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Why Does the Government Provide Tax Deductions for the Individual's Net Losses?

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Abstract

This paper shows that providing tax deductions for individuals' net losses could be optimal for the government when (1) the government's aggregated utility function is more risk-averse than that of the representative individual, or (2) the insured is overly optimistic with regard to the loss probability. The results of this paper could be further applied to explain why a government provides supplementary public insurance or government relief.

Keywords: Tax deduction, insolvency risk, non-expected utility

JEL classification: G22, H24, D50

摘 要

本論文提出兩個理由以解釋爲何政府提供個人淨損失稅額抵減爲社會最適決策，此兩個理由爲: (1) 政府的總和效用函數較代表性個人的效用函數更爲風險趨避；(2) 代表性個人對於損失發生機率的看法過於樂觀，本論文的發現可以更近一步解釋爲何政府應提供補助性公共保險以及社會救助。

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1. Introduction

In the United States and many other countries in the world, individuals are able to deduct some of their net losses while filing their income tax returns. The total amount of the tax deductions for individuals' medical and dental expenses in the United States was about $61.5 billion in 2004, and that for casualty and theft loss deductions was about $3.5 billion. However, the literature has shown that tax deductions for individuals' net losses are unnecessary when a private insurance market is available. Specifically, Kaplow (1992b) indicates that a non-zero tax deduction rate for individuals' net losses is Pareto-inferior to a zero tax deduction rate even after considering moral hazard, administration costs, and some imperfections in the private insurance market. Thus, an interesting research question that arises owing to this gap between the reality and the literature is as follows: Why would a government provide tax deductions for the individual's net losses?

This paper attempts to put forward two reasons for this. We show here that the government could have the incentive to provide tax deductions for individuals' net losses when (1) the government's aggregated utility function is more risk-averse than that of the representative individual, or (2) the insured is overly optimistic with regard to the loss probability.[1]

The first case is concerned with the government's risk attitude. There are several reasons that a government may have its own objective function, and it may differ from individual's objective function. The first one is aggregation problem. A social planner would experience a huge utility deduction if large numbers of individuals were to suffer from a catastrophe at the same time. However, individuals could ignore others' utility and might make their decisions only on the basis of maximizing their own utility. Indeed, a government could intend to maximize the aggregated utility of all individuals. Thus, when the risks faced by individuals are highly correlated, such as a huge tornado, an earthquake, or terrorist attacks, a government may become more risk averse and wish to provide more protection to individuals. Therefore, the government's objective function may differ from that of the representative individual. The second one is agency

[1] The main purpose of this paper is to provide rationale for a government to establish a tax deduction system in the absence of asymmetric information. Researchers concern this issue in the presence of adverse selection could refer to Kim and Schlesinger (2005).

problem. In a democracy country, the party which protects individuals from risk may win in elections. In order to win elections, the government could be more risk averse than individuals while making decisions. These concerns motivate us to alter Kaplow's (1992b) assumption. Kaplow (1992b) assumed that there is a representative individual, and that the government's objective is to maximize that individual's utility function. In this paper, we consider the social utility function to be a function of the representative agent's utility function, and show that the government would like to provide more insurance to the public, if the social utility function is a concave transformation of the insured's utility.

The second case concerns the insured's risk assessment. Kaplow (1992b) assumed that the individual makes an insurance decision on the basis of objective loss probability. However, the insured could make decisions regarding his/her own subjective probability rather than his/her objective probability, as proposed by Ramsey (1931) and Savage (1954). If the individual's risk assessment is more optimistic than the objective probability but the insurer prices the insurance on the basis of the objective probability, the individual could purchase less than full coverage, even under an actuarially fair price. If the government has access to information on the objective loss probability, insurance coverage in the private market may not be sufficient from the social planner's point of view. Thus, the government would "correct" the insured's choice and provide additional insurance by allowing tax deductions for net losses.

