BODY FORCE EFFECT ON CONSOLIDATION OF POROUS ELASTIC MEDIA DUE TO PUMPING

Tung-Lin Tsai*, Kuo-Chyang Chang, and Liang-Hsiung Huang

ABSTRACT

In this study, the linear poro-elasticity theory is applied to examine the body force effect on consolidation of porous media due to pumping. The steady-state solutions of displacement and incremental effective stress for a stratum of clay sandwiched between sandy strata are analytically given. The effect of body force could be represented by the body force number that depends on Lame's constants, porosity, and the thickness of porous media. The consolidation of clay is significantly related not only to the body force number but also to the ratio of water table depression in upper and lower sandy strata due to pumping. The neglect of body force will severely underestimate the displacement and incremental effective stress when porous media are soft or thick, or both. This might lead to potential flaws in engineering practice, such as the calculation of soil settlement.

Key Words: porous media, consolidation, body force, linear poro-elasticity theory, pumping.

I. INTRODUCTION

The conventional one-dimensional Terzaghi consolidation theory of porous elastic media has been widely used to estimate soil consolidation (Das, 1990; Lambe and Whitman, 1979), but the effect of body force (i.e., the variation of self weight) is not considered in it. This is because of the assumption that the incremental effective stress is identical to the dissipative pore pressure (i.e., the invariance of total stress) as consolidation proceeds. Gibson *et al.* (1967, 1981) used finite strain theory and Lagrangian coordinates to analyze soil consolidation with body force effect caused by surface loading. Mei (1985) employed small strain theory and Eulerian coordinates to formulate the consolidation of elastic media with consideration of the body force effect on a single soil layer subjected to surface loading. The results show that the conventional Terzaghi consolidation theory should be only valid for a thin or stiff soil layer.

In addition to surface loading, the change of water table due to groundwater pumping is another major cause of soil consolidation. From the view point of hydrogeology, a soil stratum could be considered as the composition of alternating layers of highly porous sand (aquifers) and highly impervious clay (aquitards). This is usually called a multiaquifers system. Due to significant differences of permeability and compressibility between sand and clay, a two-step procedure (Gambolati and Freeze, 1973) is used to analyze the soil consolidation in a multiaquifers system. In the two-step procedure, a hydrological model based on the standard groundwater flow equation is first employed to calculate the variations of hydraulic head. With the calculated hydraulic head in the aquifers as boundary conditions of the aquitards, one-dimensional Terzaghi consolidation theory (Terzaghi, 1954) is then applied to compute vertical deformation of clay. The two-step procedure, as compared with the fully three-dimensional consolidation model (Lewis and Schrefler 1978; Ng and Mei 1995) in which the soil deformation and hydraulic head in the sand and clay are coupled for calculation,

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has been often used to analyze soil consolidation in multiaquifers systems, especially for practical applications, due to its efficiency and convenience (Helm, 1975; Gambolati *et al.*, 1991; Onta and Gupta, 1995; Larson *et al.*, 2001).

In this study, based on the two-step procedure concept and the linear poro-elasticity theory (Biot, 1941; Helm, 1987; Gambolati *et al.*, 1991; Fallou *et al.*, 1992; Gutierrez and Lewis, 2002), the effect of body force on soil consolidation due to water table depression in a multiaquifers system is examined. In the following sections, the governing equations for consolidation of porous elastic media, considering the body force effect, are derived first. The steady-state solutions of displacement and incremental effective stress are then obtained analytically. Finally, the comparison studies of results with and without body force effect are conducted.

II. GOVERNING EQUATIONS

The mass conservation of fluids and solids in saturated porous media (Bear and Corapcioglu, 1981) can be respectively written as

$$\frac{\partial(n\rho_w)}{\partial t} + \nabla\cdot(n\rho_w \boldsymbol{V}_w) = 0 \tag{1}$$

and

$$\frac{\partial[(1-n)\rho_s]}{\partial t} + \nabla\cdot[(1-n)\rho_s \boldsymbol{V}_s] = 0 \tag{2}$$

where n is porosity. ρ_w and ρ_s represent densities of fluid and solid. $\boldsymbol{V}_w$ and $\boldsymbol{V}_s$ denote velocities of fluid and solid.

