



ELSEVIER



Insurance: Mathematics and Economics ■ (■■■■) ■■■-■■■

INSURANCE
 MATHEMATICS & ECONOMICS
www.elsevier.com/locate/ime

Government-provided annuities under insolvency risk

 Rachel J. Huang^{a,*,1}, Jeffrey T. Tsai^b, Larry Y. Tzeng^c
^a *Department of Finance, Yuan Ze University, 135 Yuan-Tung Road, Chung-Li, Taoyuan 32003, Taiwan*
^b *Yuanpei University, Taiwan*
^c *National Taiwan University, Taiwan*

Received August 2007; received in revised form October 2007; accepted 15 October 2007

Abstract

This paper seeks to determine whether governments should intervene in the private annuity market by directly providing public insurance in the form of annuities when both the government and the insurance companies could default. It is found that, although the government could default, intervening by means of an annuity can improve social welfare if the insurance companies could default and the expected return on the public annuity is greater than the rate of return on a risk-free bond. We also find that, under actuarially fair pricing, the government should provide more in terms of a public annuity than the optimal amount of the annuity that the individual purchases in the private market if the government is less likely to default on the public annuity than an insurance company would in the case of a private annuity.

© 2007 Elsevier B.V. All rights reserved.

Keywords: Annuity puzzle; Optimal annuitization; Insolvency risk; Public annuity

1. Introduction

The importance of annuity markets has rapidly increased in many parts of the world because of the aging of the population. To take care of people in their old age, the governments of most developed countries intervene in the private annuity market by directly providing indemnity with a mode of annuity. For example, individuals aged 65 and over are able to receive benefits in the form of life annuities under the Old-Age and Survivors Insurance program in the United States. In addition, retired workers can receive retirement benefits from social security and public pension programs. Approximately 34 million retired workers and their dependants received social security benefits in 2006. The average monthly retirement benefit has been estimated at \$1044 for each worker and \$1713 for a couple in 2007.²

However, there has been a long debate in the literature on whether the government intervention could improve social

welfare given the existence of a private insurance market. Papers that are against social insurance argue that social insurance may distort the individual's incentive in the private market and further crowd out the private market.³ For example, [Cutler and Gruber \(1996\)](#) estimated that, in the United States, about 50% of the increase in Medicaid coverage was associated with a reduction in private insurance coverage over the 1987–1992 period.

On the other hand, there are three main arguments in support of public intervention: transaction costs, redistribution, and asymmetric information. [Diamond \(1992\)](#) and [Mitchell \(1998\)](#) argued that the public pension system has lower transaction costs than that in the private sector because of the small scale of private insurers and their advertising costs. As for the second reason, [Brown \(2003\)](#) examined the redistribution effect on mandatory annuitization, and found that a mandatory annuity could make the individuals in all groups better off.⁴ With regard to asymmetric information, most papers in this field focus

* Corresponding author. Tel.: +886 930 553 988; fax: +886 3 435 4624.

E-mail addresses: rachel1308@gmail.com, rachel@saturn.yzu.edu.tw (R.J. Huang).

¹ Huang gratefully acknowledges David Blake, Richard MacMinn, and Thomas Post for valuable comments and suggestions.

² The data are obtained from the website of the National Committee to Preserve Social Security and Medicare (<http://www.ncpsm.org>).

³ Please see [Kaplow \(1992\)](#) and [Selden \(1993\)](#), for example.

⁴ [Rochet \(1991\)](#) and [Cremer and Pestieau \(1996\)](#) have found that social insurance is a desirable redistributive device when risk is negatively correlated with income. [Coronado et al. \(2000\)](#), [Gustman and Steinmeier \(2001\)](#) and [Liebman \(2002\)](#) focus their attention on the social security system in the United States.

on the adverse selection problem. [Akerlof \(1970\)](#) indicated that the U.S. Medicare program could overcome the adverse selection problem in the private market. Numerous papers⁵ follow [Akerlof \(1970\)](#) and point out that public intervention could deal with adverse selection and/or moral hazard problems in the private sector.

Although the previous literature has provided many ingenious findings, until recently, relatively few studies have considered insolvency risk, which is a very important factor⁶ that might cause market failure and further influence the annuitization decision. In particular, [Babbel and Merrill \(2006\)](#) used a continuous model to examine the individual's annuitization behavior if the insurers were to become insolvent. By simulation, they find that the individual's annuity purchasing decisions are quite sensitive to the risk of default. [Schulze and Post \(2007\)](#) also mentioned that the default risk of the insurance companies is crucial to the individual's annuitization. It should be noted that both papers exogenously assume the existence of government intervention. Whether or not the social welfare maximizing government should set up a public system to take care of old people when the insurance companies might default remains an open question. This paper intends to provide an answer to it.

At the outset, it should be noted that the government intervention that we focus on takes the form of an annuity. This type of intervention can be observed in several systems, such as social security, public pensions and mandatory longevity insurance program. Thus, we refer to these programs as “public annuities” in order to compare them with private annuities.

We adopt a two-stage game. In the first stage, the government decides whether or not to provide a public annuity. In the second stage, individuals, who can only live for two periods, choose to allocate their savings among private annuities and risk-free bonds.⁷ Since insurers and public annuity providers might become insolvent, we assume that there exists default risk for both private and public annuities. Furthermore, as in [Yaari \(1965\)](#), we assume that the individuals are expected utility maximizers with intertemporally separable utility and have no bequest motive.

We focus on the equilibrium conditions for the existence of the public annuity and the crowding out effect of the public annuity on the private annuity. In the classical literature, [Yaari \(1965\)](#) demonstrated that full annuitization was the optimal allocation for retirement savings when the individuals had no bequest motive and were expected utility maximizers with intertemporally separable utility. Recently, [Davidoff et al. \(2005\)](#) confirmed [Yaari's \(1965\)](#) results by relaxing the

expected utility and intertemporal separability assumptions. However, the observed levels of private annuitization are much smaller than scholars' expectations. This low rate of annuitization gives rise to the “annuity puzzle”. One possible explanation for the annuity puzzle is the crowding out effect caused by social insurance (see [Mitchell et al. \(1999\)](#), [Brown and Poterba \(2000\)](#)).⁸ Thus, we intend to study the condition when the existence of a public annuity is socially optimal as well as the condition when the crowding out effect takes place.

