An Increase in Background Risk and Demand for Loss Reduction

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Abstract: This paper extends the research about the impact of an increase in background risk from cases with one decision variable to those with two decision variables. We apply the results of Eeckhoudt and Kimball (1992) to examine the comparative statics of an increase in background risk on demand for loss reduction that depends on market insurance and self-insurance together. We find that individuals with decreasing absolute risk aversion and decreasing absolute prudence demand more loss reduction in the face of an increase in background risk, although they may not demand more market insurance or self-insurance. [Key words: background risk, market insurance, self-insurance, loss reduction, absolute prudence]

INTRODUCTION

Arrow (1965) and Pratt (1964) introduced measures of risk aversion for von Neumann-Morgenstern utility functions. Specifically, for a von Neumann-Morgenstern utility function u, they proposed $r = \frac{u''}{u'}$ to be an absolute measure of risk aversion. Since then, the celebrated Arrow-Pratt measures of risk aversion have become indispensable tools for analysis of decisions under uncertainty in the expected utility framework. However, the Arrow-Pratt theory of risk aversion "tends to" compare risky situations with situations of certainty, while actual economic agents are more likely to be comparing two situations of uncertainty. Indeed, it is known that the

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Arrow-Pratt measures of risk aversion are too weak for making comparisons between risky situations (Cass and Sriglitz, 1972; Hart, 1975). Ross (1981) addresses this problem and proposes a stronger measure of risk aversion to handle comparisons between risky situations. Ross's strong risk aversion condition, though powerful, is very stringent. Kihlstrom, Romer, and Williams (1981) discuss the same question from a somewhat different perspective. They try to show that the Arrow-Pratt theory of risk aversion can be applied to make comparison between risky situations if some more conditions are imposed. Whence opens a stream of research on background risks.

In ordinary life, risks are, in fact, multiple. Since markets (e.g., capital, insurance) are simply incomplete, the risks can be divided to endogenous risks, from which an agent can deliberately choose his exposure level, and exogenous risks or background risks, which are not under the control of the agent. As a consequence, choices about endogenous risks usually are made as the agent faces one or more exogenous background risks simultaneously. For example, consider the problem of investment in risky assets. The individual has random wealth, $\tilde{x}+\tilde{y}$, where $\tilde{x}$ is wealth from his portfolio of risky assets and has an endogenous risk—the individual decides his investment level—and is $\tilde{y}$ income subject to human-capital risks or endowment subject to social risks, which are background risks since the individual has no way to protect himself from such risks.

Kihlstrom, Romer, and Williams (1981) and Nachman (1982) show that the Arrow-Pratt theory of risk aversion applies not only to risks imposed on a world of certainty, but also to risks added to preexisting uncertainty (background risk), as long as the following two conditions hold. The first is a statistics condition of risks: the added risk must be independent of the preexisting uncertainty. The second is a behavior condition of utility functions: at least one of the utility functions involved should exhibit decreasing absolute risk aversion.

Pratt and Zeckhauser (1987) introduce the concept of proper risk aversion or properness, which guarantees that an undesirable risk—that is, an $\tilde{x}$ such that $Eu(\omega+\tilde{x}) \le u(\omega)$—always remains undesirable in the presence of an independent undesirable background risk, $\tilde{y}$, whether the wealth ω is fixed or independently random. However, properness has some disadvantages: The global necessary and sufficient conditions for properness are complicated; the local necessary condition, $r''(\omega) \ge r'(\omega)r(\omega)$, is simple but not globally sufficient. (Anyway, it is worth remembering that all mixtures of risk-averse exponential, power, and logarithmic functions exhibit properness.) Besides the inconvenience in tractability, properness has another disadvantage: it does not imply that being forced to face an undesirable risk always reduces the optimal investment in a security with an independent return. Kimball's standardness will remedy this shortage.

