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Accurate Statistical Process Data Modeling using Complex Gaussian Polynomials

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***Abstract*— The most recent research of SSTA requires accurate non-Gaussian data processing modeling. Although quadratic Gaussian forms have been proposed to data modeling, limitations are imposed to ensure real coefficients. However, there are even more difficult distributions such as uniform distribution which can not be modeled by previous one. In this paper, we are going to solve these problems by allowing complex coefficients and higher order Gaussian polynomials with a PDF recovering scheme. Experimental results show how our methods and new algorithms expose some enhancements in both accuracy and versatility.**

I. Introduction

As the technology feature sizes are getting smaller than the wave length of optical lithography light source, the process variation issues are also getting significant and must be taken into consideration during design [1]. Classical corner-based timing analysis produces timing predictions that are often too pessimistic and grossly conservative because we have only few chances to get parameters of all gates working on their corner values. Statistical static timing analysis (SSTA) that characterizes time variables as statistical random variables offers a better approach for more accurate and realistic timing prediction.

Many existing SSTA algorithms were built upon Gaussian distributions due to its simplicity while dealing with maximum operation which is essential during timing analysis [2] [3]. However, the modeling capability of a signal Gaussian is quite limited and may not be able to deal with various non-Gaussian process distributions. Recently, the authosrs in [4] [5] [6] and [7] have proposed the non-Gaussian models used in SSTA to solve these problems. As the technology feature size are getting smaller and smaller, the spatial correlation analysis becomes critical in SSTA. The authors in [8] has proposed a mathematical approach to model this phenomenon. For both these reasons, the authors in [7] has proposed to use a quadratic Gaussian polynomial, $Y = aX^2 + bX + c$, where X is respective Gaussian random variable, and $a, b,\ c$ are respective real numbers. Although in most cases, this method can match the first 3 moments, it requires the skewness is under a limitation to ensure real coefficients. Furthermore, this method is limited in only matching the first 3 moments and thus the forth moment *Kurtosis* and even higher order moments are not matched. Note that Kurtosis is one of the key moments to measure the peak of a distribution. Figure 1 is the threshold voltage distribution which we got from the industry. We can see the Kurtosis is different in different locations. So, to model the Kurtosis accurately is very important.

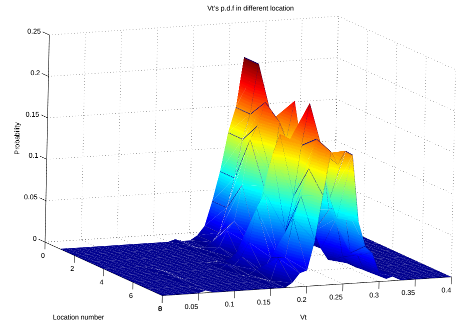

Fig. 1. The threshold voltage distribution in the chip between different measurement locations under 90 nm technology.

In this paper, we present several effective and accurate methods to general models which are compatible with SSTA and can match more board types of distribution such as normal and high skewness by several novel techniques. First, by allowing the coefficients of Gaussian polynomials to be complex, we can match distributions with high skewness which could not be achieved by previous approaches. Second, we expand the power of the Gaussian polynomials from 3 to 4 and even above to be matched with the up to 4_{th} moment and beyond.

The rest of the paper is organized as following: Session II reviews the pros and cons of existing statistical process data modeling methods and basic definitions of statistical moments. Session III presents our newly discovery to solve those problems. Session IV shows how to use CQGP to the ADD and MAX operations in SSTA. Session V shows the experimental results. Finally, in session VI, we conclude this paper.

II. Preliminary

We introduce the basic definition of moments and the moment matching problem in the statistics domain in this session. Later on, we will also briefly introduce the existing process variation modeling techniques.

A. Moments in the Statistics Domain

According to the scope of application, there are various definitions of moments. In the VLSI timing analysis, we usually use the "scaled" raw moments as moments. However, in the statistics domain, we usually use the central moments such as variance and skewness as moments. To make our discussion more clear, we formally introduce our terminology in the following.