The deformation of porous media is attributed to the rolling and slipping of grains with respect to each other. Hence, ρ_s in Eq. (2) remains unchanged in consolidation. Denoting $\boldsymbol{u}$ and P as the solid displacement and pore pressure, and assuming $\boldsymbol{V}_s = \partial\boldsymbol{u}/\partial t$ and $|\partial P/\partial t| >> \boldsymbol{V}_s \cdot \nabla P$, the flow equation of deforming porous media without the compressibility of fluid by combing Eqs. (1)-(2) (Bear and Corapcioglu, 1981) can be written as

$$\nabla\cdot\boldsymbol{q}_r + \frac{\partial}{\partial t}\nabla\cdot\boldsymbol{u} = 0 \tag{3}$$

where $\boldsymbol{q}_r = n(\boldsymbol{V}_w - \boldsymbol{V}_s)$ is Darcy's flux velocity.

In the absence of inertial force, the equilibrium of forces for saturated porous media (Biot, 1941; erruijt, 1969; Bear and Corapcioglu, 1981) can be expressed as

$$\nabla\cdot\overline{\overline{\boldsymbol{\sigma}'}} + \boldsymbol{f} = \nabla P \tag{4}$$

where $\overline{\overline{\boldsymbol{\sigma}'}}$ is the effective stress tensor. $\boldsymbol{f} = [\rho_w n + (1 - n)\rho_s]\boldsymbol{g}$ denotes the body force. $\boldsymbol{g}$ represents the gravitational acceleration.

Denoting $\overline{\overline{\boldsymbol{\sigma}'}}$, P, $\boldsymbol{f}$, and n as the sum of initial steady values $\overline{\overline{\boldsymbol{\sigma}'^0}}$, P^0, $\boldsymbol{f}^0$, $\boldsymbol{n}^0$ and consolidation-producing incremental values $\overline{\overline{\boldsymbol{\sigma}'^e}}$, P^e, $\boldsymbol{f}^e$, n^e, the equations for equilibrium of forces shown in Eq. (4) can be divided into initial steady-state equations

$$\nabla\cdot\overline{\overline{\boldsymbol{\sigma}'^0}} + \boldsymbol{f}^0 = \nabla P^0 \tag{5}$$

and incremental equations

$$\nabla\cdot\overline{\overline{\boldsymbol{\sigma}'^e}} + \boldsymbol{f}^e = \nabla P^e \tag{6}$$

where $\boldsymbol{f}^e = -(\rho_s - \rho_w)n^e\boldsymbol{g}$ represents the perturbation of body force due to the variation of porosity as the porous media deforms.

Similarly, the flow equation shown in Eq. (3) can also be decomposed into an initial steady-state equation

$$\nabla\cdot\boldsymbol{q}_r^0 = 0 \tag{7}$$

and an incremental equation

$$\nabla\cdot\boldsymbol{q}_r^e + \frac{\partial}{\partial t}\nabla\cdot\boldsymbol{u} = 0 \tag{8}$$

The deformation of porous media takes place as a result of the change in the effective stress. The constitutive relationship between the effective stress and displacement for linear poro-elastic media with small strain can be written as

$$\sigma_{ij}'^e = G\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \lambda\left(\frac{\partial u_k}{\partial x_k}\right)\delta_{ij} \quad i, j, k = 1, 2, 3 \tag{9}$$

where G and λ are the well-known Lame's constants.

From Eq. (2), the consolidation-producing incremental porosity, i.e., n^e, can be represented as

$$n^e = (1 - n^0)\nabla\cdot\boldsymbol{u} \tag{10}$$

The detailed derivation of Eq. (10) is shown in the Appendix.

Substituting Eqs. (9)-(10) into Eq. (6) yields the equilibrium of forces in incremental state as follows:

$$G\nabla^2\boldsymbol{u} + (G+\lambda)\nabla(\nabla\cdot\boldsymbol{u}) - \Delta\rho(1-n^0)(\nabla\cdot\boldsymbol{u})\boldsymbol{g} = \nabla P^e \tag{11}$$

where $\Delta\rho = \rho_s - \rho_w$.

Using Darcy's law, that is,

$$\boldsymbol{q}_r^e = -\frac{\overline{\overline{\boldsymbol{K}}}\cdot\nabla P^e}{\rho_w g} \tag{12}$$

the flow equation of deforming porous media in the incremental state shown in Eq. (8) becomes

$$\nabla \cdot (\overline{\overline{\boldsymbol{K}}} \cdot \nabla P^e) + \rho_w g \frac{\partial}{\partial t} \nabla \cdot \boldsymbol{u} = 0 \tag{13}$$

where $\overline{\overline{\boldsymbol{K}}}$ is the hydraulic conductivity tensor.

Governing Eqs. (1), (2), (4) with constitutive relation Eq. (9) now are rewritten as Eqs. (13), (10), and (11) for incremental equations. And Eqs. (11) and (13) are the governing equations of soil consolidation used herein.

