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Interaction between a Production Decision and the Demand for Insurance under Revenue Uncertainty and Background Risk

營業風險與營業外風險下廠商生產決策與保險決策的互動關係

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摘要

本論文使用 Gollier (1995)和 Eeckhoudt, Gollier, and Schlesinger (1996)的研究成果，將 Machnes (1993)的研究延伸到同時包含生產決策與保險決策的狀況。本論文發現公司營業風險或營業外風險的增加不必然一定使得一個趨避風險的廠商減少生產或是增加保險，本論文進一步提供廠商減少生產或是增加保險的條件，更重要的是本論文的結果可以推廣到包含兩個決策變數的其他問題。

關鍵詞：風險優勢、生產決策、保險需求、背景風險。

Abstract

This paper extends the research about the impact of an increase in risk and background risk from cases with one decision variable to those with two decision variables. Using the results of Gollier (1995) and Eeckhoudt, Gollier, and Schlesinger (1996), we generalize Machnes' s study (1993) to analyze the interaction between a firm' s production decision and the demand for insurance. We find that an increase in revenue risk and background risk may not always lead a risk-averse firm to produce less or insure more. We further provide conditions for unambiguous comparative statics of increases in firm' s revenue risk and background risk.

Keywords: Central Risk Dominance, Background Risk, Production Decision, Demand for Insurance.

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1. Introduction

Rothschild and Stiglitz (1970, 1971) found that risk-averse individuals may not take less risky assets after an increase in risk. Although their results were not determinant, their question stimulated many researchers and led to two approaches for generating unambiguous comparative statics. Under the first approach, a number of researchers (Meyer and Ormiston, 1983, 1985; Black and Bulkley, 1989; and Eeckhoudt and Hansen, 1980, 1983) considered an increase in risk with certain constraints when decision makers are still assumed to be risk-averse. At the same time, other researchers (Dreze and Modigliani, 1972; Diamond and Stiglitz, 1974; Dionne and Eeckhoudt, 1987; and Briys, Eeckhoudt, and Dionne, 1989) considered a more confined utility function in which the underlined assets still face an increase in risk. A major contributor to this issue is Gollier (1995), who showed that central risk dominance is the necessary and sufficient condition for unambiguous comparative statics of an increase in risk for all risk-averse individuals.

On the other hand, when making a decision, an individual usually faces multiple risks in his ordinary life. However, the comparative statics of adding one more risk had not been examined very closely until recent research successfully identified the linkage between an increase in background risk and an increase in risk aversion. Gollier and Pratt (1996) show that an individual facing a statistically independent increase in background risk becomes more risk-averse if and only if the individual's preference is risk-vulnerable. They also show that proper risk aversion—defined by Pratt and Zeckhauser (1987)—and standard risk aversion—defined by Eeckhoudt and Kimball (1992)—are special cases of risk vulnerability but that risk vulnerability is a sufficient condition of decreasing absolute risk aversion. Eeckhoudt, Gollier, and Schlesinger (1996) show that if and only if the individual's preference is to decrease absolute risk aversion in the sense of Ross, then a second-order stochastic deterioration in background wealth makes the individual more risk-averse.

Although these previous studies have provided many ingenious findings on the impact of an increase in risk and background risk, most of the literature has analyzed this issue when there is only one decision variable. This paper intends to extend the research on the impact of an increase in risk and background risk from cases with one decision variable to those with two decision variables. The results of this paper can be applied to many problems, such as interaction of self-insurance and market insurance (Ehrlich and Becker, 1972) and portfolio decisions of insurance and investments (Mayers and Smith, 1982).

To analyze the comparative statics of an increase in risk with two decision variables, this paper generalizes the production problem provided by Machnes (1993). Machnes found that any firm, regardless of its preference towards risk, increases its production when there is a monotone likelihood ratio change in the firm's demand uncertainty. He also found that any firm with decreasing absolute risk aversion increases its production level when there is a monotone likelihood ratio change in the firm's background assets.