The nth raw moment m'_k (i.e., moment about zero) of a distribution $Y(x)$ is defined by

$$m'_k = \langle x^k, \rangle$$

where

$$\begin{aligned}\langle f(x)\rangle = & \sum f(x)Y(x), \quad \text{discrete distribution} \\ = & \int f(x)Y(x)dx, \text{continuous distribution.}\end{aligned}$$

m'_1, the mean is usually simply denoted as $\mu = m_1$. If the moment is instead taken about a point α, then

$$\begin{aligned}m_k(\alpha) = & \langle (x-\alpha)^k \rangle \\ = & \sum (x-\alpha)^k Y(x).\end{aligned}$$

The statistical moments are most commonly taken about the mean.These so-called central moments and are denoted as m_k and are defined by

$$\begin{aligned}m_k &= \langle (x-\mu)^k \rangle \\ &= \int (x-\mu)^n Y(x)dx.\end{aligned}$$

B. *AWE-type Statistical Moment Matching*

Due to the different definition of moment definition, we now define the statistical moment matching (SMM) method. SMM is almost the same as the AWE method [9] except that moment definitions are different and the target function is usually probability density function (PDF). Thus, it is required that PDF is always positive and the total area below PDF is equal to one.

Given a PDF $y(t)$, we can obtain its Laplace transform $Y(s) = \int_0^\infty y(t)\, e^{-st} dt$. Expanding e^{-st} about $s = 0$ to yield

$$\begin{aligned}Y(s) &= \int_0^\infty y(t)\left[1 - st + \frac{1}{2}s^2t^2 - \frac{1}{6}s^3t^3 + \ldots\right] \\ &= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} s^k \int_0^\infty t^k y(t)\, dt \\ &= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} s^k m'_k. \qquad (1)\end{aligned}$$

We can see the statistical moment definition is different than the AWE method by a division of a factorial, $m_k^{awe} = m'_k/k!$. To obtain an approximation of $Y(s)$, we utilize $Pad\check{e}$ approximation as follows:

$$\bar{Y}(s) = \frac{1 + a_1 s + a_2 s^2 + \ldots + a_n s^n}{1 + b_1 s + b_2 s^2 + \ldots + b_m s^m}, \qquad (2)$$

where m and n are positive integers.

We now use 3^{rd} approximation to illustrate the SMM process. Comparing these two equations (2) and (1) and considering a 3^{rd} order approximation, i.e., m = 3 and n $\leq$ 2, we obtain the following equation:

$$\begin{bmatrix} m'_0 & -m'_1 & \frac{1}{2!}m'_2 \\ -m'_1 & \frac{1}{2!}m'_2 & -\frac{1}{3!}m'_3 \\ \frac{1}{2!}m'_2 & -\frac{1}{3!}m'_3 & \frac{1}{4!}m'_4 \end{bmatrix} \begin{bmatrix} b_3 \\ b_2 \\ b_1 \end{bmatrix} = - \begin{bmatrix} -\frac{1}{3!}m'_3 \\ \frac{1}{4!}m'_4 \\ -\frac{1}{5!}m'_5 \end{bmatrix}.$$

Once the $b'_j s$ are obtained, we commutate a_l by the following equations:

$$\begin{aligned}1 &= m'_0 \\ a_1 &= m'_0 b_1 - m'_1 \\ a_2 &= m'_0 b_2 - m'_1 b_1 + \frac{1}{2!}m'_2 \qquad (3)\end{aligned}$$

Finally, using $a's$ and $b's$, we can represent the approximated PDF in terms of exponentials,

$$y(t) \approx k_1 e^{p_1 t} + k_2 e^{p_2 t} + k_3 e^{p_3 t}. \qquad (4)$$

Issues of AWE type SMM:

There are several issues of SMM besides the traditional AWE methods such as stability issues. One of the problem is about the initial condition issues. AWE methods are traditionally assume zero initial condition. However, as a matter of fact, AWE can not guarantee zero initial condition. As a result, the resulting function may have big errors in the starting point. We now illustrate a failure example of using AWE to approximate a $3-sigma$ shifted Gaussian approximation in the following figure. It clearly shows that the whole curve is deviated from the shifted Gaussian a lot.

