A Robust Timing Synchronization Scheme in Multiple Antenna Systems With Doppler Frequency Shifts

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***Abstract*—In this letter, we propose a timing synchronization scheme for a dual antenna system in Rayleigh-fading environments. Instead of assuming the channel gain to be constant during the training duration, we consider the time-variant nature of the multiplicative distortions and formulate them with linear combinations of eigenfunctions. Then, we derive a formula to find the maximum-likelihood estimation of the channel timing and simulate the performance with both ideal and nonideal channel state information. The results show that this approach outperforms the conventional ones especially when the Doppler spread is severe.**

***Index Terms*—Antenna arrays, fading channels, MIMO, timing synchronization.**

I. Introduction

IN A MULTIPLE antenna system, timing error can cause severer degradation than the single antenna case since intersymbol interference affects all antennas at the same time. Traditionally, symbol timing recovery can be obtained based on the maximum-likelihood (ML) criterion and implemented in the open-loop or closed-loop form. For example, the well-known early-late gate method and the wave different method (WDM) are closed-loop timing synchronizers [1]. On the other hand, the timing estimator developed in [2] is an open-loop timing estimator for a dual antenna system. However, these estimators are obtained under the assumption of quasi-static channel in the training duration. When the Doppler frequency shift is severe, the quasi-static assumption may result in significant error such that the performance deteriorates with broadened Doppler bandwidth. In this paper, we take the time variation of the channel gains into consideration and formulate the multiplicative distortions with linear combinations of their eigenfunctions. Based on this scheme, we derive the ML estimate of the symbol timing. The performance of this scheme is numerically simulated and tested under nonideal channel state information (CSI) and channel model mismatch. The results show that the proposed scheme outperforms the conventional one.

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II. System Description and the Eigenfunction Decomposition Approach

The communication system under consideration has two transmitting antennas and two receiving antennas. The transmitting antennas send the training signals as

$$x_a(t) = \sum_{k=1}^{L} x_{ak} p(t - kT_s), \qquad a = 1, 2 \tag{1}$$

where L denotes the length of the training sequence, a is the index number of transmitting antenna, T_s is the symbol time duration, x_{ak} is the training sequence, and $p(t)$ is the square-root raised cosine pulse shaping function with roll-off factor ζ. The received signal $z_b(t)$ at receiving antenna is modeled as

$$z_b(t) = \sum_{a=1}^{2} x_a(t - \varepsilon)\alpha_{ab}(t - \varepsilon) + n_b(t), \qquad b = 1, 2 \tag{2}$$

where $\alpha_{ab}(t)$ is multiplicative distortion, $n_b(t)$ is the additive noise and ε is channel delay. $\alpha_{ab}(t)$ is modeled as zero mean cyclic symmetric complex Gaussian random process having identical autocorrelation function [2] as

$$R_\alpha(t, u) = E[\alpha_{ab}(t)\alpha_{ab}^*(u)] = \sigma_\alpha^2 J_0(2\pi f_d(t - u)) \tag{3}$$

where $E[\cdot]$ denotes the expectation over the ensembles, $*$ means complex conjugate, $J_0(\cdot)$ represents the zero order Bessel function of the first kind, f_d is the maximum Doppler frequency, and $n_b(t)$ is cyclic symmetric white Gaussian random processes with autocorrelation function $R_n(t, u) = N_0\delta(t - u)$. In addition, all of these random processes are assumed to be independent. As in [3], it is possible to find the eigenfunctions, $f_k(t)$, which satisfy

$$\int_{T_i}^{T_f} R_\alpha(t, u) f_k(u) du = \lambda_k f_k(t) \tag{4}$$

where λ_k are the corresponding nonnegative real eigenvalues. T_i and T_f are the initial time and final time of the observation duration, respectively. $f_k(t)$ and λ_k are obtained numerically by approximating (4) with a discrete one with sufficient resolution. Then, we transform it into the matrix form and find the eigenvectors and eigenvalues of the autocorrelation matrix [4], [5]. Assuming $\lambda_k \geq \lambda_{k+1}$, $\alpha_{ab}(t)$ can be expressed as