Kimball (1993) develops the concept of standard risk aversion or standardness, which guarantees that a loss-aggravating risk always aggravates an independent undesirable risk. The results also hold whether wealth is fixed or independently random. Kimball shows that, given monotonicity and concavity of the utility function, decreasing absolute risk aversion and decreasing absolute prudence are jointly necessary and sufficient for the property of standardness.[1] Thus, the inconvenience of properness argues for standardness: not only are the conditions for standardness easy to check, but standardness also entails unambiguous comparative statics results when facing introduction of an independent undesirable risk.

Gollier and Pratt (1996) define risk vulnerability as a property of utility functions that guarantees that adding an unfair background risk—an $\tilde{x}$ with $E(\tilde{x}) \leq 0$—in wealth makes risk-averse individuals behave in a more risk-averse way with respect to any other independent risk, whether the background wealth is fixed or independently random. Risk vulnerability not only entails, like standardness, unambiguous comparative statics results when facing introduction of an independent unfair risk, it is in fact the necessary and sufficient condition to ensure such result.

Gollier and Pratt obtain a necessary and sufficient condition for global risk vulnerability. The best sufficient condition the authors found was the local properness condition, $r''(\omega) \geq r'(\omega)r(\omega)$. It is very convenient and worth remembering that all risk-averse HARA utility functions[2] are risk vulnerable (indeed proper).

So far, the exogenous background risks are considered to be immutable. What is considered is the effect of introducing a background risk. A more general problem would be to consider the effect of a change in the background risk. It seems natural that an exogenous deterioration in background risk, say background wealth, will cause an individual to take more care elsewhere. When the increase in background risk is not an addition of an independent risk, what conditions entail that risk-averse individuals will take more care elsewhere? Eeckhoudt, Gollier, and Schlesinger (1996) address this problem. Specifically, they examine the deterioration of background risks in the form of general first-degree stochastic dominance (FSD) and second-degree stochastic dominance (SSD). They derive the necessary and sufficient conditions on utility function for each of these two types of background risk changes to imply more risk-averse behavior on the part of the individual. For the case of FSD changes, the condition is decreasing absolute risk aversion in Ross's stronger sense. For the case of SSD changes, the condition is locally risk-vulnerable preference in Ross's stronger sense. The conditions are fairly restrictive upon preferences. However, as Eeckhoudt et al. (1996) noted, "they [the conditions] place canonical limits upon appropriate utility representations if it is believed that individual acts in a more risk-averse manner whenever the distribution of background wealth deteriorates." Guiso, Jappelli, and Terlizzese (1996) lend some empirical supports for such belief from Italian survey data.

Besides the papers cited above, which are more theoretical in nature, several articles analyze applications of the comparative statics of an increase in background risk. Doherty and Schlesinger (1983a, 1983b) discuss whether full insurance is optimal for risk-averse individuals in the presence of background risk. Duffie and Zariphopoulou (1993) find that agents facing uncertainty in labor income reduce their investments in risky assets. Eeckhoudt and Kimball (1992) show that an individual with decreasing absolute risk aversion and decreasing absolute prudence increases his demand for insurance when faced with an increase in background risk.

Although previous research has provided many ingenious findings on the impact of an increase in background risk, most of the literature discussing this issue focuses on cases with just one decision variable. This paper intends to extend the research on the impact of an increase in background risk from cases with one decision variable to those with two decision variables. Specifically, we examine the comparative statics of an increase in background risk on demand for loss reduction, which depends on an individual's decisions regarding market insurance and self-insurance. We find that an individual will demand greater loss reduction when facing an increase in background risk if and only if his preference exhibits decreasing absolute risk aversion and decreasing absolute prudence. This result is analogous to the results of Eeckhoudt and Kimball (1992), which discusses cases with one random variable. We further find that whether the individual demands more market insurance or self-insurance depends on whether market insurance and self-insurance are normal factors[3] for producing loss reduction.