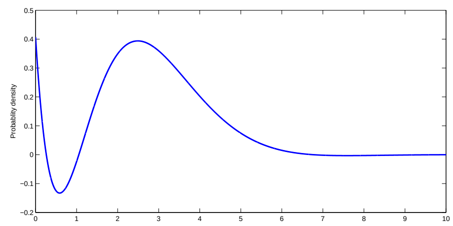

Fig. 2. Initial condition mismatches of AWE-SMM

C. *Quadratic Gaussian Polynomial (QGP)*

To facilitate SSTA, [2], [10], [11] propose the following canonical timing model for a given delay function D:

$$D = \mu + \alpha R + \sum_i \beta_i Y_i,$$

where $Y_i's$ are parameter distributions. To model the nonlinearity of the delay dependency to $Y_i's$, the authors in [7] proposed to add quadratic terms to the above linear canonical form as follows:

$$D = m + \alpha R + \sum_i \beta_i Y_i + \sum_{i,j} \gamma_{ij} Y_i Y_j. \qquad (5)$$

Both canonical and quadratic time model assume parameter variations to be Gaussian distributions which are not always applicable in practice. In the cases when a parameter variation, Y, can not be properly modeled by a simple Gaussian, [7] proposes to express it as a quadratic polynomials in terms of independent Gaussian random variable X (QGP) as follows:

$$Y = aX^2 + bX + c, \ \ (QGP) \tag{6}$$

where a, b, and c are real numbers.

To get proper a, b, and c, [7] uses moment matching methods to match the first three moments. The moment matching equations are as follows:

$$m_1 = \mu_Y = E\{Y\} = a + c \tag{7}$$
$$m_2 = \sigma_Y^2 = E\{(Y - E\{Y\})^2\} = 2a^2 + b^2 \tag{8}$$
$$m_3 = \kappa_Y^3 = E\{(Y - E\{Y\})^3\} = 8a^3 + 6ab^2 \tag{9}$$

Issues of Quadratic Gaussian Polynomial Model:
The limitation of this previous work is that it can only match the first three moments - mean, variance and skewness. Also, due to the limitations of real coefficients, it can not model high skew distribution when the following conditions are not satisfied.

$$|\kappa_Y| \leq \sqrt{2}\sigma_Y. \tag{10}$$

III. Complex Gaussian Polynomial(CGP) Model

In this session, we will present our enhancement to use more general forms to model the process variation data. We will first analyze the high skewness distribution issues and then the ways to deal with this issue such as least-square fitting and complex quadratic Gaussian polynomial. Cubic as well as higher order polynomial forms will also be discussed in details.

Let us first illustrate our complex Gaussian polynomial process data modeling flow as shown in Figure 3. First, assume the process data and/or its characteristics are given, we use our CGP modeling method to obtain accurate abstract form of those data. Afterward, we pass the CGP to SSTA for statistical timing analysis. Once SSTA is done, we then recover the distribution by the modified statistical moment matching method.

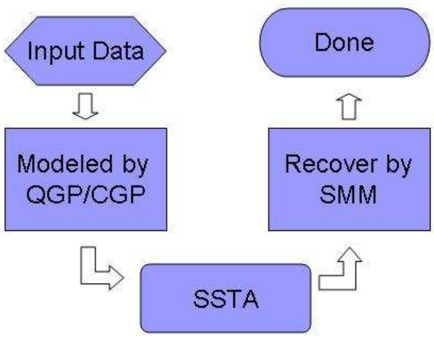

Fig. 3. Complex Gaussian Polynomial Modeling Flow for SSTA

A. *Least Square Fitting Based Quadratic Gaussian Polynomials*

Before we resort to complex extension, let's make a last effort to the (real) quadratic Gaussian polynomial. When there is no real root which can simultaneously satisfy equation (8) and (9), the QGP approach fails. It is, nevertheless, still possible to find some real coefficients which approximately satisfy the two equations. This problem is formulated as a least fitting problem as follows:

$$Minimize \ \ \epsilon_1^2 + \epsilon_2^2$$

where $\epsilon_1 = \frac{2a^2+b^2-m_2}{m_2}$, and $\epsilon_2 = \frac{8a^3+6ab^2-m_3}{m_3}$.