We find that in the cases where (1) both the insurer and the government will never default,⁹ or (2) only one party among the insurer and the government might be insolvent, full annuitization is still optimal. Our results are consistent with the findings of [Yaari \(1965\)](#). In these cases, individuals should invest all their savings either in the social annuity or private annuity, depending on which one provides a higher expected return. If the expected return on the public annuity is greater than that on the private annuity, building up the public annuity system could make the individuals better off. However, the private annuity will be totally crowded out by the public annuity.

In the case where both the insurer and the government have positive insolvency probabilities, we find that partial annuitization is optimal and the individuals always purchase risk-free bonds. Our finding is consistent with [Babbel and Merrill \(2006\)](#) and [Schulze and Post \(2007\)](#). Moreover, we find that if the expected return on the public annuity is greater than the risk-free bond return, then providing the public annuity could increase social welfare. The private annuity will be totally crowded out only if the expected return on the private annuity is less than that on risk-free bonds. If both the expected returns on the private and public annuities are greater than the returns on risk-free bonds, then it is optimal for private and public annuities to co-exist in the market. We further find that under the assumption of actuarially fair pricing, the optimal solution is for both private and public annuities to co-exist since the expected returns on the annuities are greater than that on risk-free bonds under actuarially fair pricing. Moreover, in this case, we find that the government should make it mandatory for the individuals to invest more in public annuities when the government has a lower insolvency probability than the insurer.

The paper is organized as follows. In Section 2, we set up a two-stage model. Section 3 provides the equilibrium conditions. Section 4 discusses the optimal public annuity under different

⁸ The literature has provided many other explanations for the annuity puzzle, including the bequest motive ([Yaari, 1965](#)); unfairly priced annuities due to adverse selection or transaction costs ([Friedman and Warshawsky, 1988](#); [Mitchell et al., 1999](#); [Milevsky and Young, 2007](#)); intrafamily risk sharing ([Kotlikoff and Spivak, 1981](#); [Post et al., in press](#)); mortality risk ([Olivieri, 2001](#); [Milevsky et al., 2005](#)); and the default-risk of the annuity-providing insurer ([Babbel and Merrill, 2006](#)).

⁹ We analysis the cases where the insurer and the public annuity provider has independent default risk. The government is considered as the public annuity provider. In the cases where the government is insolvent, it means that the sector in the government providing public annuity has insolvency problem. However, the sector in the government who issues risk-free bonds does not have any default risk. For example, the social security system in the United States faces significant default risk recently, but the Fed does not have that problem.

⁵ For example, see [Wilson \(1977\)](#), and [Eckstein et al. \(1985\)](#). Recently, there are some papers ([Einav et al., 2007](#); [Netzer and Scheuer, 2007](#)) showing that the welfare improvement by public intervention is limited in the present of asymmetric information.

⁶ According to the A. B. Best company insolvency study, the annual impairment frequency of private annuities in the United States averaged 1.02% during 1976–2002, which was higher than the average rate for the life/health industry of 0.92%.

⁷ Here we assume that the individuals cannot directly invest in risky assets as in [Yaari \(1965\)](#).

assumptions regarding the probability of insurer insolvency. Section 5 concludes the paper.

2. Model approach

A two-stage game is employed in this paper. The structure of the game is as follows:

Stage 1. The government considers establishing a public annuity system.

Stage 2. Individuals make their asset allocation decisions.

In the first stage, a benevolent government considers establishing a public annuity system. In this system, individuals are forced to invest in the public annuity when they are young, and obtain coverage from it when they are old. In the second stage, by knowing their obligation in the public annuity system, individuals can further make asset allocation decisions that involve private annuities based on a perfectly competitive private insurance market and a risk-free bond.

For simplicity, we assume that the individuals are identical and that they can only live for two periods as in Davidoff et al. (2005). Since the literature has demonstrated that the bequest motive could be one of the reasons for the partial annuitization, we further assume that the individuals have no bequest motive in order to clarify the effect of insurer insolvency on partial annuitization.

As in Yaari (1965), we assume that the individuals are expected utility maximizers with intertemporally separable utility $U(C_t)$ with $U' > 0$ and $U'' < 0$, where C_t denotes the consumption level in period t , and $t = 1, 2$. We further follow the assumption in Davidoff et al. (2005) that zero consumption is extremely bad, i.e.,

$$\lim_{C_t \rightarrow 0} U'(C_t) = \infty, \quad t = 1, 2. \quad (1)$$

In the first period, the representative individual is definitely alive with initial wealth w . The individual is required to invest T dollars, $T \in [0, w]$, in the public annuity, which can generate a gross return R_T in period two. Furthermore, the individual can also allocate his/her wealth among private annuities and risk-free bonds with dollar amounts A and B , respectively. These two assets will respectively generate gross returns R_A and R_B in the second period. As assumed in Davidoff et al. (2005), we assume that the return on the annuity is greater than that on the bonds. Thus,

$$R_A > R_B \quad \text{and} \quad R_T > R_B.$$

We further impose the short sale constraint:

$$A \geq 0 \quad \text{and} \quad B \geq 0. \quad (2)$$

In period two, the individual faces a probability of death $p \in (0, 1)$. Meanwhile, the insurance companies and the public annuity system have a probability of insolvency $q \in [0, 1)$ and $r \in [0, 1)$ in this period, respectively. Here we assume that the insurer and the public annuity provider have independent default probabilities. We also assume that the individuals cannot obtain any indemnity from the insolvent

institution.¹⁰ Thus, if the individual is alive in the second period, he/she will face four different states. The first state is where both the insurer and the government are solvent, in which case the individual obtains $R_A A + R_B B + R_T T$. The second state is where the insurer is solvent but the government is insolvent. The individual gets $R_A A + R_B B$ in this case. The third state is that the insurer is insolvent but government is solvent, so that the individual receives $R_B B + R_T T$ from the investment. The last state is where both the insurer and the government are insolvent. In this state, the individual has only the risk-free bond payment $R_B B$. If the individual is not alive in the second period, he/she will obtain zero utility. For simplicity, we further assume that the time preference discount factor is zero.

3. Equilibrium conditions

This model could be solved by backward induction. In the second stage of the game, given an arbitrary mandatory annuitization level T , the individual maximizes his/her expected utility by deciding the investment in the private annuity A and risk-free bond B under the short sale constraint (2):

$$\begin{aligned} \text{Max}_{A, B} EU &= U(w - A - B - T) + (1 - p)(1 - q) \\ &\times (1 - r)U(R_A A + R_B B + R_T T) \\ &+ (1 - p)(1 - q)rU(R_A A + R_B B) \\ &+ (1 - p)q(1 - r)U(R_B B + R_T T) \\ &+ (1 - p)qrU(R_B B) \end{aligned} \quad (3)$$

s.t $A \geq 0$ and $B \geq 0$.