$$\alpha_{ab}(t) = \lim_{M \to \infty} \alpha_{abM}(t) \tag{5}$$

where

$$\alpha_{abM}(t) = \sum_{k=1}^{M} c_{abk} f_k(t) \tag{6}$$

and c_{abk} is complex Gaussian random variable with variance λ_k. If $\lambda_k = 0$ for some k, the limiting operation in (5) is removed and M is taken as the maximum integer that renders λ_k positive. Now, our objective is to estimate the channel delay ε given the received signal, $z_b(t)$, $b = 1, 2$. By an extension to the derivation in [6] and that $n_b(t)$ and c_{abk} are independent, we can get the joint probability density function of c_{abk} and ε, conditioned on $z_b(t)$ and $x_a(t)$. By taking logarithm of the probability density function and neglecting irrelevant constants and scaling, we obtain the likelihood function Γ as

$$\Gamma = \lim_{M\to\infty} -\left\{\sum_{b=1}^{2}\left(\frac{1}{N_0}\int_{T_i}^{T_f}|n_b(t)|^2 dt + \sum_{k=1}^{M}\sum_{a=1}^{2}\frac{|c_{abk}|^2}{\lambda_k}\right)\right\}. \quad (7)$$

Now, we can apply (2)(5)(6) and rewrite the last term of (7) in matrix form to obtain

$$\Gamma = \lim_{M\to\infty}\Gamma_M, \quad (8)$$

$$\Gamma_M = -\left\{\sum_{b=1}^{2}\left(\frac{1}{N_0}\int_{T_i}^{T_f}\left|z_b(t+\varepsilon) - \sum_{a=1}^{2}x_a(t)\alpha_{abM}(t)\right|^2 dt + \sum_{a=1}^{2}C_{abM}^H\Lambda_M^{-1}C_{abM}\right)\right\} \quad (9)$$

where C_{abM} and Λ_M are a column vector and a diagonal matrix given by $C_{abM} = [\,c_{ab1} \quad c_{ab2} \quad \ldots \quad c_{abM}\,]^T$ and $\Lambda_M = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_M)$. The superscripts T and H denote the transpose and Hermitian transpose, respectively. Here, we have used the fact that a shift in $n_b(t)$ does not change its statistical property. To maximize Γ_M, we obtain the ML channel estimate as

$$C_{bM} = (R + N_0\Lambda_{M2}^{-1})^{-1}V_{bM}(\varepsilon) \quad (10)$$

where $C_{bM} = [\,C_{1bM}^T \quad C_{2bM}^T\,]^T$ and $\Lambda_{M2} = \mathrm{diag}\,[\,\Lambda_M \quad \Lambda_M\,]$. R is a $2M \times 2M$ matrix with elements r_{mn} given by

$$r_{mn} = \int_{T_i}^{T_f} x_{a_1}(t)f_{k_1}(t)x_a^*(t)f_k^*(t)dt \quad (11)$$

where $m = (a-1)M + k, n = (a_1 - 1)M + k_1, a, a_1 = 1 \sim 2$, $k, k_1 = 1 \sim M$. $V_{bM}(\varepsilon)$ is defined as

$$V_{bM}(\varepsilon) = [\,v_{b11}\; v_{b12} \ldots v_{b1M} \quad v_{b21}\; v_{b22} \ldots v_{b2M}\,]^T \quad (12)$$

where

$$v_{bak} = \int_{T_i}^{T_f} z_b(t+\varepsilon)x_a^*(t)f_k^*(t)dt. \quad (13)$$

Based on (5) through (13), we find the ML estimate of ε in (14), shown at the bottom of the page.

$$\varepsilon_{ML} = \arg\max_{\varepsilon}\left\{\lim_{M\to\infty}\sum_{b=1}^{2}\left[V_{bM}^H(\varepsilon)(R + N_0\Lambda_{M2}^{-1})^{-1}V_{bM}(\varepsilon) - \int_{T_i}^{T_f}|z_b(t+\varepsilon)|^2 dt\right]\right\}. \quad (14)$$