To get further understanding of these two equations and the least square fitting approach, we now illustrate their behaviors in detail. Figure 4 plots these solution set of the two equation in the 2D plane. The X-axis and Y-axis are a and b, respectively. The blue and red curves show the solution set of equation (8) and equation (9), respectively. The upper picture shows the relation of a and b in one PDF when variance is equal to 4 and skewness is equal to 3. Actually, the solution sets of equation (8) exactly forms an ellipse. The bottom picture shows another PDF which variance is equal to 4 and skewness is equal to 300. The ratio of variance and skewness violates equation (10) so there is no intersection between these two curves. Generally speaking, when the variance becomes larger, the major axis in the ellipse will become larger too. As the skewness becomes larger, the turning point will be getting far away from the ellipse. These two characteristics explain the reason why real solutions are not exist. When the two curves are reasonably close by, we anticipate the least square fitting work perfectly. Otherwise, the fitting results may not be able to meet your expectation. Figure 5 shows the inconsistency of the fitting approach. To get more flexibility during equation solving, complex coefficients must be allowed.

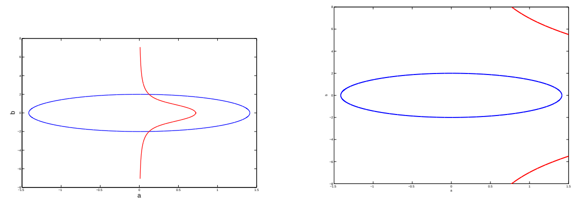

Fig. 4. The solution sets diagram of equation (7) and equation (8) with different variance and skewness.

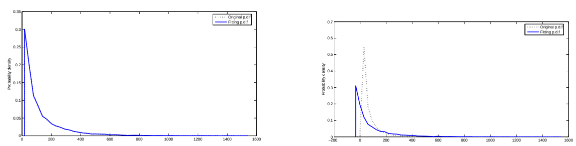

Fig. 5. Two results obtained from Least Square Fitting Algorithm. (a)Good. (b) Bad

B. *Complex Quadratic Gaussian Polynomials (CQGP)*

The key to derive a more robust modeling approach is to allow $a - c$ to take complex values. When complex numbers are allow, the 3 moment equations all can be simultaneously satisfied. We now present our new complex quadratic Gaussian polynomial (CQGP) form as follows:

$$Y = aX^2 + bX + c, \ \ (CQGP) \tag{11}$$

where a, b, and c are complex numbers. We have the following theorem to show the effectiveness of the CQGP representation.

Theorem 1: Given any distribution Y, the complex quadratic Gaussian polynomial (CQGP) form can always match the first three moments of Y.

As a matter of fact, to satisfy those moment equations when the skew condition is violated, a must be a real number while b must be an imaginary number. We now prove this theorem as follows:

Theorem 2: When condition (10) is violated, a and c must be real while b is an imaginary number.

proof: From equation (8), we know that $2a^2 = m_2 - b^2$. Substituting into equation (9), we get $a = m_3/(5b^2 + 4m_2)$. Therefore, no matter what value b will take, a must be real. Also, from $b = \pm\sqrt{m_2 - 2a^2}$, we know that when a is real, b must take imaginary values.

