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Revisiting the Demand for Insurance
保險需求的再探討

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Abstract

This paper first derives the Hicksian demand for insurance and further applies the famous Slutsky Equation to link the Marshallian and Hicksian demands for insurance. It shows that the Slutsky Equation of insurance can provide additional explanations for insurance markets. The connection between the income effect and the substitution effect in insurance markets is discussed. We further demonstrate how to apply our results both in the theory and empirical studies. We also provide some implications of our results in liability insurance market.

Key Words: Demand for Insurance, Wealth Effect, Substitution Effect.

摘　要

本篇論文首先推導保險的 Marsharllian 需求與 Hicksian 需求，並利用 Slutsky 定理重新檢視保險市場中的所得效果與替代效果，論文進一步討論本文在理論與實證上可能的應用，最後我們提供應用的實例，並說明此一實例在責任保險上可能的意涵。

關鍵字：保險需求，財富效果，替代效果。

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1. Introduction

Mossin (1968) pioneered how to derive the demand for insurance and demonstrated that a risk-averse insured chooses to buy less than a full amount of coverage if the price of the insurance is higher than the actuarially fair price. Moreover, he found that a decreasing risk-averse insured considers insurance an inferior good. Since then, many studies—such as those by Hoy and Robson (1981) and Briys, Dionne, and Eeckhoudt (1988)—have applied Mossin's model to derive the demand for insurance and have found that insurance may be a Giffen good.

At the same time, a number of other studies—including those by Outreville (1990), Truett and Truett (1990), Cleeton and Zellner (1993), Browne and Kim (1993), Showers and Shotick (1994), Eisenhauer (1997), Meier (1998), and Enz (2000)—have used Mossin's model to generate predictions of income and price effects on insurance and have examined them by means of empirical data. The empirical results on whether insurance is a Giffen good have been mixed. Most of the research has found that people tend to purchase more insurance with respect to an increase in wealth and a decrease in premium. Thus, empirical studies seem to support that insurance is a normal good and could be an ordinary good, whereas the theory predicts that, for a decreasing risk-averse insured, insurance is an inferior good and may be a Giffen good.

Although these theoretical papers[1] have generated many insights for analyzing insurance demand, they have never demonstrated that the demand for insurance generated by Mossin's work (1968) is indeed a Marsharllian demand. It is well known in the literature that the slope of a Marshallian demand may not always be negative. Thus, it is no surprise that previous empirical studies testing a Marsharllian demand for insurance have displayed

1 For readers who are interested in more on insurance economics, Dionne and Harrington (1992, pp. 1-48) provide a survey of the literature, including demand for insurance.

mixed results.

This paper intends to derive the Hicksian demand for insurance. Conventionally, Marshallian demand can be derived by the model, which maximizes individual's utility subject to a budget constraint. On the other hand, Hicksian demand is derived by the model, which minimizes consumer's expenditure subject to a level of utility. Although the concept of a Hicksian demand is well recognized in the economics literature, no research has ever specifically derived a Hicksian demand for insurance, as far as we know.

Both Marsharllian demand and Hicksian demand have their own places in economics theory. Which of them is more useful depends on the problem we intend to study. Hicksian demand generally provides essential insights on cost-benefit analysis in public economics where insurance also plays an important role. Thus, the study on Hicksian demand for insurance can help the theoretical development of cost-benefit analysis in insurance.

On the other hand, Marsharllian demand and Hicksian demand generate different measurements for income effect and substitution effect. Both income effect and substitution effect on demand for insurance are key issues in empirical studies in insurance. Thus, Hicksian demand for insurance could provide another approach to exam the empirical data and could produce more understanding on individual's behavior for purchasing insurance.

In this paper, we show that the slope of a Hicksian demand for insurance is always negative, consistent with the findings in the economics literature. We further generate the Slutsky Equation in insurance markets. We demonstrate that, consistent with Mossin's model based on a Marsharllian demand for insurance, a Hicksian demand and the Slutsky Equation for insurance provide further understanding of income and price effects on insurance in both theory and empirical research. We also provide examples to illustrate possible applications of the paper and some implications in liability insurance market.