This paper differs from Machnes's study (1993) in two ways. First, we assume that the firm needs to make both production and insurance decisions rather than a production decision alone. Second, unlike Machnes's study, which used monotone likelihood ratio dominance to characterize increases in risk and background risk, we use central risk dominance — proposed by Gollier (1995) — and second-order stochastic dominance — analyzed by Eeckhoudt, Gollier, and Schlesinger (1996) — to define increases in revenue risk and background risk, respectively. It is worth to recognize that monotone likelihood ratio dominance is a special case of second-order stochastic dominance. Thus, some results in the paper are considered to be more general than those in Machnes (1993). Moreover, theorems found by Gollier (1995) and Eeckhoudt, Gollier, and Schlesinger (1996) are necessary and sufficient conditions, whereas those in Machnes (1993) are sufficient conditions. Thus, by using results of Gollier (1995) and Eeckhoudt, Gollier, and Schlesinger (1996), we can generate the necessary and sufficient conditions in Theorems 3 and 4. Furthermore, many papers (Eeckhoudt and Kimball, 1992; and Gollier 1995) have applied central risk dominance and second-order stochastic dominance in the research of insurance. Thus, the findings of the paper can be further integrated with the literature of research in future studies.

In this paper, we find that an increase in revenue risk and background risk may not always lead a risk-averse firm to produce less or insure more. Instead, a risk-averse firm reduces its possible losses when its revenue faces an increased risk with central risk dominance. On the other hand, we find that an increase in the background risk with second-order stochastic dominance makes a firm with Ross's decreasing absolute risk aversion reduce its possible losses. Furthermore, we also find that when a firm's revenues or background assets face an increase in risk, the firm reduces its production level (increases its insurance purchasing) if production (insurance) is a normal (inferior) factor for maximizing its possible profits.

2. The Basic Model

Let us assume that a firm faces revenue uncertainty and risks of background assets. The initial wealth of the firm is $W + X$, where W is a fixed amount and $X \in [\underline{x}, \overline{x}]$ is a random variable and represents the background risk of the firm. Respectively, $h(X)$ and $H(X)$ denote the probability density function and accumulative density function of X. Let Q denote the output of the firm. Both the firm's revenue $R(Q)$ and the production costs $C(Q)$ are functions of the firm's output. Because the firm's revenue faces a random situation, this uncertainty is expressed as $(1-Y)R(Q)$, where $Y \in [0,1]$ denotes the revenue uncertainty. Let $f(Y)$ and $F(Y)$ denote, respectively, the probability density function and the accumulative density function of Y. The firm can purchase insurance, $a \in [0,1]$, to reduce its revenue uncertainty. Therefore, the firm's revenue after insurance is $(1-(1-a)Y)R(Q)$. The insurance premium is equal to an actuarially fair price plus expense loading, l, and is denoted as $(1+l)amR(Q)$, where m is the mean of Y. Assume that there exists a time lag between the occurrence of losses and payments of insurance premium and the time value caused by interest rate r can be measured by $[W-(1+l)amR(Q)](1+r)$ also uncertain. Thus, the final wealth of the firm, Z, is $W(1+r)+X+(1-(1-a)Y)$ $R(Q)-C(Q)-(1+l)amR(Q)(1+r)$. It is assumed that the firm will choose optimal production and insurance to maximize its expected utility, u, as shown in the following model:

$$\underset{Q,a}{Max} \quad EU=\int_{\underline{x}}^{\bar{x}}\left[\int_0^1 u(Z)\,dF(Y)\right]dH(X)\,. \tag{1}$$

The optimal solutions of the firm can be solved by the following first-order conditions:

$$\frac{\partial EU}{\partial Q}=\int_{\underline{x}}^{\bar{x}}\int_0^1 \frac{\partial Z}{\partial Q}u'(Z)\,dF(Y)\,dH(X)=0, \tag{2}$$

where $\dfrac{\partial Z}{\partial Q}=(1-(1-a)Y)R'(Q)-C'(Q)-(1+l)amR'(Q)(1+r)$.

$$\frac{\partial EU}{\partial a}=\int_{\underline{x}}^{\bar{x}}\int_0^1 \frac{\partial Z}{\partial a}u'(Z)\,dF(Y)\,dH(X)=0, \tag{3}$$

where $\dfrac{\partial Z}{\partial a}=YR(Q)-(1+l)mR(Q)(1+r)$.