III. ONE-DIMENSIONAL CONSOLIDATION

A stratum of clay sandwiched between sandy strata that are highly permeable and much stiffer than the clay is shown in Fig. 1. In Fig. 1, water table depression h_1 and h_2 due to pumping take place in sandy strata above and below the clay. Due to significant differences of permeability and compressibility between sand and clay, excessive pore pressure only exists in the clay as consolidation proceeds. And nearly all of the consolidation happens due to the volume change within the clay, while the sandy strata may be considered rigid media as compared with the clay. In addition, the horizontal dimension is much larger than the thickness of the consolidation stratum. Hence, one-dimensional consolidation is well assumed herein. This leads to the flow and the strain occurring only in the vertical direction. Furthermore, the steady-state solutions, which are usually applied in engineering practice, will be considered.

Because we are dealing with one-dimensional steady-state consolidation, the flow equation of deforming porous media shown in Eq. (13) for an isotropic and homogeneous clay can be simplified as

$$K \frac{\partial^2 P^e}{\partial z^2} = 0 \tag{14}$$

The equations for equilibrium of forces given by Eq. (11) can also be reduced to

$$(2G + \lambda) \frac{\partial^2 u_z}{\partial z^2} + \Delta\rho g (1 - n^0) \frac{\partial u_z}{\partial z} = \frac{\partial P^e}{\partial z} \tag{15}$$

One can clearly see that these two equations are in fact decoupled. The incremental pore pressure of the flow equation can be given first. The displacement is then obtained by solving the equation for equilibrium of forces with the known incremental pore pressure. From Eq. (15), ignoring the body force effect, i.e., $\Delta\rho g(1 - n^0)\partial u_z/\partial z$ can obtain $P^e = (2G + \lambda)\partial u_z/\partial z$. With the assumption of one-dimensional consolidation, the constitutive relationship between the incremental effective stress and displacement shown in Eq. (9) for clay becomes $\sigma'^e_{zz} = (2G + \lambda)\partial u_z/\partial z$. It can be seen

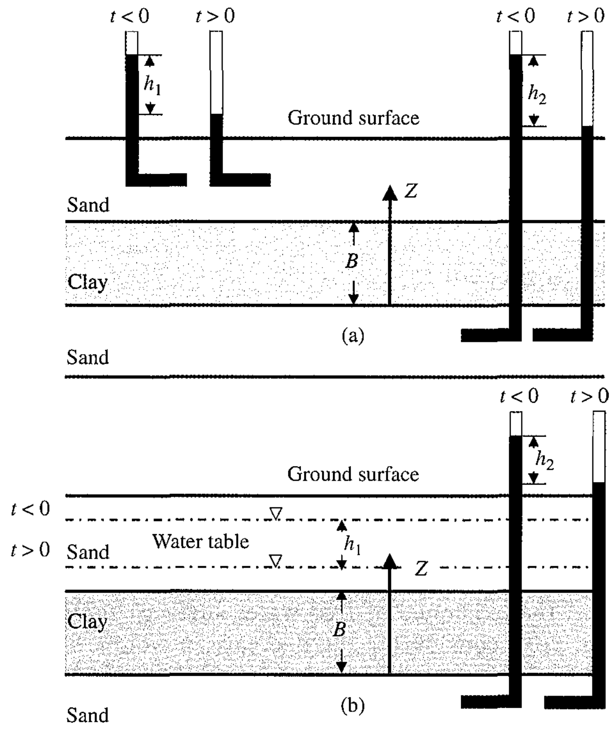

Fig. 1 Sketches of soil consolidation due to water table drpression: (a) confined case, (b) unconfined case

from the above that neglecting the body force effect, Eq. (15) is identical to the conventional one-dimensional Terzaghi consolidation theory in which the incremental effective stress equals the dissipative pore pressure, i.e., $P^e = \sigma'^e_{zz}$.

In Fig. 1, water table depression h_1 and h_2 due to pumping happen in sandy strata above and below the clay. As mentioned above, the water table depression in the aquifer can be obtained by using a groundwater model to simulate a miltiaquifers system. Applying the continuity of pore pressure, the incremental pore pressure at the bottom and top boundaries of the clay can be respectively expressed as

$$P^e|_{z=0} = -\rho_w g h_2 \tag{16a}$$

and

$$P^e|_{z=B} = -\rho_w g h_1 \tag{16b}$$

From Eqs. (14) and (16), the incremental pore pressure in the clay stratum is

$$P^e = -\rho_w g \left(h_2 + \frac{h_1 - h_2}{B} z\right) \tag{17}$$

Thus, the equation for equilibrium of forces shown in (15) becomes

$$(2G+\lambda)\frac{\partial^2 u_z}{\partial z^2}+\Delta\rho g(1-n^0)\frac{\partial u_z}{\partial z}=\frac{-\rho_w g(h_1-h_2)}{B} \tag{18}$$