For a given arbitrary mandatory annuitization level T , the Lagrangian function in the optimization problem (3) is

$$\begin{aligned} \mathcal{L} &= U(w - A - B - T) + (1 - p)(1 - q)(1 - r)U(R_A A \\ &+ R_B B + R_T T) + (1 - p)(1 - q)rU(R_A A + R_B B) \\ &+ (1 - p)q(1 - r)U(R_B B + R_T T) \\ &+ (1 - p)qrU(R_B B) + \lambda_A A + \lambda_B B, \end{aligned} \quad (4)$$

where λ_A and λ_B denote the Lagrangian multipliers, and are non-negative constants. At this stage, we have four possible solutions: $A = B = 0$; $A > B = 0$; $B > A = 0$; $A > 0$ and $B > 0$. Note that, under the assumption $U' > 0$ and $U'' < 0$, the first-order conditions are not only necessary but also sufficient conditions. Thus, the necessary and sufficient conditions for these four cases are as follows:

1. $A = B = 0$ if $\lambda_A > 0, \lambda_B > 0$,

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial A} \Big|_{A=B=0} &= -U'(w - T) + (1 - p) \\ &\times (1 - q)(1 - r)R_A U'(R_T T) \\ &+ (1 - p)(1 - q)rR_A U'(0) + \lambda_A = 0, \end{aligned} \quad (5)$$

¹⁰ The main results of this paper still hold if partial indemnity from the insolvent institution is allowed.

and

$$\frac{\partial \mathcal{L}}{\partial B} \Big|_{A=B=0} = -U'(w - T) + (1 - p)(1 - r)R_B U'(R_T T) + (1 - p)rR_B U'(0) + \lambda_B = 0. \quad (6)$$

2. $A > B = 0$ if $\lambda_A = 0, \lambda_B > 0$,

$$\frac{\partial \mathcal{L}}{\partial A} \Big|_{B=0} = -U'(w - A - T) + (1 - p)(1 - q) \times (1 - r)R_A U'(R_A A + R_T T) + (1 - p)(1 - q)rR_A U'(R_A A) = 0, \quad (7)$$

and

$$\frac{\partial \mathcal{L}}{\partial B} \Big|_{B=0} = -U'(w - A - T) + (1 - p)(1 - q) \times (1 - r)R_B U'(R_A A + R_T T) + (1 - p)(1 - q)rR_B U'(R_A A) + (1 - p)q(1 - r)R_B U'(R_T T) + (1 - p)qrR_B U'(0) + \lambda_B = 0. \quad (8)$$

3. $B > A = 0$ if $\lambda_A > 0, \lambda_B = 0$,

$$\frac{\partial \mathcal{L}}{\partial A} \Big|_{A=0} = -U'(w - B - T) + (1 - p)(1 - q) \times (1 - r)R_A U'(R_B B + R_T T) + (1 - p)(1 - q)rR_A U'(R_B B) + \lambda_A = 0, \quad (9)$$

and

$$\frac{\partial \mathcal{L}}{\partial B} \Big|_{A=0} = -U'(w - B - T) + (1 - p)(1 - r)R_B U' \times (R_B B + R_T T) + (1 - p)rR_B U'(R_B B) = 0. \quad (10)$$

4. $A > 0$ and $B > 0$ if $\lambda_A = 0, \lambda_B = 0$,

$$\frac{\partial \mathcal{L}}{\partial A} = -U'(w - A - B - T) + (1 - p)(1 - q) \times (1 - r)R_A U'(R_A A + R_B B + R_T T) + (1 - p)(1 - q)rR_A U'(R_A A + R_B B) = 0, \quad (11)$$

and

$$\frac{\partial \mathcal{L}}{\partial B} = -U'(w - A - B - T) + (1 - p)(1 - q)(1 - r) \times R_B U'(R_A A + R_B B + R_T T) + (1 - p)(1 - q)rR_B U'(R_A A + R_B B) + (1 - p)q(1 - r)R_B U'(R_B B + R_T T) + (1 - p)qrR_B U'(R_B B) = 0. \quad (12)$$

Let $A^* = A^*(T)$ and $B^* = B^*(T)$ denote the optimal strategy for the individual in the second stage. In the first stage, knowing that the individual's decision depends on T , the benevolent government makes a decision regarding the mandatory annuitization level T to maximize social welfare:

$$\begin{aligned} \text{Max}_T SW &= U(w - A^* - B^* - T) \\ &+ (1 - p)(1 - q)(1 - r)U(R_A A^* + R_B B^* + R_T T) \\ &+ (1 - p)(1 - q)rU(R_A A^* + R_B B^*) \end{aligned}$$

$$\begin{aligned} &+ (1 - p)q(1 - r)U(R_B B^* + R_T T) \\ &+ (1 - p)qrU(R_B B^*) \end{aligned}$$

s.t $T \geq 0$. (13)

Thus, the Lagrangian function at this stage becomes

$$\begin{aligned} \mathcal{L}_g &= U(w - A^* - B^* - T) + (1 - p)(1 - q) \\ &\times (1 - r)U(R_A A^* + R_B B^* + R_T T) \\ &+ (1 - p)(1 - q)rU(R_A A^* + R_B B^*) \\ &+ (1 - p)q(1 - r)U(R_B B^* + R_T T) \\ &+ (1 - p)qrU(R_B B^*) + \lambda_T T, \end{aligned} \quad (14)$$

where λ_T is the Lagrangian multiplier in this optimization problem. Therefore, by the Envelope Theorem, we have:

1. $T = 0$, if $\lambda_T > 0$ and

$$\begin{aligned} \frac{\partial \mathcal{L}_g}{\partial T} \Big|_{T=0} &= -U'(w - A^* - B^*) + (1 - p)(1 - q) \\ &\times (1 - r)R_T U'(R_A A^* + R_B B^*) \\ &+ (1 - p)q(1 - r)R_T U'(R_B B^*) + \lambda_T = 0. \end{aligned} \quad (15)$$

2. $T > 0$, if $\lambda_T = 0$ and

$$\begin{aligned} \frac{\partial \mathcal{L}_g}{\partial T} &= -U'(w - A^* - B^* - T) + (1 - p)(1 - q) \\ &\times (1 - r)R_T U'(R_A A^* + R_B B^* + R_T T) \\ &+ (1 - p)q(1 - r)R_T U'(R_B B^* + R_T T) = 0. \end{aligned} \quad (16)$$

Let T^* denote the optimal strategy for the government.