From Equations (2) and (3),

$$\frac{C^{'}(Q)}{R^{'}(Q)}=1-(1+l)m. \tag{4}$$

Let us assume that the revenue uncertainty is shifted from $f(Y)$ to $g(Y)$ due to a mean preserving spread. On the basis of Gollier (1995), we can define the central risk dominance of an increase in risk as follows:

Definition 1

An increase in risk from $f(Y)$ to $g(Y)$ is called central risk dominance, $CD_g(a)$, if and only if there exists a $g \ni t_g(Y) \le g t_f(Y), \forall Y$, where

$$\boldsymbol{t}_f(Y) = \int_0^Y [t-(1+l)(1+r)m]\ f(t)dt \quad \textbf{\textit{and}}$$
$$\boldsymbol{t}_g(Y) = \int_0^Y [t-(1+l)(1+r)m] g(t)dt\,.$$

Thus,

Theorem 1

If a firm's revenue faces a mean preserving spread with $CD_g(a)$, the risk-averse firm increases its insurance coverage while maintaining its production level.

Proof of Theorem 1

Because the increase in risk is mean-preserved, it is obvious from Equation (4) that even a risk-averse firm maintains the same level of production. Moreover, the first-order conditions, Equations (2) and (3), are deemed to be equivalent to Equations (3) and (4). Since the firm's production level remains the same, a central risk dominance with $CD_g(a)$ provides unambiguous comparative statics in Equation (3) for a risk-averse firm to increase its insurance coverage on the basis of Gollier (1995). ***Q. E. D.***

Let us assume that a firm's background assets face an increase in risk with second-order stochastic dominance (SSD), as defined by Rothschild and Stiglitz (1970, 1971). We then find that

Theorem 2

An SSD increase in background risk makes a firm with Ross's decreasing absolute risk aversion increase its insurance coverage while maintaining its production level.

Proof of Theorem 2

Obviously, Equation (4) is independent of background risk. Thus, an increase in background risk leads the firm to maintain the same level of production. Moreover, on basis of Eeckhoudt, Gollier, and Schlesinger

(1996), if the individual's preference is to decrease absolute risk aversion in the sense of Ross, then a second-order stochastic deterioration in background wealth makes the individual more risk-averse. It is a well-known application of Diamond and Stiglitz (1974) that a more risk-averse firm demands more insurance. Thus, on basis of Eeckhoudt, Gollier, and Schlesinger (1996) and Diamond and Stiglitz (1974), it follows from Equation (3) that an SSD increase in background makes a firm of Ross's decreasing absolute risk aversion more risk-averse and leads it to demand more insurance. ***Q. E. D.***

3. The General Model

In this section, we relax the assumptions that the insurance available is a proportional insurance and that the insurance premium is an actuarially fair price plus expense loading. We also allow that the purchase of insurance may have certain impacts on the firm's revenue. Let $\pi(Q,\alpha)$ and $L(Q,\alpha)$ denote the possible profits and losses of the firm, respectively. Other assumptions are the same as those in the previous section on the basic model. Thus, the firm's final wealth can be expressed as $Z = W(1+r) + X + \pi(Q,\alpha) - L(Q,\alpha)Y$. The firm chooses optimal production and insurance to maximize its expected utility as in the following model:

$$\underset{Q,\alpha}{Max} \quad EU = \int_a^b [\int_0^1 u(Z)dF(Y)]dH(X)\,. \tag{5}$$

The firm's optimal solutions can be solved by the following first-order conditions:

$$\frac{\partial EU}{\partial Q} = 0 \text{ ,and} \tag{6}$$

$$\frac{\partial EU}{\partial \alpha} = 0\,. \tag{7}$$

From Equations (6) and (7),

$$\frac{p_Q}{p_a} = \frac{L_Q}{L_a} . \tag{8}$$

Although Equation (8) is similar to Equation (4), we cannot conclude that the firm' s production level is fixed. Thus, the comparative statics of revenue uncertainty and an increase in background risk are not determined generally.