Note that the last term in (14) is independent of ε for proper choices of T_i and T_f. As a result, (14) is simplified as

$$\varepsilon_{\mathrm{ML}} = \arg\max_{\varepsilon}\left[\lim_{M\to\infty}\sum_{b=1}^{2}V_{bM}^H(\varepsilon)(R + N_0\Lambda_{M2}^{-1})^{-1}V_{bM}(\varepsilon)\right]. \quad (15)$$

III. Simulation Results and Discussion

Depending on the bandwidth of the random process, it is sufficient to well approximate $\alpha_{ab}(t)$ with a small M [3]. Therefore, with an appropriate value of M, the limit operation in (15) can be removed with only negligible degradation in performance. We find that for normalized maximum Doppler frequency, $f_n = f_d T_s$, less than 0.05, the first two eigenvalues consist of more than 85% of the total energy. In the following simulations, we show that with $M = 2$, the estimation error is greatly reduced compared with the conventional ML based on quasi-static assumption for normalized maximal Doppler frequency f_n as large as 0.06.

A. Comparison With the Conventional Approach

The conventional scheme uses simple correlation to acquire the estimate of ε[2]. This is equivalent to have $M = 1$ and approximate $f_1(t)$ as a constant function in (6) without the limit operation in (5). In Fig. 1, we compare the proposed scheme with $M = 2$ to the conventional correlation approach. The parameters of the simulation are given as follows. The roll-off factor is 0.3, and the pulse shaping function $p(t)$ is truncated to $\pm 3T_s$ around $t = 0$. The continuous waveform is represented by 32 samples per symbol. The acquisition interval is assumed to be $\pm 1.5T_s$ around the correct timing delay. The orthogonal training sequence pair is $x_{1k} = [+ - - - - + - - + + + -]$ and $x_{2k} = [+ + - + + - - - - + - ++]$. We can see that the conventional approach deteriorates seriously as the maximum Doppler frequency shift increases, while the proposed method retains reasonable performance. We can see that for normalized frequency as large as 0.05, the proposed method with $Es/No = 4$ dB has the same performance as the conventional method with $Es/No = 10$ dB. The improvement is more significant at high SNR.

B. Sensitivity to Non-Ideal CSI and Channel Model Mismatch

As suggested in (15), the timing synchronization requires the knowledge of signal to noise ratio and the maximum Doppler frequency shift. Thus, we investigate sensitivity of the proposed method to the nonideal CSI. In Fig. 1, we plot the performance of the proposed method with nonideal information of SNR. The systems are designed with Es/No 8 dB or 20 dB larger than

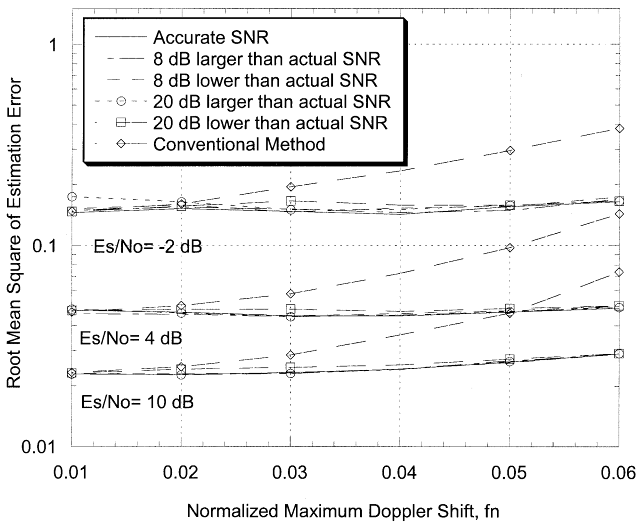

Fig. 1. Performance comparison with the conventional scheme for various inaccurate SNR values.