4. The optimal public annuity

In this section, we would like to discuss the socially optimal annuity under four cases: $q = r = 0$; $q > r = 0$; $r > q = 0$; and $r > 0$ and $q > 0$. It is obvious that, in all cases, $T^* = A^* = B^* = 0$ can be excluded in equilibrium by assuming that $\lim_{C_2 \rightarrow 0} U'(C_2) = \infty$.

The first case we focus on is where $q = r = 0$. In this case, the government and insurance companies will never be insolvent. In other words, all assets are risk-free. According to our assumption that $R_A > R_B$ and $R_T > R_B$, the bond market is dominated by both annuities. Therefore, $B^* = 0$ is in equilibrium. The following proposition shows the condition for the existence of a public annuity:

Proposition 1. Given $q = r = 0$,

1. $A^* > T^* = B^* = 0$ if $R_T < R_A$.
2. $T^* > 0, A^* = B^* = 0$ if $R_T > R_A$.
3. $T^* > 0, A^* > B^* = 0$ if $R_T = R_A > R_B$.

Proof. Please see Appendix A. ■

Proposition 1 shows that the individuals should invest in the asset with the higher return between private annuity and public annuity when the insurer and the government will never default. Our proposition is consistent with the finding in Davidoff et al. (2005) that the individuals should annuitize all of their savings. However, we find that the individuals do not necessarily purchase private annuities. If $R_A > R_T$, individuals

should invest all their savings in a private annuity and the government should walk away. If $R_T \geq R_A$, the government should provide a public annuity and the private annuity will be totally crowded out by the public annuity if $R_T > R_A$. This result confirms the finding of Diamond (1992) and Mitchell (1998).

The second case is that $q > r = 0$, where the insurers may be insolvent, but the government will never default. In the state where the insurers are insolvent, individuals receive $R_B B + R_T T$ in the second period. Since $\lim_{C_2 \rightarrow 0} U'(C_2) = \infty$, it is obvious that $T^* = B^* = 0$ is not optimal. Furthermore, when $r = 0$, the public annuity is also a risk-free asset. Thus, the risk-free asset will be dominated by the public annuity, since we assume that $R_T > R_B$. In other words, the bond market is totally crowded out by the public annuity.

Proposition 2. Given $q > r = 0$,

1. $T^* > 0, A^* = B^* = 0$ if $R_T > (1 - q)R_A$.
2. $T^* > 0, A^* > B^* = 0$ if $R_T < (1 - q)R_A$.

Proof. See Appendix B. ■

Proposition 2 shows that the government should always set up a public annuity system when the government does not have an insolvency problem but the insurers have. However, whether or not the private annuity is totally crowded out by the public annuity depends on the return on the public annuity and the expected return on the private annuity, $(1 - q)R_A$. If the expected return on the private annuity is greater than that on the public annuity, the private annuity will still exist. On the other hand, even if $R_A > R_T$, the public annuity will totally crowd out private annuity in the optimum as long as $R_T > (1 - q)R_A$.

The third case is $r > q = 0$, where the government may be insolvent but the insurers will never be insolvent. If the government is insolvent, the individuals will receive $R_A A + R_B B$ in the second period. It is obvious that $A^* = B^* = 0$ is not an equilibrium since we assume that $\lim_{C_2 \rightarrow 0} U'(C_2) = \infty$. In this case, the bond market is dominated by a private annuity since $R_A > R_B$. Thus, full annuitization will be optimal.

Proposition 3. Given $r > q = 0$,

1. $A^* > T^* = B^* = 0$ if $(1 - r)R_T < R_A$.
2. $T^* > 0, A^* > B^* = 0$ if $(1 - r)R_T > R_A$.

Proof. Please see Appendix C. ■

Proposition 3 concludes that the government should provide a public annuity if the expected return on the public annuity, $(1 - r)R_T$, is greater than R_A when the government has a positive default rate but the insurers have zero default rate. In this case, the private and public annuities may co-exist in the market when $(1 - r)R_T > R_A$.

The last case is where $r > 0$ and $q > 0$. In this case, both the government and insurers may be insolvent. If the government and insurers are insolvent, individuals receive $R_B B$ in the second period. Thus, $B^* = 0$ is not an equilibrium due to $\lim_{C_2 \rightarrow 0} U'(C_2) = \infty$. Therefore, when both the insurer and the government may default, even though the probability

is small, individuals should always invest in risk-free bonds. In this case, partial annuitization is optimal. The result is consistent to Babel and Merrill (2006) and Schulze and Post (2007).

Proposition 4. Given $r > 0$ and $q > 0$,

1. $T^* = 0, A^* > 0, B^* > 0$ if $(1 - r)R_T < R_B < (1 - q)R_A$.
2. $T^* = 0, B^* > A^* = 0$ if $R_B > \max\{(1 - r)R_T, (1 - q)R_A\}$.
3. $T^* > 0, B^* > A^* = 0$ if $(1 - r)R_T > R_B > (1 - q)R_A$.
4. $T^* > 0, A^* > 0, B^* > 0$ if $(1 - r)R_T > R_B$ and $(1 - q)R_A > R_B$.

Proof. Please see Appendix D. ■

Proposition 4 indicates that full annuitization is not optimal in the case where $r > 0$ and $q > 0$. In the first two conditions in Proposition 4, the government should not intervene in the insurance market, since the expected return on the public annuity is less than that on the bonds. However, if $(1 - r)R_T > R_B$, government intervention could increase social welfare, even when the expected return on the public annuity might be less than that on the private annuity, i.e., $(1 - r)R_T < (1 - q)R_A$. Kaplow (1992) and other researchers find that government intervention will decrease social welfare when the government is less efficient than the private market, i.e., $(1 - r)R_T < (1 - q)R_A$. However, we find that the above conclusion should be modified if the insolvency risk is involved. Even if the government is less efficient than the private market, government intervention could still make the individuals better off, because providing a public annuity could help the individuals in the state where insurance companies are insolvent. Furthermore, the private market will be crowded out if the expected return on the private annuity is less than that on the bonds.

In the case where $r > 0$ and $q > 0$, we further analyze the equilibrium under the assumption of actuarially fair pricing. With positive q and r , and an actuarially fair pricing return $R_A = \frac{R_B}{(1-p)(1-q)}$ and $R_T = \frac{R_B}{(1-p)(1-r)}$, the equilibrium will be $T^* > 0, A^* > 0, B^* > 0$ by Proposition 4. Moreover, the higher the probability of insolvency in the private annuity market, the higher the amount of a public annuity should be provided as shown by the following proposition:

Proposition 5. Under actuarially fair pricing, the government should provide more in the way of a public annuity ($T^* > A^*$) if the government has a lower insolvency probability ($q > r$).