In both cases, the objective function of the government is not the same as the objective function of the representative individual. Is this possible? The answer is a resounding "Yes". For example, a government needs the support of voters, as pointed out by many social theorists (for example, see McKelvey, 1979; Coughlin and Nitzan, 1981; Caplin and Nalebuff, 1988 and 1991, etc.). Thus, it will maximize a social welfare function, which is a combination of all citizens' utilities, but is not necessarily equal to the simple summation of them as demonstrated by Diamond and Mirrlees (1979). Therefore, in our first case, we consider the government's objective function to be a concave transformation of the individual's utility function. In the second case, the government is concerned with the representative agent's utility. However, the government's objective function is formed according to the objective risk probability, rather than the subjective probability as in the case of the agent.

The results of this paper are applied further to analyze the rationale behind supplementary public insurance and government relief. Although our paper focuses mainly on whether tax deductions for individuals' net losses should be optimal for the government, our results can be easily extended to explain, at least in part, why a government provides supplementary social insurance or relief for catastrophic losses.

The remainder of this paper is organized as follows. Section 2 discusses how the government's aggregate utility function affects optimal tax deduction decisions. Section 3 analyzes the impact of the government's risk assessment on the government's decisions regarding tax deduction. Section 4 discusses the applications of this paper and concludes the paper.

2. The Government's Risk Attitude

To find the optimal tax deduction for net losses, a two-stage game is constructed. In the first stage, the government provides a proportional tax deduction rate t for an individual's net losses; and the deduction is self-financed by a lump-sum tax τ, as in Kaplow (1992b).

In the second stage, in observing the level of the tax deduction rate, the representative individual and the insurance company sign an optimal insurance contract under a perfectly competitive insurance market. Assume that the insured suffers a fixed loss amount L with a probability π. With complete information, the insurance premium P is $\lambda \pi Q$, where λ is a loading factor and Q denotes the amount of coverage the insured chooses. Assume that $\lambda > 1$ and $Q \leq L$. The insured is strictly risk-averse with a von Neumann-Morgenstern utility of wealth, represented by a twice differentiable function u with $u' > 0$, $u'' < 0$. Let w be the insured's initial wealth; thus the insured's final wealth is

$$\begin{cases} w - \lambda \pi Q - \tau & \text{if no loss occurs;} \\ w - (L - Q)(1 - t) - \lambda \pi Q - \tau & \text{if loss occurs.} \end{cases}$$

The optimal tax deduction can be solved by means of backward induction. In the second stage, the insured chooses an optimal level Q, taking t and τ as given[2], to maximize his/her expected utility,

[2] In the second stage, t and τ is observed by individuals. Thus, we do not need to consider the government's budget constraint.

$$Eu = (1-\pi)u(w - \lambda\pi Q - \tau) + \pi u(w - (L-Q)(1-t) - \lambda\pi Q - \tau)\,. \tag{1}$$

The first-order condition for an interior solution is

$$\Gamma = -\lambda\pi(1-\pi)u'_N + \pi(1-t-\lambda\pi)u'_L = 0\,, \tag{2}$$

where the subscripts N and L denote the no-loss state and loss state, respectively. The second-order condition holds and is equal to

$$\frac{\partial\Gamma}{\partial Q} = \lambda^2\pi^2(1-\pi)u''_N + \pi(1-t-\lambda\pi)^2 u''_L < 0\,. \tag{3}$$

From Equation (2), we know that if $t \geq 1-\lambda\pi$, then the private insurance will be totally destroyed by the tax deduction system.

Now, let us move to the first stage: the government's decision. Most of the previous research has examined the optimal fiscal policy by assuming a utility-maximizing government. Standing in a neutral role, the government cares only about the representative individual. However, the government is a complex institution and frequently generates fiscal decisions across different minds, as Wagner (1997) argued. Adopting this consideration, we assume the government's utility function is a function (v with $v' > 0$) of the insured's utility, not necessarily equal to the representative insured's utility function. Thus, the analysis of Kaplow (1992b) will be a special case where v is a linear function of the insured's utility.