2. Models

2.1 Mossin's Model

Let the insured with initial wealth W cope with an insurable loss L, with loss probability π[2]. The insured pays a premium $P \cdot Q$, where Q is the insurance coverage[3] and P is the unit price per insurance coverage. The insured receives a payoff Q when loss occurs. Let us assume that the insured makes decisions so as to maximize his expected utility. Let the insured possess a twice-differentiable utility function, where the utility function is strictly increasing and strictly concave downward. If the insured selects the demand for insurance to maximize his expected utility, Mossin's (1968) model can be rewritten as:

$$\underset{Q}{MAX} \quad EU = \pi u[W - PQ - L + Q] + (1 - \pi)u[W - PQ]. \tag{1}$$

The first order condition of the above model can be written as:

$$\pi(-P + 1)u' \, [W - PQ - L + Q] - (1 - \pi)Pu'[W - PQ] = 0. \tag{2}$$

Equation (2) can be rewritten implicitly as:

$$D(Q; P, W, \pi, L) = 0, \tag{3}$$

where D denotes the Marshallian demand for insurance.

Although the above model is very straightforward, it is not the standard form in economics that can generate a Marshallian demand curve. A Marshallian

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2 The process to derive a Hicksian demand for insurance may appear obvious and trivial when the loss distribution follows a Bernoulli distribution. Indeed, the process is robust for a continuous loss distribution. Without the insight generated from the case, where the loss follows a Bernoulli distribution, a Hicksian demand for insurance for a continuous loss distribution may not be easy to derive. For clear demonstration, we keep the assumption of Bernoulli distribution.

3 For simplicity, insurance is considered as the only one decision variable in the model. In real practice, individual may need to cope with multiple decisions (Mayers and Smith, 1983), such as saving and insurance (Moffet, 1975, 1977; Dionne and Eeckhoudt, 1984) and market insurance and self-insurance (Ehrlich and Becker, 1972). To generate the Hicksian demand for insurance, the paper focuses only on insurance decision itself. Any generalization from one decision (insurance only) to multiple decisions could provide fruitful results and deserve further research.

demand is commonly derived through a model in which the decision maker maximizes his/her utility function subject to a budget constraint. While equation (2) is not the standard form in economics that can generate a Marshallian demand curve, equation (3) can be used to determine a Marshallian demand for insurance.

2.2 A Marshallian Demand for Insurance

It is easy to generate a Marshallian demand for insurance through equation (1).

Let $X = Q$, and

$Y = W - PQ$.

X and Y can be considered as two goods the individual consumes. Obviously, X is the insurance coverage. On the other hand, Y is individual's final wealth when no loss happens. Moreover, $PX + Y = W$ plays a role like a budget constraint where insurance price P denotes the relative price between wealth and insurance coverage. Thus, by variable transformation, Marshallian demand for insurance can be derived from the following model:

$$\begin{aligned} &\underset{X,Y}{MAX} \quad EU = \pi u[X+Y-L] + (1-\pi)u[Y], \\ &\text{such that } \mathrm{P} \cdot X + Y = W. \end{aligned} \tag{4}$$

Although the above model is just a deviated model from Mossin (1968), it provides a way to generate a Hicksian demand for insurance. Equation (4) demonstrates that Marshallian demand for insurance (X) is derived by maximizing individual's utility on goods X and Y ($\pi u[X+Y-L] + (1-\pi)u[Y]$) subject to a budget constraint ($PX + Y = W$). Therefore, Hicksian demand for insurance can be derived by minimizing individual's expenditure ($PX + Y$) on goods X and Y subject to a level of utility ($\pi u[X+Y-L] + (1-\pi)u[Y]$).

2.3 A Hicksian Demand for Insurance

Equation (4) can be transferred into a model to analyze a Hicksian demand for insurance that may not be easy to do directly from Mossin (1968). A Hicksian demand for insurance can be derived from the following model:

$$\underset{X,Y}{MIN} \quad E = P \cdot X + Y, \qquad (5)$$
$$such\, that \;\; \pi u[X + Y - L] + (1-\pi)u[Y] = \bar{U},$$

where E = total expenditure and

$\bar{U}$ = a level of utility.

Let $Q_X(P,W)$ and $H_X(P,\bar{U})$ denote a Marshallian demand and a Hicksian demand for insurance, respectively. That is, $Q_X(P,W)$ and $H_X(P,\bar{U})$ are the solutions of equations (4) and (5), respectively.

Proposition 1

A sufficient but not necessary condition for a downward sloping Marshallian demand curve for insurance is that the insured is non-decreasing absolute risk-averse.