Proof. See Appendix E. ■

The government with a lower probability of insolvency should offer more in the way of public annuity than a private annuity. This means that the public annuity may squeeze the private market, since the default rate of the government is generally believed to be less than that of the insurer. This finding is consistent to Mitchell et al. (1999), and Brown and Poterba (2000). However, the public annuity should never crowd out the private annuity except where $r = 0$.

5. Conclusions

This paper contributes to the literature in that it indicates that insolvency risk is one possible rationale for the government to provide public annuities. We set up a two-stage game and show that, without a bequest motive, partial annuitization could be optimal. In the cases where at most one party, the insurer or the government, may default, individuals should invest all their savings in the annuity with the highest expected return, which means that full annuitization is still optimal. If the expected return on the public annuity is greater than that on the private annuity, building up the public annuity system could make the individuals better off and the public annuity will totally crowd out the private annuity. On the other hand, in the case where both parties may be insolvent, partial annuitization is optimal. The government should provide a public annuity if the expected return on the public annuity is greater than that on the risk-free bond. Furthermore, we find that, when the price of the annuity is actuarially fair, the government should provide more of the public annuity than the amount of the private annuity that individuals buy, if the default rate of the government is lower than that of the insurer.

The first implication of our paper is that we provide a rationale for the government to intervene in the private annuity market since the insurers who provide the private annuity could default. It should be noted that, by providing the public annuity, the government could improve social welfare even when it could default itself on the public annuity. This finding is consistent with the finding of Huang and Tzeng (in press) who observed that it is socially optimal for the government to provide a tax deduction for net losses as a result of the catastrophe, because the private insurers may go bankrupt when the catastrophe takes place.

On the basis of Proposition 5, the problem can be further condensed by considering whether the probability of defaulting on the public annuity is smaller than that of defaulting on the private annuity. On the one hand, as asserted by Brown and Orszag (2006), it may be more appropriate for the government rather than the private insurer to provide annuity coverage because the default rate of the government may be lower than the insurer since the government could reduce its default rate through intergenerational risk sharing. On the other hand, it is very important to recognize, as pointed out by Milevsky et al. (2006), that the public annuity may not always benefit from the law of large numbers if the numbers of policyholders in regard to the public annuity are greater than those in relation to the private annuity. Milevsky et al. (2006) have showed that through the law of large numbers, the insurer can diversify mortality risk only if mortality rates are deterministic. However, it has been well-recognized in the literature (Olivieri, 2001; Cairns et al., in press; Brown and Orszag, 2006; Schulze and Post, 2007) that the mortality rates are stochastic and that the improvement in the mortality rate cannot be ignored. Therefore, the default rate of the government may not be lower than that of the insurers if the numbers of policyholders are larger in the public annuity system. A study on whether the default rate in

relation to the public annuity is smaller than that in regard to the private insurer should certainly be fruitful in the future.

Appendix A. Proof of Proposition 1

From the equilibrium conditions in the previous section, we can find the optimal solution.

- From Eqs. (7), (8) and (15)), we have the solution $A^* > T^* = B^* = 0$ if the following conditions hold:

$$-U'(w - A^*) + (1 - p)R_A U'(R_A A^*) = 0,$$

$$-U'(w - A^*) + (1 - p)R_B U'(R_A A^*) + \lambda_B = 0,$$

and

$$-U'(w - A^*) + (1 - p)R_T U'(R_A A^*) + \lambda_T = 0.$$

Since $\lambda_B > 0$, $\lambda_T > 0$ and $R_A > R_B$, the sufficient conditions for $A^* > T^* = B^* = 0$ is $R_A > R_T$.

- From Eqs. (5), (6) and (16)), $T^* > 0$, $A^* = B^* = 0$ if

$$-U'(w - T^*) + (1 - p)R_A U'(R_T T^*) + \lambda_A = 0,$$

$$-U'(w - T^*) + (1 - p)R_B U'(R_T T^*) + \lambda_B = 0,$$

and

$$-U'(w - T^*) + (1 - p)R_T U'(R_T T^*) = 0.$$

Alternatively, $R_T > R_A$, since $\lambda_A > 0$ and $\lambda_B > 0$.

- From Eqs. (7), (8) and (16), the sufficient condition for $T^* > 0$, $A^* > B^* = 0$ are

$$-U'(w - A^* - T^*) + (1 - p)R_A U'(R_A A^* + R_T T^*) = 0,$$

$$-U'(w - A^* - T^*) + (1 - p)R_B U'(R_A A^* + R_T T^*) + \lambda_B = 0,$$

and

$$-U'(w - A^* - T^*) + (1 - p)R_T U'(R_A A^* + R_T T^*) = 0.$$

In other words, $R_T = R_A > R_B$.

Appendix B. Proof of Proposition 2

The proof is similar to that in Proposition 1.

- From Eqs. (5), (6) and (16), $T^* > 0$, $A^* = B^* = 0$ if

$$-U'(w - T^*) + (1 - p)(1 - q)R_A U'(R_T T^*) + \lambda_A = 0,$$

$$-U'(w - T^*) + (1 - p)R_B U'(R_T T^*) + \lambda_B = 0,$$

and

$$-U'(w - T^*) + (1 - p)R_T U'(R_T T^*) = 0.$$

Or, $R_T > (1 - q)R_A$.

- From Eqs. (7), (8) and (16), $T^* > 0$, $A^* > B^* = 0$ if

$$-U'(w - A^* - T^*) + (1 - p)(1 - q)R_A U'(R_A A^* + R_T T^*) = 0, \tag{A.1}$$

$$-U'(w - A^* - T^*) + (1 - p)(1 - q)R_B U'(R_A A^* + R_T T^*) + (1 - p)qR_B U'(R_T T^*) + \lambda_B = 0, \tag{A.2}$$

and

$$\begin{aligned}
 & -U'(w - A^* - T^*) + (1 - p)(1 - q)R_T U' \\
 & \quad \times (R_A A^* + R_T T^*) \\
 & \quad + (1 - p)qR_T U'(R_T T^*) = 0. \tag{A.3}
 \end{aligned}$$

Subtracting Eq. (A.1) from (A.3) yields

$$\begin{aligned}
 (1 - q)(R_A - R_T)U'(R_A A^* + R_T T^*) & = qR_T U'(R_T T^*) \\
 & > qR_T U'(R_A A^* + R_T T^*),
 \end{aligned}$$

since $U'' < 0$. Thus, we have $R_T < (1 - q)R_A$.