The rest of the paper is organized as follows: Section 2 explains the model. Section 3 discusses the comparative statics results, followed by the conclusion.

MODEL

We now consider the following situation. The initial wealth of the insured has a fixed component W and a random component y, which follows a distribution function $g(y, \rho)$, $y \in [a, b]$, where an increase in ρ denotes adding a negative noise—that is, Prob $[\rho \leq 0] = 1$—in risk of $g(y, \rho)$.[4] An interpretation may be that the insured has random wealth, which may be income subject to human-capital risks or endowment subject to social risks—in a word, uninsurable risks. The insured also faces a random loss $x \in [0, L]$, which follows a distribution function $f(x)$. The individual can either purchase market insurance or spend his money on self-insurance to reduce his loss, but $g(y, \rho)$ is uninsurable. Market insurance and self-insurance are defined according to the usage of Ehrlich and Becker (1972), where market insurance is a proportional insurance coverage and self-insurance is an expenditure to reduce the loss size. We adopt this usage and structure our model in a broader way. First, Ehrlich and Becker assume that the loss follows a Bernoulli distribution, which has only a fixed amount of loss. In our model, we allow a general form of loss distribution and hence allow partial losses. However, we assume that market insurance and self-insurance reduce the loss size proportionally. Note that, from the definitions we adopt, market insurance and self-insurance reduce the loss size but do not change the shape of the net loss.[5] Second, Ehrlich and Becker assume that the price of market insurance is independent of the amount of self-insurance—that is, they assume that an individual's effort for self-insurance has no impact on the market insurance premium. However, insurance companies sometimes provide premium discounts for the risk-management effort of their insured. So, in our model, we do not limit the functional form of the total expenditure of market insurance and self-insurance, allowing the market insurance premium and the expenditure for self-insurance to interact. Our model would reduce to that of Ehrlich and Becker's if the loss distribution is specified to follow the Bernoulli distribution and the premium on market insurance is independent of the expenditure of self-insurance.

Formally, let Q and C denote the amount of market insurance purchased and the efforts for self-insurance, respectively. Let $E(Q, C)$ be the total expenditure on market insurance and self-insurance, and let $V(Q, C)\tilde{x}$ be the total effects of insurance coverage and loss reduction due to market insurance and self-insurance. It is natural to assume that both $E(Q, C)$ and $V(Q, C)$ are increasing functions of Q and C. Thus, the final wealth of the insured, Z, is $W + \tilde{y} - \tilde{x} + V(Q, C)\tilde{x} - E(Q, C)$, since we assume that the market insurance and self-insurance reduce the loss size proportionally. Assume that the insured chooses the optimal insurance amount Q^* and self-insurance amount C^* to maximize his expected utility $u(Z)$, where $u'(\cdot) > 0$ and $u''(\cdot) < 0$. The model can be written as:

$$\underset{Q,\,C}{Max}\, EU(Q, C; u, Z, f, g) = \tag{1}$$
$$\int_a^b \int_0^L u(W + y - x + V(Q, C)x - E(Q, C)) f(x) g(y, \rho) dx dy.$$

Assume that the second-order conditions of Equation (1) hold to guarantee interior solutions.[6] The optimal insurance and self-insurance amount can be determined by the following first-order conditions of Equation (1):

$$EU_Q = \int_a^b U_Q(y, Q^*, C^*) g(y, \rho) dy = 0\,, \tag{2}$$

where

$$U_Q(y, Q^*, C^*) = \tag{3}$$
$$\int_0^L [V_Q x - E_Q] u'(W + y - x + V(Q^*, C^*)x - E(Q^*, C^*)) f(x) dx.$$

And

$$EU_C = \int_a^b U_C(y, Q^*, C^*) g(y, \rho) dy = 0 \tag{4}$$

where

$$U_C(y, Q^*, C^*) = \tag{5}$$
$$\int_0^L [V_C x - E_C] u'(W + y - x + V(Q^*, C^*)x - E(Q^*, C^*)) f(x) dx.$$

In the above equations, and throughout the paper, all subscripts denote partial derivatives and Q^* and C^* denote the optimal amount of Q and C, respectively.