The bottom boundary of the clay is connected to the nearly rigid sandy stratum. The displacement of clay at $z = 0$ can be expressed as

$$u_z|_{z=0}=0 \tag{19}$$

If the overlaying sandy stratum shown in Fig. 1(a) is confined, the top boundary of the clay $z = B$ is subjected to an incremental effective stress of $-\rho_w g h_1$ due to water table depression, i.e.,

$$(2G+\lambda)\frac{\partial u_z}{\partial z}\bigg|_{z=B}=-\rho_w g h_1 \tag{20}$$

However, for the unconfined overlaying stratum (i.e., the existence of free water surface) shown in Fig. 1(b), considering the decrease in weight by releasing pore water (Corapcioglu, and Bear, 1983), the incremental effective stress at the top boundary of the clay becomes

$$(2G+\lambda)\frac{\partial u_z}{\partial z}\bigg|_{z=0}=-\rho_w g h_1^* \tag{21}$$

where $h_1^* = (1 - n)h_1$ represents effective water table depression and n is porosity in the overlaying sandy stratum. In the following derivation, the unconfined case could be easily determined by replacing h_1 shown in Eq. (20) with h_1^*, hence considering only the confined case is enough.

The exact solution for Eqs. (18)-(20) is

$$\begin{aligned}u_z=\frac{-\rho_w gB}{2G+\lambda}\{&h_2 z'+(h_1-h_2)\frac{z'^2}{2}\\&+h_1[\frac{z'}{M}-\frac{z'^2}{2}+\frac{(M-1)}{M^2}(e^M-e^{M(1-z')})]\\&+h_2[\frac{-z'}{M}-z'+\frac{z'^2}{2}+\frac{1}{M^2}(e^M-e^{M(1-z')})]\}\end{aligned} \tag{22}$$

where

$$M=\frac{\Delta\rho g(1-n^0)B}{(2G+\lambda)} \tag{23}$$

in which M represents the body force number and $z' = z/B$ is the nondimensional coordinate in the z direction. One can clearly observe from Eq. (23) that the body force number becomes large when the soil is soft or thick, or both. For example, a soft and thick clay stratum with $B = 40$ m, $\Delta\rho g = 1.62 \times 10^4$ N/m^3, $n^0 = 0.15$, and $(2G + \lambda) = 1.5 \times 10^6$ N/m^2 yields $M = 0.2754$, which is not small. However, in the following discussion, it is quite safe to let $0 \le M \le 1$

Taking $z' = 1$ and using L'Hopital's rule for $M \to 0$ in Eq. (22), the vertical deformation at the top boundary of the clay, without body force effect, i.e., $(\Delta z)_{nb} = -u_z|_{M\to 0,\ z'=1}$ can be represented as

$$(\Delta z)_{nb}=\frac{\rho_w gB(h_1+h_2)}{2(2G+\lambda)} \tag{24}$$

The nondimensional displacement in the clay can be obtained by introducing $u_z^* = u_z/[-(\Delta z)_{nb}]$ as follows:

$$\begin{aligned}u_z^*=&\frac{r}{1+r}z'^2+\frac{1}{1+r}(2z'-z'^2)\\&+\frac{2r}{1+r}[\frac{z'}{M}-\frac{z'^2}{2}+\frac{M-1}{M^2}(e^M-e^{M(1-z')})]\\&+\frac{2}{1+r}[-\frac{z'}{M}-z'+\frac{z'^2}{2}+\frac{1}{M^2}(e^M-e^{M(1-z')})]\end{aligned} \tag{25}$$

where $r = h_1/h_2$ represents the ratio between the water table depression in upper and lower sandy strata.