Furthermore, although CQGP allows complex coefficients, the moments of CQGP are still **real**. So, the moment based statistical timing operations still can be applied without modification! So, we can still handle operations such as Add and Max operations. Furthermore, the correlations in between can also be easily preserved by the following theorem which is similar to [7]. Following the approach in [7], we get:

Theorem 3: Assuming CQGP $Y_1 = a_1X_1^2 + b_1X_1 + c_1$ and $Y_2 = a_2X_2^2 + b_2X_2 + c_2$ are timing parameters in gates 1 and 2 and assuming X_1 and X_2 are computational Gaussian random variables at those gates. ρ_x is the correlation coefficient between X_1 and X_2 and is equal to $cov\left(X_1, X_2\right)$. Then

$$cov\left(Y_1, Y_2\right) = \rho_x b_1 b_2 + 2\rho_x^2 a_1 a_2 \tag{12}$$

The process to prove this theorem does not depend on b_1 and b_2 if they are complex numbers or not. Because b is a pure image number, the result for b_1b_2 is just a real number. This theorem still useful when we get a complex number solution.

After knowing the covariance between Y_1 and Y_2, as we estimated from the measurement data, we are able to solve the correlation coefficient between X_1 and X_2 which can be used in statistical timing analysis. Therefore, we can construct a covariance matrix - **S** and S_{ij} denotes $cov\left(Y_i, Y_j\right)$. Furthermore, we can do eigen value decomposition to S. Let $\lambda_1, \lambda_2, \cdots, \lambda_p$ are eigenvalues and $e_1, e_2, \cdots, e_p$ are eigenvectors of S. The solid ellipsoid of X value is satisfying $\left(X - \bar{X}\right)' S^{-1} \left(X - \bar{X}\right) \leq \chi_p^2\left(\alpha\right)$ has $1 - \alpha$ probability and $\bar{X}$ the center, the direction and the length of major axes are e_1 to e_p and $\sqrt{\chi_p^2\left(\alpha\right) * \lambda_1}$ to $\sqrt{\chi_p^2\left(\alpha\right) * \lambda_p}$, respectively. Any observation out of the ellipse can treat as an outlier.

Figure 6 shows two high skewness and Kurtosis examples which CQGP can not be modeled very accurate. So, we need CCGP to handle this situation.

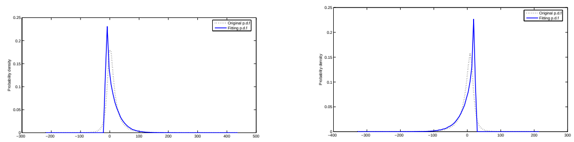

Fig. 6. CQGP Modeling for High Skewness Distributions, positive and negative.

C. *Complex Cubic Gaussian Polynomial (CCGP)*

CQGP matches any distribution up to the 3 moments- mean, variance, skewness. Sometimes, in the case when the fourth moment, Kurtosis, is needed, we propose to use complex cubic Gaussian polynomials (CCGP).

$$Y = aX^3 + bX^2 + cX + d, \tag{13}$$

where a, b, c, and d are complex numbers. Following the moment definitions, we have the following theorem:

Theorem 4: Given the first 4 moments $m_1 - m_4$ of any distribution, a CCGP $Y = aX^3 + bX^2 + cX + d$ will match those four moments if and only if $a - d$ satisfies the following equations:

$$m_1 = b + d \tag{14}$$

$$m_2 = 15a^2 + 6ac + 2b^2 + c^2 \tag{15}$$

$$\begin{aligned} m_3 = {} & 300ba^2 + 24abc + 15a\left(-2ba + 2bc\right) + 12b\left(2ca + b^2\right) \\ & + 3c\left(-2ba + 2bc\right) + 2b\left(-2b^2 + c^2\right) - 2c^2b \end{aligned} \tag{16}$$

$$\begin{aligned} m_4 = {} & 10425a^4 + 945\left(4ca + 2b^2\right)a^2 + 3810b^2a^2 \\ & + 105\left(-4b^2 + 2c^2\right)a^2 + 105\left(-8ba + 8bc\right)ba \\ & + 105\left(2ca + b^2\right)^2 - 120b^2ca + 6b^2\left(2ca + b^2\right) \\ & + 15\left(-2ba + 2bc\right)^2 + 15\left(-4b^2 + 2c^2\right)\left(2ca + b^2\right) \\ & - 12bc\left(-2ba + 2bc\right) + 3\left(-2b^2 + c^2\right)^2 \\ & + 2b^2\left(-2b^2 + c^2\right) + 4b^2c^2 + b^4 \end{aligned} \tag{17}$$