The optimal tax deduction problem, as described in Equations (4) and (5), becomes

$$\max_t \quad (1-\pi)v\big(u(w-\lambda\pi Q^* - \tau)\big) + \pi v\big(u(w-(L-Q^*)(1-t) - \lambda\pi Q^* - \tau)\big), \tag{4}$$

$$s.t. \qquad \tau = \lambda\pi(L-Q^*)t\,, \tag{5}$$

where Q^* is the optimal insurance coverage in the second stage, which satisfies Equation (2), and Equation (5) represents the government's budget constraint. Here we assume that the government loading is the same as the insurance loading. Since Kaplow (1992b) has documented that efficiency could be one of the reasons for government to provide tax deduction, assuming the same loading could have a clear view about the effect of government's risk attitude on optimal tax deduction.

The first-order condition for the optimization problem is

$$\Lambda = -(1-\pi)(\lambda\pi \frac{\partial Q^*}{\partial t} + \frac{\partial \tau}{\partial t}) u'_N v'_N$$
$$+ \pi \left[L - Q^* + (1-t-\lambda\pi)\frac{\partial Q^*}{\partial t} - \frac{\partial \tau}{\partial t} \right] u'_L v'_L \,, \tag{6}$$

$$= \left(-\lambda\pi(1-\pi)u'_N v'_N + \pi(1-t-\lambda\pi)u'_L v'_L \right) \frac{\partial Q^*}{\partial t}$$
$$- \left((1-\pi)u'_N v'_N + \pi u'_L v'_L \right) \frac{\partial \tau}{\partial t} + \pi(L-Q^*)u'_L v'_L \,, \tag{7}$$

where

$$\frac{\partial \tau}{\partial t} = \lambda\pi(L-Q^*) - \lambda\pi \frac{\partial Q^*}{\partial t} t \,. \tag{8}$$

From Equation (7), Proposition 1 is developed.

Proposition 1

Given that $Q^* - \left.\frac{\partial Q^*}{\partial t}\right|_{t=0} < L$ *and* $\lambda\pi < 1$, *if the government's utility is a strictly concave transformation of the insured's utility, then a non-zero tax deduction is socially optimal.*

Proof Substituting Equation (2) into Equation (7) yields

$$\Lambda = \pi(1-t-\lambda\pi)u'_L(v'_L - v'_N)\frac{\partial Q^*}{\partial t} + \lambda\pi\left((1-\pi)u'_N v'_N + \pi u'_L v'_L\right)\frac{\partial Q^*}{\partial t} t$$
$$+ \pi u'_L (L-Q^*)\left(-(1-t-\lambda\pi)v'_N + (1-\lambda\pi)v'_L \right), \tag{9}$$

Evaluating Equation (9) at $t = 0$, we have

$$\Lambda\big|_{t=0} = \pi(1-\lambda\pi)u'_L(v'_L - v'_N)(L - Q^* + \left.\frac{\partial Q^*}{\partial t}\right|_{t=0}) \,. \tag{10}$$

Since we assume

$$Q^* - \left.\frac{\partial Q^*}{\partial t}\right|_{t=0} < L \text{ and } \lambda\pi < 1, \tag{11}$$

the condition that leads to the existence of a non-zero tax deduction rate is

$$v'_L > v'_N . \tag{12}$$

Condition (12) holds for a strictly concave function of v.

Q.E.D.

Proposition 1 demonstrates a rationale for the existence of the tax deduction system. The optimal tax deduction rate will be positive if the government's utility is a strictly concave transformation of the individual's utility. In other words, the government is more risk-averse than the individual. It should be noted that the conditions in Proposition 1, $Q^* - \left.\frac{\partial Q^*}{\partial t}\right|_{t=0} < L$ and $\lambda\pi < 1$, generally hold for most insurance, since, in reality, insurance coverage is usually less than the full loss and the premium rate times the loading factor cannot be greater than one.

One reason why the government behaves in a more risk-averse manner stems from the aggregation problem. When there is more than one insured in the market, the government's decision depends on the aggregation of all the individuals' utilities. When the aggregation is not a simple summation of all identical utilities, we cannot assume that the government's utility is the same as that of the representative individual when solving the government's optimization problem. Indeed, in some non-linear aggregation cases, the government's utility might look like a utility that is a strictly concave transformation of the representative agent's utility, especially in a dependent risk case.