Comparative statics results

We generate comparative statics by taking a derivative with respect to ρ in Equations (2) and (4):

$$\begin{bmatrix} EU_{QQ} & EU_{QC} \\ EU_{CQ} & EU_{CC} \end{bmatrix} \begin{bmatrix} Q^*_\rho \\ C^*_\rho \end{bmatrix} = \begin{bmatrix} -EU_{Q\rho} \\ -EU_{C\rho} \end{bmatrix}. \tag{6}$$

Thus, from Equation (6),

$$Q^*_\rho = \frac{\begin{vmatrix} -EU_{Q\rho} & EU_{QC} \\ -EU_{C\rho} & EU_{CC} \end{vmatrix}}{\begin{vmatrix} EU_{QQ} & EU_{QC} \\ EU_{CQ} & EU_{CC} \end{vmatrix}}. \tag{7}$$

$$C^*_\rho = \frac{\begin{vmatrix} EU_{QQ} & -EU_{Q\rho} \\ EU_{CQ} & -EU_{C\rho} \end{vmatrix}}{\begin{vmatrix} EU_{QQ} & EU_{QC} \\ EU_{CQ} & EU_{CC} \end{vmatrix}}. \tag{8}$$

From the second-order conditions of Equation (1), which are assumed to hold, we have

$$\begin{vmatrix} EU_{QQ} & EU_{QC} \\ EU_{CQ} & EU_{CC} \end{vmatrix} > 0. \tag{9}$$

Thus, the signs of Q^*_ρ and C^*_ρ are determined by the signs of nominators of Equations (7) and (8) respectively. That is,

$$sign(Q^*_\rho) = sign(EU_{C\rho}EU_{QC} - EU_{Q\rho}EU_{CC}). \tag{10}$$

$$sign(C^*_\rho) = sign(EU_{Q\rho}EU_{CQ} - EU_{C\rho}EU_{QQ}). \tag{11}$$

From Equations (2) and (4), at the optimal level of Q and C, we have

$$\frac{EU_Q}{V_Q} = 0\text{, and} \tag{12}$$

$$\frac{EU_C}{V_C} = 0. \tag{13}$$

By comparing Equations (12) and (13), we can find that at the optimal level of Q and C,

$$\frac{E_Q}{V_Q} = \frac{E_C}{V_C}. \tag{14}$$

From Equations (12), (13), and (14),

$$\frac{EU_{Q\rho}}{V_Q} = \frac{EU_{C\rho}}{V_C} \tag{15}$$

at the optimal level of Q and C.

Let $\frac{EU_{Q\rho}}{V_Q} = \frac{EU_{C\rho}}{V_C} = \tau$. Thus, Equations (10) and (11) can be rewritten as

$$sign(Q_\rho^*) = sign(\tau) \times sign(V_C EU_{QC} - V_Q EU_{CC}). \tag{16}$$

$$sign(C_\rho^*) = sign(\tau) \times sign(V_Q EU_{CQ} - V_C EU_{QQ}). \tag{17}$$

Theorem 1

If the preference of the individual exhibits decreasing absolute risk aversion and decreasing absolute prudence, then an increase in background risk implies $sign(\tau) > 0$.