Taking $z' = 1$ in Eq. (25), the nondimensional displacement at the top boundary of the clay, i.e., $(\Delta z)^* = u_z^*|_{z'=1}$, is

$$\begin{aligned}(\Delta z)^*=1&+\frac{2r}{1+r}[\frac{1}{M}-\frac{1}{2}+\frac{M-1}{M^2}(e^M-1)]\\&+\frac{2}{1+r}[-\frac{1}{M}-\frac{1}{2}+\frac{1}{M^2}(e^M-1)]\end{aligned} \tag{26}$$

From Eqs. (9) and (22), the incremental effective stress in the clay stratum can be written as .

$$\begin{aligned}\sigma'^e_{zz}=-\rho_w g\{&h_1 z'+h_2(1-z')\\&+h_1[e^{M(1-z')}-\frac{1}{M}(e^{M(1-z')}-1)-z']\\&+h_2[\frac{1}{M}(e^{M(1-z')}-1)-1+z']\}\end{aligned} \tag{27}$$

Applying L'Hoptial's rule for $M \to 0$ in Eq. (27), the incremental effective stress without the body force effect, i.e., $(\sigma'^e_{zz})_{nb} = \sigma'^e_{zz}|_{M\to 0}$, can be written as

$$(\sigma'^e_{zz})_{nb}=-\rho_w g[h_1 z'+h_2(1-z')] \tag{28}$$

One can see from Eq. (28) that the incremental effective stress agrees with the incremental pore pressure shown in Eq. (17) due to the ignorance of body force effect (i.e., the invariance of total stress), which is used in the conventional Terzaghi consolidation theory of porous elastic media.

If h_1 is larger than h_2 the nondimensional incremental effective stress can be obtained by introducing $(\sigma'^e_{zz})^* = \sigma'^e_{zz}/(-\rho_w g h_1)$ as follows:

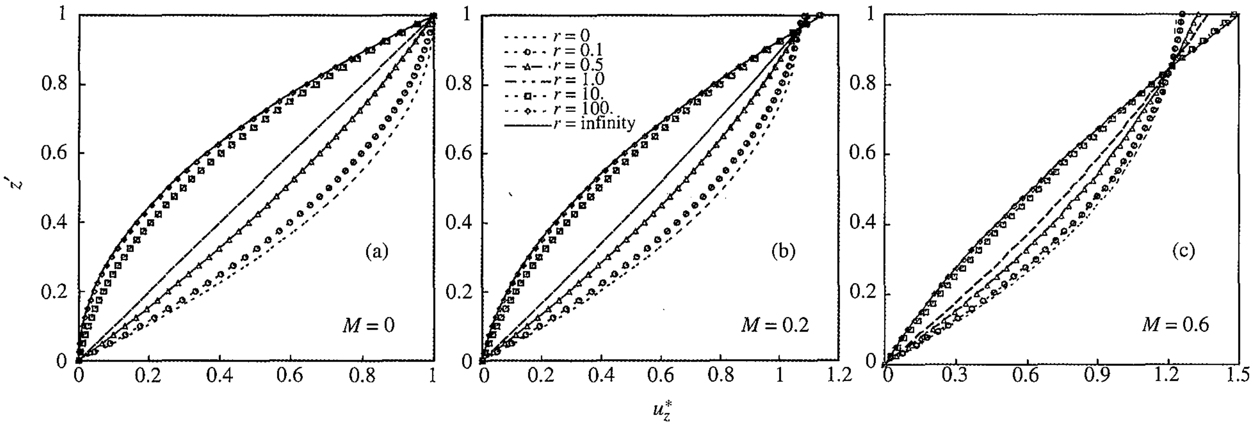

Fig. 2 The nondimensional displacement of clay

$$(\sigma_{zz}^{\prime e})^* = z' + \frac{1-z'}{r} + [e^{M(1-z')} - \frac{1}{M}(e^{M(1-z')} - 1) - z'] + \frac{1}{r}[\frac{1}{M}(e^{M(1-z')} - 1) - 1 + z'] \quad (29)$$

On the contrary, when h_2 is larger than h_1 the nondimensional incremental effective stress can be given with the introduction of $(\sigma_{zz}^{\prime e})^{**} = \sigma_{zz}^{\prime e}/(-\rho_w g h_2)$ as follows:

$$(\sigma_{zz}^{\prime e})^{**} = rz' + (1-z') + r[e^{M(1-z')} - \frac{1}{M}(e^{M(1-z')} - 1) - z'] + [\frac{1}{M}(e^{M(1-z')} - 1) - 1 + z'] \quad (30)$$

IV. DISCUSSIONS

The nondimensional displacements of the clay stratum shown in Eq. (25) for the body force numbers of 0, 0.2, and 0.6 are displayed in Figs. 2(a)-(c), respectively. The nondimensional displacement at the top boundary of the clay shown in Eq. (26) is depicted in Fig. 3. One can observe from Figs. 2 and 3 that the consolidation of clay is significantly related to not only the body force number but also the ratio of water table depression. Without the body force effect, the nondimensional displacement is a quadratic polynomial distribution

$$u_z^*|_{M\to 0} = \frac{r}{1+r}z'^2 + \frac{1}{1+r}(2z' - z'^2) \quad (31)$$

If water table depression only appears in the upper stratum, i.e., $r \to \infty$, the nondimensional displacement becomes

$$u_z^*|_{r\to 0} = (2z' - z'^2) + 2[-\frac{z'}{M} - z' + \frac{z'^2}{2} + \frac{1}{M^2}(e^M - e^{M(1-z')})] \quad (32)$$