Let us use an example to demonstrate the case described above. In a market with two individuals who are insured, assume that the government aggregates the utility of agents by means of a Cobb-Douglas form, i.e.,

$$v(u_1, u_2) = u_1^a u_2^a, \quad a < \frac{1}{2}, \tag{13}$$

where u_i represents the utility function of agent i, $i = 1,2$. If we assume that the utility functions of these two agents are identical, i.e., $u_1(z) = u_2(z) = u(z)$, and their losses always occur at the same time, then the government’s expected utility will become

$$(1-\pi)\big(u(w-\lambda\pi Q-\tau)\big)^{2a} + \pi\big(u(w-\lambda\pi Q-(L-Q)(1-t)-\tau)\big)^{2a}. \tag{14}$$

The expected utility in Equation (14) can be expressed as the expected utility of a government whose utility is equal to u^{2a}, a strictly concave transformation of the representative agent’s utility u.

3. The Insured’s Risk Assessment

Adopt the game structure and all the notations in the previous section. Now, let us assume that the insured’s optimal choice is determined by his/her subjective probability of risk occurrence, $g(\pi)$, rather than the objective probability π. However, the insurer and the government price the insurance and finance the tax deductions based on the objective probability. Thus, the insured’s expected utility is

$$Eu = (1-g(\pi))u(w-\lambda\pi Q-\tau) + g(\pi)u(w-(L-Q)(1-t)-\lambda\pi Q-\tau). \tag{15}$$

In the second stage, under a given t and τ, the insured chooses an optimal insurance amount to maximize his/her expected utility. The optimal insurance contract Q^* in the private insurance market satisfies the following equation:

$$\Gamma = -\lambda\pi(1-g(\pi))u'_N + g(\pi)(1-t-\lambda\pi)u'_L = 0. \tag{16}$$

As predicted by Kaplow (1992b), in this case the optimal coverage in the private insurance market will still decrease due to the existence of the tax deduction by adopting the implicit function theory, i.e.,

$$sign\{\frac{\partial Q^*}{\partial t}\} = sign\{\frac{\partial\Gamma}{\partial t}\} = sign\Big\{g(\pi)\Big[-u'_L + u''_L(L-Q^*)(1-t-\lambda\pi)\Big]\Big\} < 0. \tag{17}$$

In the first stage, the government will choose an optimal tax deduction rate to maximize its own objective function. For simplicity, assume that the government’s objective function is the same as the representative individual’s utility. Since the government makes decisions based upon the objective probability, the optimal tax deduction problem becomes

$$\max_{t} \quad (1-\pi)u(w-\lambda\pi Q^{*}-\tau)+\pi u(w-(L-Q^{*})(1-t)-\lambda\pi Q^{*}-\tau)\ , \tag{18}$$

$$s.t. \qquad \tau=\lambda\pi(L-Q^{*})t\ , \tag{19}$$

where Q^{*} satisfies Equation (16), the first-order condition in the private insurance market. Thus, we can find the conditions that make a non-zero tax deduction rate optimal.

Proposition 2

Given that $Q^{*}-\left.\frac{\partial Q^{*}}{\partial t}\right|_{t=0}<L$, *if* $g(\pi)<\pi$, *then a non-zero tax deduction is socially optimal.*

Proof The first-order condition of the optimal taxation problem is

$$\Lambda=(1-\pi)u_N'\left[-\lambda\pi\frac{\partial Q^{*}}{\partial t}-\frac{\partial\tau}{\partial t}\right]+\pi u_L'\left[(1-t-\lambda\pi)\frac{\partial Q^{*}}{\partial t}-\frac{\partial\tau}{\partial t}+(L-Q^{*})\right], \tag{20}$$

$$=\left(-\lambda\pi(1-\pi)u_N'+\pi(1-t-\lambda\pi)u_L'\right)\frac{\partial Q^{*}}{\partial t}-\left((1-\pi)u_N'+\pi u_L'\right)\frac{\partial\tau}{\partial t}+\pi u_L'(L-Q^{*})\ , \tag{21}$$

where

$$\frac{\partial\tau}{\partial t}=\lambda\pi(L-Q^{*})-\lambda\pi\frac{\partial Q^{*}}{\partial t}t\ . \tag{22}$$