Proof of Theorem 1

From Equation (15),

$$\tau = \int_a^b \int_0^L \left(x - \frac{E_Q}{V_Q}\right) u'(Z(Q, C, x, y)) f(x) g_\rho(y, \rho) dx dy. \tag{18}$$

Let

$$v(Z(Q, C, x), \rho) = \int_a^b u(Z(Q, C, x, y)) g(y, \rho) dy = u(Z(Q, C, x) - \pi(\rho)), \tag{19}$$

where $\pi(\rho)$ is the risk premium that the individual is willing to pay to avoid the revenue uncertainty.
Obviously,

$$v'(Z(Q, C, x), \rho) = u'(Z(Q, C, x) - \pi(\rho)). \tag{20}$$

$$v''(Z(Q, C, x), \rho) = u''(Z(Q, C, x) - \pi(\rho)). \tag{21}$$

Thus, from equations (20) and (21),

$$\frac{v''(Z(Q, C, x), \rho)}{v'(Z(Q, C, x), \rho)} = -\frac{u''(Z(Q, C, x) - \pi(\rho))}{u'(Z(Q, C, x) - \pi(\rho))}.$$

An increase in ρ represents an increase in background risk. Thus, an increase in ρ implies an increase in $\pi(\rho)$ if the preference of the individual exhibits decreasing absolute risk aversion and decreasing absolute prudence by Eeckhoudt and Kimball (1992).[7] Further, since the preference of the individual exhibits decreasing absolute risk aversion, an increase in $\pi(\rho)$ implies an increase in risk aversion of $v(Z(Q, C, x), \rho)$. Thus, an increase in ρ implies an increase in risk aversion of $v(Z(Q, C, x), \rho)$, if the preference of the individual exhibits decreasing absolute risk aversion and decreasing absolute prudence.

Therefore, Equations (12) and (18) can be rewritten as

$\int_0^L \left(x - \frac{E_Q}{V_Q}\right) v'(Z(Q, C, x), \rho) f(x) dx = 0$, and

$$\tau = \int_0^L \left(x - \frac{E_Q}{V_Q}\right) v'_\rho (Z(Q, C, x), \rho) f(x) dx.$$

Thus, based on Theorem 4 of Diamond and Stiglitz (1974), if there exists an x^* such that $\frac{\partial Z}{\partial Q} \le 0$ for $x \le x^*$ and $\frac{\partial Z}{\partial Q} \ge 0$ for $x \ge x^*$, then $\int_0^L \left(x - \frac{E_Q}{V_Q}\right) v'_\rho\,(Z(Q, C, x), \rho) f(x) dx > 0$. Since $\frac{\partial Z}{\partial Q} = V_Q x - E_Q$ and $\frac{\partial^2 Z}{\partial Q \partial x} = V_Q > 0$, $\frac{\partial Z}{\partial Q}$ increases with respect to x. Thus, $\frac{\partial Z}{\partial Q} \le 0$ for $x \le x^*$ and $\frac{\partial Z}{\partial Q} \ge 0$ for $x \ge x^*$ when $\frac{\partial Z(x^*)}{\partial Q} = 0$. Therefore, we can conclude Theorem 1.

Q. E. D.

In fact, Equations (12) and (13) are similar to the first-order condition of Eeckhoudt and Kimball (1992, their Equation 8), where they analyze background risk and demand for insurance. Since we show that $\frac{E_Q}{V_Q} = \frac{E_C}{V_C}$, Equation (18) plays a role like the comparative statics of demand for insurance with respect to background risk. That is, we find decreasing absolute risk aversion and decreasing absolute prudence are essential conditions for unambiguous comparative statics as found by Eeckhoudt and Kimball in the case of demand for insurance.

Theorem 2

If the preference of the individual exhibits decreasing absolute risk aversion and decreasing absolute prudence, then an increase in background risk implies

$$sign(Q^*_\rho) = sign[(V_{QC}E_C - E_{QC}V_C) - (V_{CC}E_Q - E_{CC}V_Q)].$$

$$sign(C^*_\rho) = sign[(V_{QC}E_Q - E_{QC}V_Q) - (V_{QQ}E_C - E_{QQ}V_C)].$$

Proof of Theorem 2

From Equation (16) and Theorem 1, we know that

$$sign(Q^*_\rho) = sign(V_C EU_{QC} - V_Q EU_{CC}).$$

From Equations (2) and (4),

$$V_C EU_{QC} =$$

$$V_C\int_a^b\int_0^L (V_{QC}x - E_{QC})u'(Z)f(x)g(y,\rho)dxdy + \tag{22}$$
$$V_QV_C^2\int_a^b\int_0^L\left(x-\frac{E_Q}{V_Q}\right)\left(x-\frac{E_C}{V_C}\right)u''(Z)f(x)g(y,\rho)dxdy\,,$$