To obtain unambiguous comparative statics, we can transfer this problem into two steps. In the first step, the firm' s possible profits are maximized, given a certain level of the possible losses, q . The model can be written as:

$$\underset{Q,a}{Max} \quad p(Q,a)$$

$$\text{s.t.} \qquad L(Q,a) = q . \tag{9}$$

From the above model, the optimal production level and insurance purchases can be expressed by q . Thus, the firm' s final wealth can be expressed as $Z(q) = W(1+r) + X + p(q) - qY$.

In the second step, the firm chooses an optimal q to maximize its expected utility.

$$\underset{q}{Max} \qquad EU = \int_a^b [\int_0^1 u(Z(q))dF(Y)]dH(X) . \tag{10}$$

The optimal solution of q can then be solved by the following first-order condition:

$$\frac{dEU}{dq} = 0 . \tag{11}$$

It is worth to recognize that the first order conditions in the two-step optimization are identical to those in the original model, i.e., that Equation (11) and the first order conditions of Equation (9) are identical to Equations (6), (7), and (8).

Let us assume that revenue uncertainty is shifted from $f(Y)$ to $g(Y)$ due to an increase in risk. On basis of Gollier (1995), we can define the central risk dominance of an increase in risk as follows:

Definition 2

An increase in risk from f(Y) to g(Y) is called central risk dominance, $CD_g(q)$***, if and only if there exists a*** $g \ni T_g(Y) \le gT_f(Y), \forall Y$ ***, where*** $T_f(Y) = \int_0^Y [p'(q) - t] f(t) dt$ ***and*** $T_g(Y) = \int_0^Y [p'(q) - t] g(t) dt$***.***

From Equation (11), we know

Theorem 3

If and only if the revenue of the firm faces an increase in risk with $CD_g(q)$***, the risk-averse firm reduces its possible losses.***

Proof of Theorem 3

The proof of Theorem 3 is simply an application of Gollier (1995). ***Q. E. D.***

It is very important to recognize that a mean-preserved spread condition is required for Theorem 1 but unnecessary for Theorem 3.

Let us assume that the firm's background assets face an increase in risk with Second-order stochastic dominance, as defined by Rothschild and Stiglitz (1970, 1971). We find that

Theorem 4

An SSD increase in background risk makes a firm with Ross's decreasing absolute risk aversion reduce its possible losses.

Proof of Theorem 4

The proof of Theorem 4 is simply an application of Eeckhoudt, Gollier, and Schlesinger (1996) and Diamond and Stiglitz (1974). ***Q. E. D.***

By transferring the direct optimization process into a two-step indirect optimization process, we find conditions for a firm to reduce its possible losses. Moreover, we can further determine unambiguous comparative statics in the firm's production decision and demand for insurance.

Theorem 5

When the revenue of the firm faces an increase in risk with $CD_g(q)$ **(*when the firm's background assets face an increase in risk of SSD*), *the risk-averse* (*Ross's decreasing absolute risk aversion*) *firm reduces its production level if*** $\frac{\partial Q}{\partial q} > 0$ ***and increases its insurance purchasing if*** $\frac{\partial a}{\partial q} < 0$**.**

Proof of Theorem 5

$p(Q,a)$ and $L(Q,a)$ are the possible profits and losses of the firm, respectively. $\frac{\partial Q}{\partial q} > 0$ and $\frac{\partial a}{\partial q} < 0$ imply that the firm increases its production level and demand for insurance if the firm takes more risks (possible losses). Since we can conclude that, under respective conditions, an increase in risk (background risk) makes the firm take fewer risks, the firm reduces its production level if $\frac{\partial Q}{\partial q} > 0$ and increases its insurance purchasing if $\frac{\partial a}{\partial q} < 0$. ***Q. E. D.***

4. Conclusion

This paper generalizes Machnes's study (1993) to analyze the comparative statics of increases in risk and background risk when the firm needs to make both production and insurance decisions rather than only a production decision. It provides conditions for unambiguous comparative statics of increases in risk on revenues and underground assets. In addition, this paper employs a more general model, where the firm's possible profits and losses are functions of the firm's production and demand for insurance, respectively. We use two-step optimization to generate useful conditions for comparative statics of an increase in revenue risk and background risk.

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