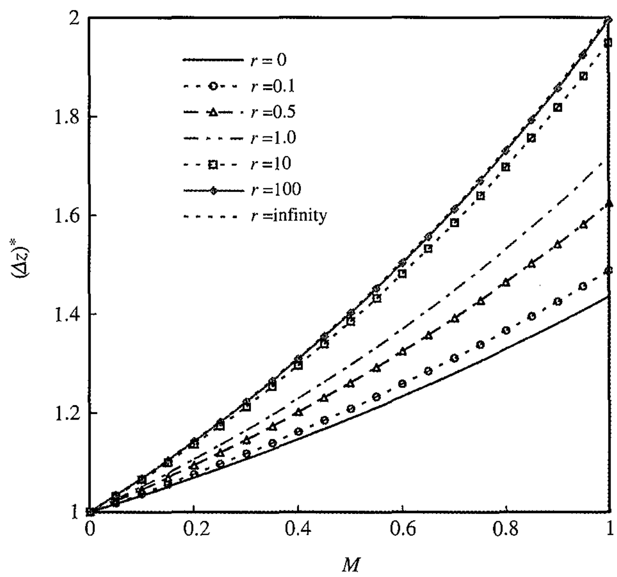

Fig. 3 The nondimensional displacement at the top boundary of the clay

On the contrary, when water table depression only exists in the lower stratum, i.e., $r \to 0$, the nondimensional displacement is

$$u_z^*|_{r\to\infty} = z'^2 + 2[\frac{z'}{M} - \frac{z'^2}{2} + \frac{M-1}{M^2}(e^M - e^{M(1-z')})] \quad (33)$$

The nondimensional incremental effective stresses in the clay stratum for $h_1 \geq h_2$ shown in Eq. (29) is depicted in Fig. 4 when the body force parameters are 0, 0.2, and 0.6. Without the body force effect, Eq. (29) becomes

$$(\sigma_{zz}^{\prime e})^*|_{M\to 0} = z' + \frac{1-z'}{r} \quad (34)$$

In addition, Fig. 5 shows the nondimensional

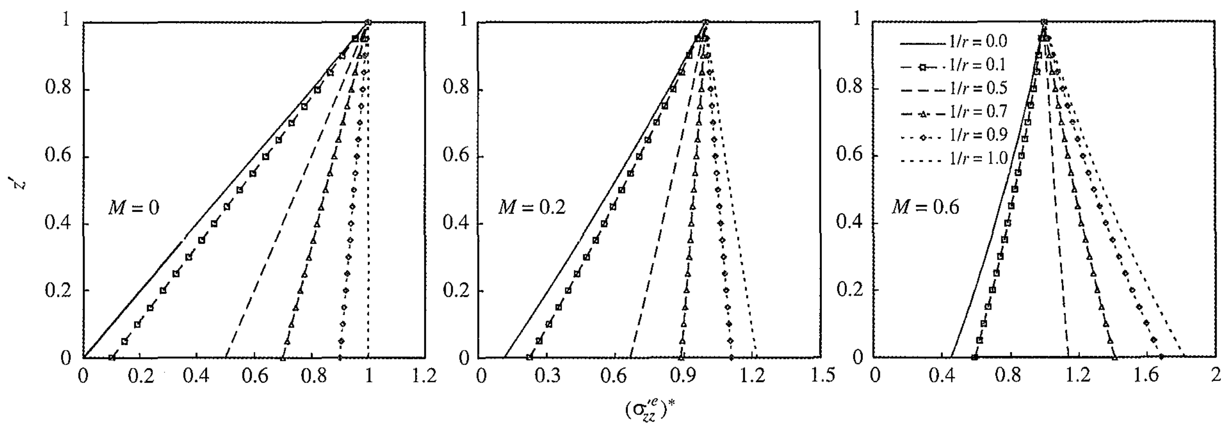

Fig. 4 The nondimensional incremental effective stress of clay for $h_1 \geq h_2$