Proof

Equation (2) can be rewritten implicitly as:

$$D(Q; P, W, \pi, L) = 0. \qquad (3)$$

Thus, from equation (3),

$$\frac{\partial D}{\partial Q} = \pi(-P+1)^2 u''[W - PQ - L + Q] + (1-\pi)P^2 u''[W - PQ] < 0. \qquad (6)$$

$$\frac{\partial D}{\partial P} = -\pi u'[W - PQ - L + Q] - (1-\pi)u'[W - PQ] - Q\frac{\partial D}{\partial W}. \qquad (7)$$

And

$$\frac{\partial D}{\partial W} = \pi(-P+1)u''[W - PQ - L + Q] - (1-\pi)Pu''[W - PQ]. \qquad (8)$$

If the insured's absolute risk-aversion index is a non-decreasing function, then

$$-\frac{u''[W - PQ - L + Q]}{u'[W - PQ - L + Q]} \le -\frac{u''[W - PQ]}{u'[W - PQ]}. \qquad (9)$$

Equation (2) can be rewritten as:

$$\pi(-P+1)u'\,[W-PQ-L+Q]=(1-\pi)Pu'[W-PQ]. \tag{10}$$

Now let us multiply equation (9) by equation (10). It then follows that:

$$\frac{\partial D}{\partial W}\geq 0. \tag{11}$$

From equations (7) and (11),

$$\frac{\partial D}{\partial P}<0.$$

By the implicit function theorem,

$$\frac{\partial Q_X(P,W)}{\partial P}=-\frac{\partial D/\partial P}{\partial D/\partial Q}.$$

Since $\frac{\partial D}{\partial Q}<0$, the sign of $\frac{\partial Q_X(P,W)}{\partial P}$ is determined by the sign of $\frac{\partial D}{\partial P}$. Therefore, $\frac{\partial Q_X(P,W)}{\partial P}<0.$ *Q. E. D.*

By the implicit function theorem, $\frac{\partial Q_X(P,W)}{\partial W}=-\frac{\partial D/\partial W}{\partial D/\partial Q}$. $\frac{\partial D}{\partial Q}<0$, if the second order conditions of the model are assumed to hold. Thus, the sign of $\frac{\partial Q_X(P,W)}{\partial W}$ is determined by the sign of $\frac{\partial D}{\partial W}$. Therefore, equation (11), $\frac{\partial D}{\partial W}\geq 0$, is actually one step away to show that non-decreasing absolute risk aversion is indeed the sufficient and necessary condition for insurance as a non-inferior good, $\frac{\partial Q_X(P,W)}{\partial W}\geq 0$, as demonstrated by Mossin (1968). Consistent with the economics literature, non-inferior goods imply non-Giffen goods, i.e., non-negative wealth effect implies positive non-negative substitution effect, Proposition 1 shows that individual demands less insurance with respect to an increase in insurance price if the risk preference of the individual is non-decreasing absolute risk aversion. It is very important to recognize that constant-absolute-risk-aversion utility function and mean-variance utility function which are commonly used in finance and insurance literature belong to the class of non-decreasing absolute risk aversion.

Proposition 2

A Hicksian demand curve for insurance is always downward sloping for all risk-averse individuals.

Proof

Let λ denote the LaGrange multiplier. The first order conditions of equation (5) can be written as:

$$P = \lambda \pi u'[X + Y - L],$$

$$1 = \lambda\{\pi u'[X + Y - L] + (1-\pi)u'[Y]\}, \text{ and}$$

$$\pi u[X + Y - L] + (1-\pi)u[Y] = \bar{U}.$$

The above first order conditions can be further rewritten as:

$$\pi(1-P)u'[X + Y - L] - P(1-\pi)u'[Y] = 0, \text{ and}$$

$$\pi u[X + Y - L] + (1-\pi)u[Y] = \bar{U}.$$

To get the comparative statics of an increase in insurance price, take the derivative with respect to P on the above first order conditions. Thus,

$$\begin{pmatrix} \pi(1-P)u''[X + Y - L] & \pi(1-P)u''[X + Y - L] - P(1-\pi)u''[Y] \\ \pi u'[X + Y - L] & \pi u'[X + Y - L] + (1-\pi)u'[Y] \end{pmatrix} \begin{pmatrix} \dfrac{\partial X}{\partial P} \\ \dfrac{\partial Y}{\partial P} \end{pmatrix}$$

$$= \begin{pmatrix} \pi u'[X + Y - L] + (1-\pi)u'[Y] \\ 0 \end{pmatrix}.$$

After some algebraic arrangement, we can get

$$\frac{\partial X}{\partial P} = \frac{\pi u'[X + Y - L] + (1-\pi)u'[Y]}{\pi(-P+1)^2 u''[X + Y - L] + (1-\pi)P^2 u''[Y]} < 0.$$