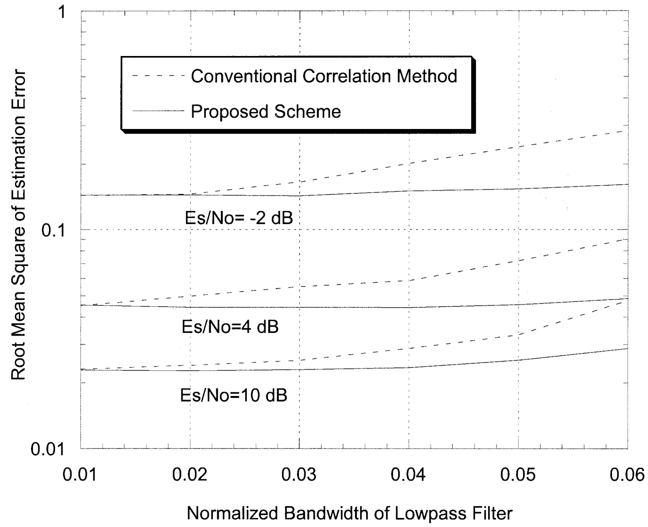

Fig. 3. Performance comparison with channel model mismatch.

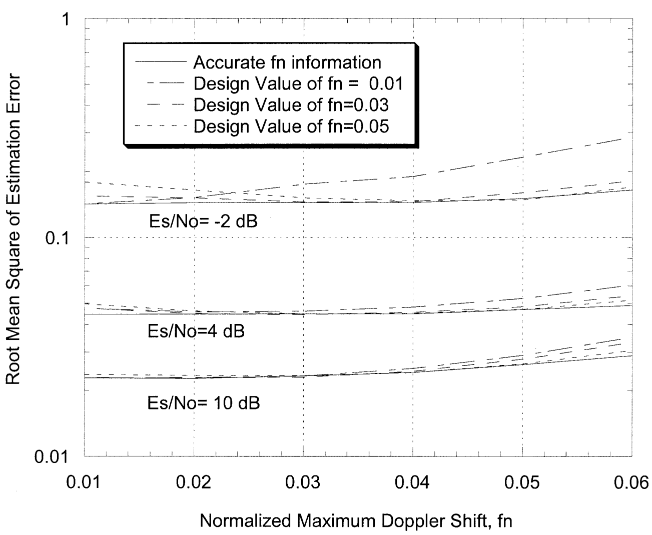

Fig. 2. Performance for various designed value of f_n.

or lower than the actual Es/No. Fig. 2 depicted the performance of the estimators with a fixed design value of f_n in various Doppler frequency spread environments. In addition, we also investigate the effect of model mismatch. We feed white noise into ideal lowpass filters to obtain the multiplicative distortion $\alpha_{ab}(t)$ while the system is designed under assumption of Jakes Doppler spectrum with $f_n = 0.03$. The results are compared to the conventional correlation approach and illustrated in Fig. 3. It shows that the proposed method is also insensitive to model mismatch. The reason for the insensitivity of model mismatch is explained as follows. The first eigenfunction is very close to a constant and the second eigenfunction is similar to a straight line. The mismatched model just changes the corresponding eigenvalues so that the importance of each component is slightly mis-weighted.

C. Implementation Complexity

Since this scheme is insensitive to model mismatch and CSI, we can choose the average value of the SNR and f_n for the target operating environment. Thus, we can find $f_k(t)$ and compute $(R+N_0\Lambda_{M2}^{-1})^{-1}$ beforehand and store them in ROM. Special care can be taken to make nondiagonal elements of $(R+N_0\Lambda_{M2}^{-1})^{-1}$ negligibly small to reduce the computation. $V_{bM}(\varepsilon)$ must be calculated via (12) and (13) in real time, which dominate the computational complexity of this scheme. Compared to the conventional correlation method with the same oversampling ratio, it takes approximately twofold multiplications and additions for $M = 2$.

IV. Conclusion

We have proposed a timing synchronization scheme using eigenfunction decomposition of the multiplicative channel distortions. The results show that the proposed scheme performs much better than the conventional approach. In addition, it is quite robust with nonideal channel state information and channel model mismatch. The derivation and discussion in this letter can be easily extended to multiple-input multiple-output systems. This scheme can be applied in burst-based transmission in multiple-input multiple-output systems in high Doppler bandwidth environments.

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