Appendix C. Proof of Proposition 3

From the equilibrium conditions in the previous section, we can find the optimal solution.

- From Eqs. (7), (8) and (15), we have the solution $A^* > T^* = B^* = 0$ if and only if the following conditions hold:

$$\begin{aligned}
 & -U'(w - A^*) + (1 - p)R_A U'(R_A A^*) = 0, \\
 & -U'(w - A^*) + (1 - p)R_B U'(R_A A^*) + \lambda_B = 0,
 \end{aligned}$$

and

$$-U'(w - A^*) + (1 - p)(1 - r)R_T U'(R_A A^*) + \lambda_T = 0.$$

Since $\lambda_B > 0$ and $\lambda_T > 0$, the necessary and sufficient condition for $A^* > T^* = B^* = 0$ is $R_A > (1 - r)R_T$.

- From Eqs. (7), (8) and (16), the necessary and sufficient conditions for $T^* > 0, A^* > B^* = 0$ are

$$\begin{aligned}
 & -U'(w - A^* - T^*) + (1 - p)(1 - r)R_A U' \\
 & \quad \times (R_A A^* + R_T T^*) \\
 & \quad + (1 - p)rR_A U'(R_A A^*) = 0, \tag{A.4}
 \end{aligned}$$

$$\begin{aligned}
 & -U'(w - A^* - T^*) + (1 - p)(1 - r)R_B U' \\
 & \quad \times (R_A A^* + R_T T^*) \\
 & \quad + (1 - p)rR_B U'(R_A A^*) + \lambda_B = 0, \tag{A.5}
 \end{aligned}$$

and

$$\begin{aligned}
 & -U'(w - A^* - T^*) + (1 - p)(1 - r)R_T U' \\
 & \quad \times (R_A A^* + R_T T^*) = 0. \tag{A.6}
 \end{aligned}$$

Subtracting Eq. (A.4) from (A.6) yields

$$\begin{aligned}
 (1 - r)(R_T - R_A)U'(R_A A^* + R_T T^*) & = rR_A U'(R_A A^*) \\
 & > rR_A U'(R_A A^* + R_T T^*),
 \end{aligned}$$

since $U'' < 0$. Thus, we have $(1 - r)R_T > R_A$.

Appendix D. Proof of Proposition 4

From the equilibrium conditions in the previous section, we can find the optimal solution.

- From Eqs. (11), (12) and (15), we have the solution $T^* = 0, A^* > 0, B^* > 0$ if

$$\begin{aligned}
 & -U'(w - A^* - B^*) + (1 - p)(1 - q)R_A U' \\
 & \quad \times (R_A A^* + R_B B^*) = 0, \tag{A.7}
 \end{aligned}$$

$$\begin{aligned}
 & -U'(w - A^* - B^*) + (1 - p)(1 - q)R_B U' \\
 & \quad \times (R_A A^* + R_B B^*) \\
 & \quad + (1 - p)qR_B U'(R_B B^*) = 0, \tag{A.8}
 \end{aligned}$$

and

$$\begin{aligned}
 & -U'(w - A^* - B^*) + (1 - p)(1 - q)(1 - r) \\
 & \quad \times R_T U'(R_A A^* + R_B B^*) \\
 & \quad + (1 - p)q(1 - r)R_T U'(R_B B^*) + \lambda_T = 0. \tag{A.9}
 \end{aligned}$$

Subtracting Eq. (A.7) from (A.8) yields

$$\begin{aligned}
 (1 - p)(1 - q)U'(R_A A^* + R_B B^*) & \\
 = (1 - p)q \frac{R_B}{R_A - R_B} U'(R_B B^*). & \tag{A.10}
 \end{aligned}$$

Since U is a strictly concave function and $R_A A^* + R_B B^* > R_B B^*$, the above equation holds if $(1 - q) > q \frac{R_B}{R_A - R_B}$, or $(1 - q)R_A > R_B$. Substituting Eq. (A.10) into (A.7) we have

$$U'(w - A^* - B^*) = (1 - p)q \frac{R_A R_B}{R_A - R_B} U'(R_B B^*). \tag{A.11}$$

Thus, substituting Eqs. (A.10) and (A.11) into Eq. (A.9) yields

$$\begin{aligned}
 & -(1 - p)q \frac{R_A R_B}{R_A - R_B} U'(R_B B^*) + (1 - p)q(1 - r) \\
 & \quad \times \frac{R_T R_B}{R_A - R_B} U'(R_B B^*) \\
 & \quad + (1 - p)q(1 - r)R_T U'(R_B B^*) + \lambda_T = 0.
 \end{aligned}$$

Or,

$$\begin{aligned}
 (1 - p)q \frac{R_A}{R_A - R_B} [-R_B + (1 - r)R_T] & \\
 \times U'(R_B B^*) + \lambda_T = 0. &
 \end{aligned}$$

Therefore, $R_B > (1 - r)R_T$, since $\lambda_T > 0$.

- From Eqs. (9), (10) and (15), we have $T^* = 0, B^* > A^* = 0$ if

$$\begin{aligned}
 & -U'(w - B^*) + (1 - p)(1 - q)R_A U' \\
 & \quad \times (R_B B^*) + \lambda_A = 0, \tag{A.12}
 \end{aligned}$$

$$-U'(w - B^*) + (1 - p)R_B U'(R_B B^*) = 0, \tag{A.13}$$

and

$$\begin{aligned}
 & -U'(w - B^*) + (1 - p)(1 - r)R_T U' \\
 & \quad \times (R_B B^*) + \lambda_T = 0. \tag{A.14}
 \end{aligned}$$