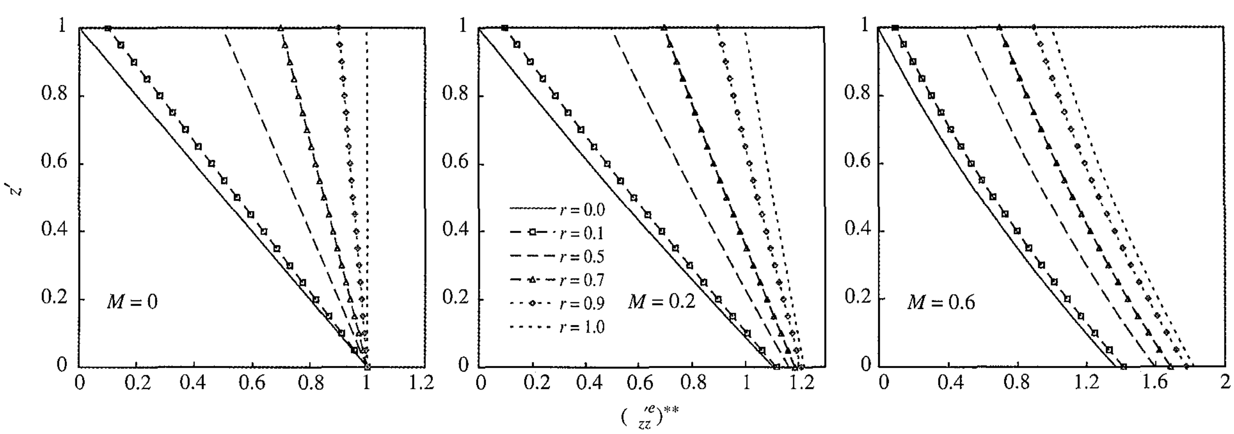

Fig. 5 The nondimensional incremental effective stress of clay for $h_2 \geq h_1$

incremental effective stress in the clay stratum for $h_2 \geq h_1$ given by Eq. (30). The nondimensional incremental effective stress in the clay without body force effect is

$$(\sigma_{zz}^{\prime e})^{**}\big|_{M \to 0} = rz' + (1 - z') \tag{35}$$

Figs. 4 and 5 show again that the consolidation of clay is strongly dependent on the effect of body force and the ratio of water table depression. In addition, one can clearly see that the incremental effective stress in the clay stratum shown in Eqs. (34)-(35) is linearly distributed when the effect of body force is neglected. However, the incremental effective stress, in general, is nonlinear due to the effect of body force.

In this study, the derived analytic solutions of displacement and incremental effective stress, simple to compute, are easy to use in engineering practice. For example, they can be applied to calculate the settlement of soil. If the water table depression h_1 and h_2 are known from using the groundwater model for a multiaquifers system, the settlement of clay without body force effect, i.e., $(\Delta z)_{nb}$, can be obtained from Eq. (24). Then, equation (26) could be used to calculate the nondimensional displacement at the top boundary of the clay $(\Delta z)^*$ with known ratio of water table depression, i.e., $r = h_1/h_2$, and body force number M shown in Eq. (23). The settlement of clay considering body force effect is then obtained from the product of $(\Delta z)_{nb}$ and $(\Delta z)^*$.

In addition, the linear poro-elasticity theory along with Eulerian coordinates is applied to investigate the body force effect on soil consolidation. The limit of small strain for soil consolidation, i.e., $\partial u/\partial z < 0.1$, has to be satisfied. Using equations (24) and (26), the small strain could be expressed as

$$\frac{\partial u}{\partial z} \approx \frac{(\Delta z)_{nb}(\Delta z)^*}{B} < 0.1 \tag{36}$$

For a critical case $M = 1$ and $r \to \infty$, i.e., $(\Delta z)^* = 2$, shown in Fig. 3, Eq. (36) could become

$$\frac{\rho_w g(h_1 + h_2)}{(2G + \lambda)} < 0.1 \tag{37}$$

From Eq. (37), for a soft clay with $\rho_w g = 9.81 \times 10^3$ N/m^3 and $(2G + \lambda) = 1.5 \times 10^6$ N/m^2 as mentioned above, we get $(h_1 + h_2) < 15.3$ m. It seems to be quite safe to apply the results derived in this study to engineering practice. It must be pointed out that the steady state solution, as derived above, is for the maximum effect of body force which seems to take a long time to develop. From Eq. (15), one can obtain

$$u_z \approx \frac{B}{2G+\lambda} P^e \tag{38}$$

From Eqs. (13) and (38), the characteristic time to steady state T needs to satisfy

$$T >> \frac{\rho_w g B^2}{(2G+\lambda)K} \tag{39}$$

For a soft and thick clay stratum with $B = 40$ m, $\rho_w g = 9.81 \times 10^3$ N/m^3, $(2G + \lambda) = 1.5 \times 10^6$ N/m^2, and $K = 1.0 \times 10^{-6}$ m/s, it will take many years for the pore pressure to be transmitted.