Thus, by variable transformation

$$\frac{\partial H_X(P,\bar{U})}{\partial P} = \frac{\pi u'[W - PQ - L + Q] + (1-\pi)u'[W - PQ]}{\pi(-P+1)^2 u''[W - PQ - L + Q] + (1-\pi)P^2 u''[W - PQ]} < 0.$$

(12)

Q. E. D.

Consistent with the economics literature, Proposition 2 shows that a Hicksian demand curve for insurance is always downward sloping. On the other hand, Propositions 1 and 2 provide the rationale to explain why the demand for insurance may not always be a normal/ordinary good under Mossin's model. Since Mossin (1968) constructed the demand for insurance on the basis of a Marshallian demand rather than Hicksian demand, Proposition 1 confirms Mossin's theorem. To explore the interaction of wealth effect and substitution effect, we further derive the Slutsky Equation of insurance, the linkage between a Marshallian demand and a Hicksian demand.

2.4 Slutsky Eqauition for Insurance and Implications for Empirical Studies

The Slutsky equation of insurance can be written as:

$$\frac{\partial Q_X(P,W)}{\partial P} = \frac{\partial H_X(P,\bar{U})}{\partial P} - \frac{\partial Q_X(P,W)}{\partial W} Q_X(P,W). \tag{13}$$

It is easy to show equation (13) given equations (8) and (12). Equation (13) provides a clear explanation for the linkage of Propositions 1 and 2. Non-decreasing absolute risk aversion is the sufficient and necessary condition for $\frac{\partial Q_X(P,W)}{\partial W} > 0$. It is not difficult to see $\frac{\partial H_X(P,\bar{U})}{\partial P} < 0$. Therefore, non-decreasing absolute risk aversion is the sufficient condition for $\frac{\partial Q_X(P,W)}{\partial P} < 0$. Indeed, this is an application of a famous theorem in economics, under which a normal good is an ordinary good but not vice versa. Although this finding is well known in economics, none of the insurance literature has ever found a way to generate a Hicksian demand for insurance and derived the Slutsky equation for insurance. This paper serves to fill this gap.

Moreover, equation (13) can be rewritten as: for any risk-averse individual, $\frac{\partial Q_X(P,W)}{\partial P} + \frac{\partial Q_X(P,W)}{\partial W} Q_X(P,W) < 0$. It is easy to show the above formula given Propositions 2 and equation (13). Thus, equation (13) can provide another hypothesis for empirical research to exam. It should be recognized that Proposition 1 holds for a subset of risk averse individuals whereas Proposition 2 holds for all risk averse individuals. Thus, equation (13) is a more robust hypothesis empirical studies can exam, since the assumption of risk aversion is more well-accepted than the assumption of non-decreasing absolute risk aversion.

Many studies examine the demand for insurance by using the following regression model:

$$Q = \alpha_0 + \alpha_P P + \alpha_W W + \alpha_\theta \theta + \varepsilon \,, \tag{17}$$

where Q is insurance demand,

P is insurance price,

W is individual's wealth, and

θ are other variables.

Most papers, therefore, have tested whether $\alpha_P < 0$ and $\alpha_W > 0$ in equation (17). However, the above two hypotheses can not hold for all risk-averse individuals. If the insured is constant risk-averse or increasing risk-averse, it is reasonable to believe that $\alpha_W > 0$ from Mossin's theorem (1968). However, if the insured is decreasing risk-averse, also suggested by Mossin (1968), the sign of α_W is negative rather than positive. Thus, depending on the assumption of individual's risk preference, the wealth effect on demand for insurance could be either positive or negative. The traditional theory seems to fail providing a precise prediction for empirical studies to exam. Moreover, if the insured is decreasing risk-averse, on basis of Proposition 1, there is no reason to believe that $\alpha_P < 0$. Again, traditional theory can not provide an unambiguous prediction on the slope of demand curve for all risk-averse individuals. In fact, for all risk-averse individuals, the hypothesis in equation (17) should be $\alpha_P + \alpha_W Q < 0$ rather than $\alpha_P < 0$ and $\alpha_W > 0$, which is supported by Proposition 2 and the Slutsky equation for insurance.