Since $\lambda_A > 0$ and $\lambda_T > 0$, we have

$$\begin{aligned}
 & -U'(w - B^*) + (1 - p)R_B U'(R_B B^*) > -U'(w - B^*) \\
 & \quad + (1 - p)(1 - q)R_A U'(R_B B^*),
 \end{aligned}$$

or,

$$R_B > (1 - q)R_A,$$

from Eqs. (A.12) and (A.13). From Eqs. (A.13) and (A.14), it is easy to obtain

$$R_B > (1 - r)R_T.$$

- From Eqs. (9), (10) and (16), $T^* > 0, B^* > A^* = 0$ if

$$\begin{aligned}
 & -U'(w - B^* - T^*) + (1 - p)(1 - q)(1 - r) \\
 & \quad \times R_A U'(R_B B^* + R_T T^*) \\
 & \quad + (1 - p)(1 - q)rR_A U'(R_B B^*) + \lambda_A = 0, \tag{A.15} \\
 & -U'(w - B^* - T^*) + (1 - p)(1 - r)
 \end{aligned}$$

$$\begin{aligned} & \times R_B U'(R_B B^* + R_T T^*) \\ & + (1-p)r R_B U'(R_B B^*) = 0, \end{aligned} \quad (\text{A.16})$$

and

$$\begin{aligned} & -U'(w - B^* - T^*) + (1-p)(1-r)R_T U' \\ & \times (R_B B^* + R_T T^*) = 0. \end{aligned} \quad (\text{A.17})$$

Subtracting Eq. (A.16) from (A.17) yields

$$\begin{aligned} & (1-p)(1-r)U'(R_B B^* + R_T T^*) \\ & = (1-p)r \frac{R_B}{R_T - R_B} U'(R_B B^*). \end{aligned} \quad (\text{A.18})$$

The above equation holds if $(1-r) > r \frac{R_B}{R_T - R_B}$, $(1-r)R_T > R_B$, since U is strictly concave. Substituting Eq. (A.18) into (A.17) we have

$$U'(w - B^* - T^*) = (1-p)r \frac{R_T R_B}{R_T - R_B} U'(R_B B^*). \quad (\text{A.19})$$

Thus, substituting Eqs. (A.18) and (A.19) into Eq. (A.15) yields

$$\begin{aligned} & -(1-p)r \frac{R_T R_B}{R_T - R_B} U'(R_B B^*) \\ & + (1-p)(1-q)r \frac{R_A R_B}{R_T - R_B} U'(R_B B^*) \\ & + (1-p)(1-q)r R_A U'(R_B B^*) + \lambda_A = 0. \end{aligned}$$

Or,

$$\begin{aligned} & (1-p)r \frac{R_T}{R_T - R_B} [-R_B + (1-q)R_A] \\ & \times U'(R_B B^*) + \lambda_A = 0. \end{aligned}$$

Thus, $R_B > (1-q)R_A$.

4. From Eqs. (11), (12) and (16), $T^* > 0$, $A^* > 0$, $B^* > 0$ if

$$\begin{aligned} & -U'(w - A^* - B^* - T^*) + (1-p)(1-q)(1-r) \\ & \times R_A U'(R_A A^* + R_B B^* + R_T T^*) \\ & + (1-p)(1-q)r R_A U'(R_A A^* + R_B B^*) = 0, \quad (\text{A.20}) \\ & -U'(w - A^* - B^* - T^*) + (1-p)(1-q)(1-r) \\ & \times R_B U'(R_A A^* + R_B B^* + R_T T^*) \\ & + (1-p)(1-q)r R_B U'(R_A A^* + R_B B^*) \\ & + (1-p)q(1-r)R_B U'(R_B B^* + R_T T^*) \\ & + (1-p)qr R_B U'(R_B B^*) = 0, \end{aligned} \quad (\text{A.21})$$

and

$$\begin{aligned} & -U'(w - A^* - B^* - T^*) + (1-p)(1-q)(1-r) \\ & \times R_T U'(R_A A^* + R_B B^* + R_T T^*) \\ & + (1-p)q(1-r)R_T U'(R_B B^* + R_T T^*) = 0. \end{aligned} \quad (\text{A.22})$$

First, we would like to show the condition for a positive T . If $T^* > 0$, this means that Eq. (A.22) evaluated at $T = 0$ is positive. That is

$$\begin{aligned} & -U'(w - A^* - B^*) + (1-p)(1-q)(1-r)R_T U' \\ & \times (R_A A^* + R_B B^*) \\ & + (1-p)q(1-r)R_T U'(R_B B^*) > 0. \end{aligned} \quad (\text{A.23})$$

Evaluating Eqs. (A.20) and (A.21) at $T = 0$ yields

$$\begin{aligned} & -U'(w - A^* - B^*) + (1-p)(1-q)R_A U' \\ & \times (R_A A^* + R_B B^*) = 0, \end{aligned} \quad (\text{A.24})$$

and

$$\begin{aligned} & -U'(w - A^* - B^*) + (1-p)(1-q)R_B U' \\ & \times (R_A A^* + R_B B^*) \\ & + (1-p)q R_B U'(R_B B^*) = 0. \end{aligned} \quad (\text{A.25})$$

Subtracting Eq. (A.24) from (A.25) yields

$$\begin{aligned} & (1-p)(1-q)U'(R_A A^* + R_B B^*) \\ & = (1-p)q \frac{R_B}{R_A - R_B} U'(R_B B^*). \end{aligned} \quad (\text{A.26})$$

Substituting Eq. (A.26) into (A.24) we have

$$U'(w - A^* - B^*) = (1-p)q \frac{R_A R_B}{R_A - R_B} U'(R_B B^*). \quad (\text{A.27})$$

Thus, substituting Eqs. (A.26) and (A.27) into Eq. (A.23) yields

$$(1-p)q U'(R_B B^*) \frac{R_A}{R_A - R_B} [(1-r)R_T - R_B] > 0.$$

The above equation holds if $(1-r)R_T > R_B$. Furthermore, from Case II, since the insurers may be insolvent, we can conclude that the individuals will invest in the private annuity if $(1-q)R_A > R_B$.

Appendix E. Proof of Proposition 5

Substituting actuarially fair pricing returns, $R_A = \frac{R_B}{(1-p)(1-q)}$ and $R_T = \frac{R_B}{(1-p)(1-r)}$, into Eqs. (A.20) and (A.22) yields

$$\begin{aligned} & -U'(w - A^* - B^* - T^*) + (1-r)R_B U'(R_A A^* \\ & + R_B B^* + R_T T^*) + r R_B U'(R_A A^* + R_B B^*) = 0, \end{aligned}$$

and

$$\begin{aligned} & -U'(w - A^* - B^* - T^*) + (1-q)R_B U'(R_A A^* \\ & + R_B B^* + R_T T^*) + q R_B U'(R_B B^* + R_T T^*) = 0. \end{aligned}$$