Substituting Equations (16) and (22) into Equation (21) yields

$$\Lambda=(1-t-\lambda\pi)u_L'\left(\pi-\frac{1-\pi}{1-g(\pi)}g(\pi)\right)\frac{\partial Q^{*}}{\partial t}+\lambda\pi\left((1-\pi)u_N'+\pi u_L'\right)\frac{\partial Q^{*}}{\partial t}t$$

$$+u_L'(L-Q^{*})\left(-\frac{1-\pi}{1-g(\pi)}g(\pi)(1-t-\lambda\pi)+\pi(1-\lambda\pi)\right). \tag{23}$$

Evaluating Equation (23) at $t=0$, we have

$$\Lambda\Big|_{t=0} = (1-\lambda\pi)u'_L\left(\pi - \frac{1-\pi}{1-g(\pi)}g(\pi)\right)\times\left[L - Q^* + \frac{\partial Q^*}{\partial t}\Bigg|_{t=0}\right]. \tag{24}$$

Since we assume that $Q^* - \frac{\partial Q^*}{\partial t}\Big|_{t=0} < L$, the condition that makes $\Lambda\big|_{t=0} > 0$ is

$$\pi - \frac{1-\pi}{1-g(\pi)}g(\pi) > 0 \tag{25}$$

or, $g(\pi) < \pi$.

Q.E.D.

Proposition 2 demonstrates that the government would provide tax deductions for the individual's net losses if the government were to think that the insured underestimates the probability of risk occurrence; or we could say that the insured is more optimistic than the insurer and the government. When the insured is more optimistic, the government will think that the individual will choose less coverage in the private market than if s/he has objective expectations. Thus, the government could have an incentive to provide the tax deduction system as additional insurance coverage.

4. Conclusion

This paper provides two possible reasons for the existence of tax deductions for individuals' net losses. The government could consider that the tax deduction system is optimal from the government's point of view when (1) the government is more risk-averse than individuals, or (2) the insured are more optimistic than the insurer or the government. In the first case, the government sets up a tax deduction system because the government's aggregated utility exhibits more risk aversion than the representative individual's utility. When the government is more risk-averse, it is willing to supply more insurance. In the second case, the existence of tax deductions for net losses depends on the deviant beliefs of the loss probability among the insured, the insurer, and the government. If the insured under-estimates the loss probability, the government may offer social insurance (tax deductions for net losses) to fix the problem.

Propositions 1 and 2 can be applied to answer, at least in part, the question as to why a government provides supplementary public insurance or government relief. The literature has a long debate on whether a government should provide supplementary public insurance or government relief. (For example, see Besley, 1989; Selden, 1993; Blomqvist and Johansson, 1997; Petretto, 1999 for public insurance and Kaplow, 1991 and 1992a for government relief.) As far as we know, most papers assume that the government maximizes the expected utility of the representative individual. It is worth discussing both policies under the assumption that the objective function of the government is not the same as the objective function of the representative agent. As shown by Kaplow (1992b), the tax deduction for individuals' net losses serves as nothing but a social insurance. Thus, we can easily transform the model of this paper to analyze the inclusion of supplementary public insurance or government relief. Our Propositions 1 and 2 can hold for the cases of supplementary public insurance or government relief.

Furthermore, our paper focuses on the tax deduction for the individual's net losses rather than the insurance premium. In the United States, the individual's health insurance premium is also tax-deductible as discussed by Eisenhauer (2002). Our model could also be applied to his model, and provides two reasons why a government would consider providing a tax deduction for the individual's net losses as well as an insurance premium.

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