3. Applications of Demand for Insurance

As mentioned above, this paper can be used to further study the relationship of wealth effect and substitution effect. As a bench mark, let us first assume the utility function of the insured is constant absolute risk aversion, i.e., an exponential function, $u(Z) = -\exp(-cZ)$, where Z is the payoff of the insured. Immediately from equation (2), $Q^* = L - \frac{1}{c}\ln[\frac{(1-\pi)P}{\pi(1-P)}]$, where Q^* denote the optimal insurance amount. We can easily verify that $\frac{\partial Q^*}{\partial P} < 0$. When the insured is constant absolute risk averse, the slope of Marshallian demand for insurance is negative as predicted by Proposition 1. Furthermore, from equation (13), the slope of Hicksian demand can be expressed as $\frac{\partial Q^*}{\partial P} + Q^* \frac{\partial Q^*}{\partial W}$. Thus, we can find that the slope of Hicksian demand is negative as predicted by Proposition 2, since $\frac{\partial Q^*}{\partial W} = 0$ as documented in Mossin (1968).

Moreover, Propositions 1 and 2 predicted that the slope of Hicksian demand is always negative, even when the slope of Marshallian demand for insurance is positive. To demonstrate this point, we should assume the insured is non-decreasing absolute risk-averse. For demonstration, let us assume that $u(Z) = \ln(Z)$. From equation (2), the optimal insurance amount can be derived as:

$$Q^* = \frac{(1-\pi)PL - (P-\pi)W}{P(1-P)}. \tag{18}$$

From equation (18), we can further find that

$$\frac{\partial Q^*}{\partial P} = \frac{(1-\pi)P^2 L - (P^2 - 2P\pi + \pi)W}{P^2(1-P)^2}. \tag{19}$$

From equation (19), the slope of Marshallian demand for insurance may not be always negative as predicted by Proposition 1. But, as predicted by Proposition 2, we can find $\frac{\partial Q^*}{\partial P} + Q^* \frac{\partial Q^*}{\partial W} < 0$, i.e., the slope of Hicksian demand is still negative. The implication of this paper can be explored further when we exam equation (19) in details.

The first remark is that $\frac{\partial Q^*}{\partial P} = \frac{\pi L - W}{\pi(1-\pi)}$, when $P = \pi$. Since the insured should have enough money for insurance, i.e., $\pi L < W$, therefore, $\frac{\partial Q^*}{\partial P} < 0$ when insurance is actuarially fair. That is, the demand for insurance decreases with respect to an increase in price as long as the insurance premium is priced at an actuarially fair level. Thus, this result seems to suggest that the demand for insurance increases with respect to an increase in price only if the insurance premium is higher than an actuarially fair price. Second, $\frac{\partial Q^*}{\partial P} = \frac{-\pi W}{P^2} < 0$, when $L = W$. This result shows that the demand for insurance always decreases with respect to an increase in price when the loss is equal to wealth. Indeed, we can further verify that $\frac{\partial Q^*}{\partial P} < 0$, when $L \leq W$. It implies that the insurance may not be a Giffen's goods if the loss is not large enough to threaten the whole wealth of the insured. In another words, the insurance may become a Giffen's goods only when the loss is large enough to threaten the whole wealth of the insured.

The above two remarks seem to suggest that the slope of demand for insurance is negative for all risk-averse individuals, when the loss is much higher than the wealth and the insurance is charged by more than an actuarially fair price. Liability insurance could fit in to this example, since liability claims sometimes may make individual go bankruptcy. Furthermore, insurance companies usually charge a higher risk premium on liability insurance and make the price of liability insurance away from actuarially fair. Thus, we may observe the liability insurance plays like a Giffen's goods. The predictions seem to be at least partially consistent to the liability crisis in the USA. During that period, the price of liability insurance increased dramatically, but the demand for liability insurance also increased.

4. Conclusions

We derive a Hicksian demand for insurance for both Mossin's model (1968) and the case of a continuous loss distribution. We show that a sufficient condition for a downward-sloping Marshallian demand curve for insurance is that the insured is non-decreasing risk-averse, whereas a Hicksian demand curve for insurance is always downward sloping. We further derive the Slutsky equation for insurance. The Slutsky equation in this paper provides a linkage between a Marshallian and a Hicksian demand for insurance and the connection between the income effect and the substitution effect in insurance markets. The empirical implications and the applications of the paper are also discussed.

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