Or,

$$\begin{aligned} & (1-r)U'(R_A A^* + R_B B^* + R_T T^*) \\ & + r U'(R_A A^* + R_B B^*) \end{aligned} \quad (\text{A.28})$$

$$\begin{aligned} & = (1-q)U'(R_A A^* + R_B B^* + R_T T^*) \\ & + q U'(R_B B^* + R_T T^*) \end{aligned} \quad (\text{A.29})$$

$$\begin{aligned} & > (1-r)U'(R_A A^* + R_B B^* + R_T T^*) \\ & + r U'(R_B B^* + R_T T^*), \end{aligned} \quad (\text{A.30})$$

where the inequality follows from $q > r$ and $U'(R_A A^* + R_B B^*) > U'(R_A A^* + R_B B^* + R_T T^*)$. Eqs. (A.29) and (A.30) are weighted averages of $U'(R_A A^* + R_B B^* + R_T T^*)$ and $U'(R_A A^* + R_B B^*)$ and Eq. (A.30) has higher weight on the smaller term $U'(R_A A^* + R_B B^* + R_T T^*)$ and so has a smaller value. If we cancel the term $(1-r)U'(R_A A^* + R_B B^* + R_T T^*)$ on both sides of Eqs. (A.28) and (A.30), we obtain

$$R_A A^* < R_T T^*.$$

Substituting the actuarially fair prices on both sides yields

$$\frac{R_B}{(1-p)(1-q)} A^* < \frac{R_B}{(1-p)(1-r)} T^*.$$

$$\text{Or, } T^* > \frac{(1-r)}{(1-q)} A^* > A^*.$$

References

- Akerlof, G., 1970. The market for lemons: Quality, uncertainty and the market mechanism. *Quarterly Journal of Economics* 74, 488–500.
- Babbel, D.F., Merrill, C.B., 2006. Rational decumulation. Wharton Financial Institutions Center Working Paper No. 06–14.
- Brown, J.R., 2003. Redistribution and insurance: Mandatory annuitization with mortality heterogeneity. *Journal of Risk and Insurance* 70 (1), 17–41.
- Brown, J.R., Orszag, P.R., 2006. The political economy of government-issued longevity bonds. *Journal of Risk and Insurance* 73 (4), 611–631.
- Brown, J.R., Poterba, J.M., 2000. Joint life annuities and annuity demand by married couples. *Journal of Risk and Insurance* 67, 527–554.
- Cairns, A.J.G., Blake, D., Down, K., 2006. Pricing death: Frameworks for the valuation and securitization of mortality risk. *ASTIN Bulletin* 36 (1), 79–120.
- Coronado, J.L., Fullerton, D., Glass, T., 2000. The progressivity of social security. NBER Working Paper, No. 7520.
- Cremer, H., Pestieau, P., 1996. Redistributive taxation and social insurance. *International Tax and Public Finance* 3, 281–295.
- Cutler, D.M., Gruber, J., 1996. Does public insurance crowd out private insurance? *Quarterly Journal of Economics* 111 (2), 391–430.
- Davidoff, Thomas, Brown, J.R., Diamond, P.A., 2005. Annuities and individual welfare. *American Economic Review* 95, 1573–1590.
- Diamond, P.A., 1992. Organizing the health insurance market. *Econometrica* 60, 1233–1254.
- Eckstein, Z., Eichenbaum, M., Peled, D., 1985. Uncertain lifetimes and the welfare enhancing properties of annuity markets and social security. *Journal of Public Economics* 26, 303–326.
- Einav, L., Finkelstein, A., Schrimpf, P., 2007. The welfare cost of asymmetric information: Evidence from the UK annuity market. NBER Working Paper No. 13228.
- Friedman, B.M., Warshawsky, M.J., 1988. Annuity prices and saving behaviour in the United States. In: Bodie, Zvi, Shoven, John B., Wise, David A. (Eds.), *Pensions in the US Economy*. The University of Chicago Press, Chicago.
- Gustman, A.L., Steinmeier, T.L., 2001. How effective is redistribution under the social security benefit formula? *Journal of Public Economics* 82, 1–28.
- Huang, R.J., Tzeng, L.Y., 2007. Insurer's insolvency risk and tax deductions for the individual's net losses. *Geneva Risk and Insurance Review* 32 (2), 129–145.
- Kaplow, L., 1992. Income tax deductions for losses as insurance. *American Economic Review* 82 (4), 1013–1017.
- Kotlikoff, L.J., Spivak, A., 1981. The family as an incomplete annuities market. *Journal of Political Economy* 89, 372–391.
- Liebman, J., 2002. Redistribution in the current U.S. social security system. In: Feldstein, M., Liebman, J. (Eds.), *Distributional Aspects of Social Security and Social Security Reform*. University of Chicago Press, Chicago, pp. 11–48.
- Milevsky, M.A., Promislow, D., Young, V., 2005. Financial valuation of mortality risk via the instantaneous sharp ratio. IFID Centre Working Paper.
- Milevsky, M.A., Promislow, D., Young, V., 2006. Killing the law of large numbers: Mortality risk premiums and the sharpe ratio. *Journal of Risk and Insurance* 73 (4), 673–686.
- Milevsky, M.A., Young, V.R., 2007. The timing of annuitization: Investment dominance and mortality risk. *Insurance: Mathematics and Economics* 40, 135–144.
- Mitchell, O., 1998. Administrative costs in public and private retirement systems. In: Feldstein, M. (Ed.), *Privatizing Social Security*. University of Chicago Press, Chicago.
- Mitchell, O.S., Poterba, J.M., Warshawsky, M.J., Brown, J.R., 1999. New evidence on the money's worth of individual annuities. *American Economic Review* 89, 1299–1318.
- Netzer, N., Scheuer, F., 2007. Taxation, insurance, and precautionary labor. *Journal of Public Economics* 91, 1519–1531.
- Olivieri, A., 2001. Uncertainty in mortality projections: An actuarial perspective. *Insurance: Mathematics and Economics* 29, 231–245.
- Post, T., Grundl, H., Schmeiser, H., 2006. Portfolio management and retirement: What is the best arrangement for a family?. *Financial Markets and Portfolio Management* 20 (3), 265–285.
- Rochet, J.C., 1991. Incentives, redistribution and social insurance. *Geneva Papers on Risk and Insurance Theory* 16, 143–165.
- Schulze, R.N., Post, T., 2007. Transferring the sticky: Individual annuity demand and demographic risk. Working Paper.
- Selden, T.M., 1993. Should the government provide catastrophic insurance? *Journal of Public Economics* 51, 241–247.
- Wilson, C., 1977. A model of insurance markets with incomplete information. *Journal of Economic Theory* 12, 167–207.
- Yaari, Menahem E., 1965. Uncertain lifetime, life insurance, and the theory of the consumer. *Review of Economic Studies* 32, 137–150.