and

$$V_QEU_{CC} =$$
$$V_Q\int_a^b\int_0^L (V_{CC}x - E_{CC})u'(Z)f(x)g(y,\rho)dxdy + \tag{23}$$
$$V_QV_C^2\int_a^b\int_0^L\left(x-\frac{E_C}{V_C}\right)\left(x-\frac{E_C}{V_C}\right)u''(Z)f(x)g(y,\rho)dxdy\,.$$

From Equation (14), the second terms in Equations (22) and (23) are the same. Thus,

$$V_CEU_{QC} - V_QEU_{CC} = \tag{24}$$
$$\int_a^b\int_0^L [(V_CV_{QC} - V_QV_{CC})x - (V_CEU_{QC} - V_QE_{CC})]u'(Z)f(x)g(y,\rho)dxdy.$$

From Equations (2) and (4),

$$\frac{\int_a^b\int_0^L xu'(Z)f(x)g(y,\rho)dxdy}{\int_a^b\int_0^L u'(Z)f(x)g(y,\rho)dxdy} = \frac{E_Q}{V_Q} = \frac{E_C}{V_C}. \tag{25}$$

From Equation (25), Equation (24) can be rewritten as:

$$V_C EU_{QC} - V_Q EU_{CC} = \tag{26}$$

$$\left[\int_a^b \int_0^L u'(Z)f(x)g(y, \rho)dxdy\right] \times$$

$$\left[V_C\left(V_{QC}\frac{E_C}{V_C} - E_{QC}\right) - V_Q\left(V_{CC}\frac{E_Q}{V_Q} - E_{CC}\right)\right].$$

Obviously, $\int_a^b \int_0^L u'(Z)f(x)g(y, \rho)dxdy$ is always positive. Thus, from Equation (26),

$$sign(Q^*_\rho) = sign[(V_{QC}E_C - E_{QC}V_C) - (V_{CC}E_Q - E_{CC}V_Q)].$$

By the same token,

$$sign(C^*_\rho) = sign[(V_{QC}E_Q - E_{QC}V_Q) - (V_{QQ}E_C - E_{QQ}V_C)].$$ **Q.E.D.**

It is very important to recognize that the comparative statics in Theorem 2 are the conditions to determine whether market insurance and self-insurance are normal factors to produce loss reduction. If we consider $V(Q,C)$ and $E(Q,C)$ to be the production function and expenditure function of loss reduction, respectively, then the minimum expenditure to produce a certain level of loss reduction can be analyzed by the following model.

$$\underset{Q,\,C}{Min} \quad E(Q, C)$$

$$s.t. \quad V(Q, C) = \theta. \tag{27}$$

It is easy to show from Equation (27) that

$$\frac{\partial Q}{\partial \theta} = (V_{QC}E_C - E_{QC}V_C) - (V_{CC}E_Q - E_{CC}V_Q), \text{ and}$$

$$\frac{\partial C}{\partial \theta} = (V_{QC}E_Q - E_{QC}V_Q) - (V_{QQ}E_C - E_{QQ}V_C).$$

The above two equations are identical to the conditions in Theorem 2. Thus, an increase in background risk increases the demand for market insurance and self-insurance depends not only on risk preference of the individual,[8] but also on whether market insurance and self-insurance are normal factors to produce loss reduction.

Moreover, if the second-order conditions of Equation (27) hold, we can show that:

Theorem 3

If the preference of the individual exhibits decreasing absolute risk aversion and decreasing absolute prudence, then an increase in background risk implies $sign(V^*_\rho(Q, C)) > \mathbf{0}$.