V. CONCLUSIONS

The effect of body force is not considered in the conventional Terzaghi consolidation theory. This study applies Biot's theory of linear poro-elasticity and the two-step procedure concept to investigate the consolidation of porous media with the effect of body force under the assumption of small strain. A case of clay stratum sandwiched between two sandy strata subjected to the water table depression due to pumping is used to conduct this examination. Closed-form solutions of one-dimensional steady-state displacement and incremental effective stress in the clay with the body force effect have been found. The consolidation of clay strongly depends on two dimensionless parameters, i.e., body force number M and the ratio of water table depression r. The body force number is a function of Lame's constants, porosity, and the thickness of soil. The body force significantly affects the magnitude of the displacement and incremental effective stress when the soil is soft or thick, or both.

NOMENCLATURE

B	thickness of clay
f	body force
f^0	initial steady value of body force
f^e	consolidation-producing incremental value of body force
g	gravitational acceleration
G	Lame constant
h	water table depression
h^*	effective water table depression
K	hydraulic conductivity
M	body force parameter
n	porosity
n^0	initial steady value of porosity
n^e	consolidation-producing incremental value of porosity
P	pore pressure
P^0	initial steady value of pore pressure
P^e	consolidation-producing incremental value of pore pressure
q_r	Darcy's velocity
q_r^0	initial steady value of Darcy's velocity
q_r^e	consolidation-producing incremental value of Darcy's velocity
r	ratio of lowering of water tables
t	time
u	displacement of solid
u_z	displacement of solid in z direction
u_z^*	nondimensional displacement of solid
V_w	velocity of fluid
V_S	velocity of solid
z	coordinate
z'	nondimensional coordinate
ρ_w	density of fluid
ρ_s	density of solid
$\Delta\rho$	difference in density between solid and fluid
β	compressibility of fluid
$\sigma'_{i,j}$	effective stress tensor
$\sigma'^0_{i,j}$	initial steady value of effective stress tensor
$\sigma'^e_{i,j}$	consolidation-producing incremental value of effective stress tensor
σ'^e_{zz}	incremental effective stress in the clay stratum
$(\sigma'^e_{zz})_{nb}$	incremental effective stress in the clay stratum without body force effect
$(\sigma'^e_{zz})^*$	nondimensional incremental effective stress for $h_1 > h_2$
$(\sigma'^e_{zz})^{**}$	nondimensional incremental effective stress for $h_2 > h_1$
λ	Lame constant
$(\Delta z)_{nb}$	the vertical deformation at the top boundary of the clay without body force effect
$(\Delta z)^*$	nondimensional displacement at the top boundary of the clay

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APPENDIX : DERIVATION OF CONSOLIDATION-PRODUCING INCREMENTAL POROSITY

The mass conservation of solid in saturated porous media can be written as

$$\frac{\partial[(1-n)\rho_s]}{\partial t} + \nabla \cdot [(1-n)\rho_s \boldsymbol{V}_s] = 0 \tag{A1}$$

where n is porosity, ρ_s is the density of solid, $\boldsymbol{V}_s$ represents velocity of solid.

The deformation of porous media is manifested by variation of porosity due to the rolling and slipping of the grains with respect to each other. Therefore, ρ_s remains unchanged in the consolidation process. As mentioned above, Eq. (A1) becomes

$$\frac{d_s(1-n)}{dt} = -(1-n)\nabla \cdot \boldsymbol{V}_s \tag{A2}$$

where d_s/dt represents the material derivative, i.e., $\partial/\partial t + \boldsymbol{V}_s \cdot \nabla$. Assuming linear variation, i.e., $|\partial(1-n)/\partial t| >> |\boldsymbol{V}_s \cdot \nabla(1-n)|$ we have

$$\frac{\partial(1-n)}{\partial t} = -(1-n)\nabla \cdot \boldsymbol{V}_s \tag{A3}$$

Dividing n into steady initial values n^0 and consolidation-producing incremental values n^e, Eq. (A3) can be linearized as follows:

$$-\frac{\partial n^e}{\partial t} = -(1-n^0)\nabla \cdot \boldsymbol{V}_s \tag{A4}$$

Denoting $\boldsymbol{u}$ as the solid displacement and letting $\boldsymbol{V}_s = \partial \boldsymbol{u}/\partial t$, Eq. (A4) can be reduced to

$$n^e = (1-n^0)\nabla \cdot \boldsymbol{u} \tag{A5}$$