15] P. Erdös and H. Hanani, "On a limit theorem in combinatorical analysis," *Publ. Math. Debrecen*, vol. 10, pp. 10–13, 1963.
[16] J. Spencer, "Asymptotic packing via a branching process," *Random Struct. Alg.*, vol. 7, pp. 167–172, 1995.
[17] V. Rödl and L. Thoma, "Asymptotic packing and the random greedy algorithm," *Random Struct. Alg.*, vol. 8, pp. 161–177, 1996.
[18] T. Ohtsuki, "Performance analysis of direct-detection optical asynchronous CDMA systems with double optical hard-limiters," *J. Lightwave Technol.*, vol. 15, pp. 452–457, Mar. 1997.
[19] J. Singer, "A theorem in finite projective geometry and some applications to number theory," *Trans. Amer. Math. Soc.*, vol. 43, pp. 377–385, 1938.

[20] J. Schönheim, “On maximal systems of k-tuples,” *Studia Sci. Math. Hungar*, vol. 1, pp. 363–368, 1966.
[21] H. Mohácsy and D. K. Ray-Chaudhuri, “A construction for infinite families of Steiner 3-designs,” *J. Combin. Theory, Ser. A*, vol. 94, pp. 127–141, 2001.
[22] V. Rödl, “On a packing and covering problem,” *Eur. J. Combin.*, vol. 5, pp. 69–78, 1985.
[23] H. M. Kwon, “Optical orthogonal code-division multiple-access system—Part I: APD noise and thermal noise,” *IEEE Trans. Commun.*, vol. 42, pp. 2470–2479, July 1994.

Chi-Shun Weng was born in Tainan, Taiwan, R.O.C., in 1976. He received the B.S. and Ph.D. degrees in electrical engineering from National Taiwan University, Taipei, Taiwan, R.O.C., in 1998 and 2002, respectively.

He is currently with Realtek Semiconductor Corporation, Hsinchu, Taiwan, R.O.C., as a Digital IC Design Engineer. His research interests include lightwave communication systems, spread-spectrum communication, and coding theory.

Mr. Weng received the Gold Award from the Asian Pacific Mathematics Olympiad in 1993.

Jingshown Wu (S’73–M’78–SM’99) received the B.S. and M.S. degrees in electrical engineering from National Taiwan University, Taipei, Taiwan, R.O.C., in 1970 and 1972, and the Ph.D. degree from Cornell University, Ithaca, NY, in 1978.

He joined Bell Laboratories in 1978, where he worked on digital network standards and performance and optical communication systems. In 1984, he joined the Department of Electrical Engineering, National Taiwan University, as a Professor, and was the Chairman of the Department from 1987 to 1989. He was also the Director of the Communication Research Center, College of Engineering, from 1992 to 1995. From 1995 to 1998, he was the Director of the Division of Engineering and Applied Science, National Science Council, Taiwan, R.O.C., on leave from the university. From 1999 to 2002, he was the Chairman of the Commission on Research and Development and the Director of the Center for Sponsor Programs of the university. Currently, he is the Vice-President of the university. He has published more than 100 journal and conference papers and holds 12 patents. His research interests include optical fiber communications, computer communications, and communication systems.

Dr. Wu is a Life Member of the Chinese Institute of Engineers, the Optical Society of China, and the Institute of Chinese Electrical Engineers. He has served as the Vice Chairman (1997–1998) and the Chairman (1998–2000) of IEEE, Taipei Section.