Proof of Theorem 3

From Theorem 2, we know that

$$sign(V^*_\rho(Q, C)) = V_Q sign(Q^*_\rho) + V_C sign(C^*_\rho). \tag{28}$$

Equation (28) is nothing but the second-order condition of Equation (27). Therefore,

$$sign(V^*_\rho(Q, C)) > 0 \quad \textbf{Q.E.D.}$$

We can consider that the individual makes his or her decision in two steps when facing an increase in background risk. In the first step, the individual decides whether to increase the demand for loss reduction. Then in the second step he or she decides how to increase loss reduction, if it is so wanted. The risk preference of the individual plays a key role, in the first step, like that in Eeckhoudt and Kimball (1992). In the second step, the normality of factors is essential to determine the final decision.

Theorem 3 shows that an individual with decreasing absolute risk aversion and decreasing absolute prudence always increases his demand for loss reduction with respect to an increase in background risk. Therefore, by means of Theorems 2 and 3 together, we can conclude that the individual increases market insurance (self-insurance) if market insurance (self-insurance) is a normal factor for producing loss reduction.

CONCLUSION

This paper applies the finding of Eeckhoudt and Kimball (1992) to analyze comparative statics of an increase in background risk when individuals need to make two decisions together. We examine the comparative statics of an increase in background risk on the demand for loss reduction, which depends on market insurance and self-insurance together. We find that an individual with decreasing absolute risk aversion and decreasing absolute prudence demands more loss reduction when faced with an increase in background risk. Moreover, the individual's demand for more market insurance and self-insurance depends on whether market insurance and self-insurance are normal factors for producing loss reduction. Our model can be extended to analyze other problems, such as the interaction between production and insurance as well as the interaction between saving and insurance.

NOTES

[1] Decreasing absolute risk aversion says that the measure of absolute risk aversion is decreasing in wealth, or formally, $\frac{\partial r(\omega)}{\partial \omega} < 0$. It is implied by such behavior as investing more in risky securities as one becomes wealthier and is almost universally considered a reasonable assumption. Kimball (1990) gives the name "prudence" to the sensibility of the optimal choice of a decision variable to risk. Analogously to Arrow-Pratt's measure of risk aversion, he gives an absolute measure of prudence $p(\omega) = -\frac{u'''(\omega)}{u''(\omega)}$.

[2] A utility function is HARA if the reciprocal of its absolute measure of risk aversion is linear in wealth. All CARA, CRRA utility functions are HARA.

[3] A normal factor is such a factor of production that the demand for this factor increases when there is an (infinitesimal) increase in output.

[4] Note that the support of $\tilde{y}$ has been set large enough to cover all relevant outcomes for all relevant ρ.

[5] Ehrlich and Becker also discuss another type of risk management, self-protection, which influences the loss distribution rather than the loss size. A further extension of our model to include self-protection is possible but may cloud the current focus of this paper. Thus, we do not consider self-protection in our model for the simplicity of demonstrating our points.

[6] The conditions may not hold. However, when the conditions fail, the maximization solution may not exist or may be a corner solution. Such insurance setting is unordinary and out of our consideration. So we set the assumption to focus attention on relevant cases.

[7] The results of Eeckhoudt and Kimball (1992) are more general than what we need here. They show that the result holds not only for adding a negative noise in background risk, but also for adding any independent undesirable background risk. More than this, most of their paper discusses a more general situation indicating that there is some "positive relationship" between background risk and the other risk—the distribution of background risk conditional upon a given level of insurable loss deteriorates in the sense of third-order stochastic dominance as the amount of insurable loss increases.

[8] In Eeckhoudt and Kimball (1992), whether an increase in background risk will increase the demand for insurance depends only on the risk preference of the individual since there is only one decision variable in their model.

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