Proof: Following the definition of Gaussian distribution, we can compute its moments as follows:

$$\begin{aligned} & E\left\{X\right\} = 0, \ E\left\{X^2\right\} = 1, \ E\left\{X^3\right\} = 0, \\ & E\left\{X^4\right\} = 3, \ E\left\{X^5\right\} = 0, \ E\left\{X^6\right\} = 15, \\ & E\left\{X^7\right\} = 0, \ E\left\{X^8\right\} = 105, \ E\left\{X^9\right\} = 0, \\ & E\left\{X^{10}\right\} = 945, \ E\left\{X^{11}\right\} = 0, \ E\left\{X^{12}\right\} = 10425 \end{aligned}$$

Using these results, the moments of Y can be evaluated as:

$$\begin{aligned}
E\{Y\} &= E\left\{aX^3 + bX^2 + cX + d\right\} \\
&= aE\left\{X^3\right\} + bE\left\{X^2\right\} + cE\{X\} + d \\
&= b + d \\
E\left\{(Y - E\{Y\})^2\right\} &= E\left\{\left(aX^3 + bX^2 + cX - b\right)^2\right\} \\
&= a^2E\left\{X^6\right\} + 2abE\left\{X^5\right\} + \left(2ac + b^2\right)E\left\{X^4\right\} \\
&\quad + (2bc - ad - ab)\,E\left\{X^3\right\} + \left(c^2 - bd - b^2\right)E\left\{X^2\right\} \\
&\quad + (-bc - cd)\,E\{X\} + bd \\
&= 15a^2 + 6ac + 2b^2 + c^2
\end{aligned}$$

The fourth moment can also be computed in similar fashion (Omited for briefly).

Given the first four moments of any distribution, we solve Equations (14)-(17) by Newton Raphson methods to obtain CCGP.

Figure 7 shows two examples modeled by CQGP and CCGP, respectively. We can see that since CCGP matches Kurtosis, it has better fitting results.

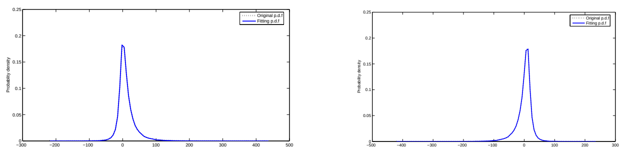

Fig. 7. CCGP Modeling Distributions the same as figure 6

D. Complex High Order Gaussian Polynomial (CHGP)

In case when the extreme accuracy is needed, we can match even higher order moments of the distribution in similar fashion. To match the first k moments, $m_1, m_2, ..., and\ m_k$, the random variable Y can be written as $Y = c_kX^k + c_{k-1}X^{k-1} + ... + c_1X + c_0$. Similar to Theorem 4, we can calculate parameters c_1, c_2 ... and c_k according to $E\{Y\}$, $E\left\{(Y - E\{Y\})^2\right\}$, ..., and $E\left\{(Y - E\{Y\})^k\right\}$.

E. Recovering PDF of our models by Modified SMM

After SSTA performs operations on our models such as CQGP, CCGP, and CHGP. The distribution can be rediscover by again the SMM (statistical moment matching) methods. However, after extensive experiments, we find out that the distribution which is recovered by SMM does not match the initial condition. Therefore, we have developed a modified SMM to solve this problem.

From the definition of Laplace Transform, the initial value theorem is as follows:

$$\lim_{s\to\infty} sf(s) = \lim_{t\to 0^+} = f\left(0^+\right)$$

Now we use a CCGP with zero initial condition as an example. In order to match zero for a 3^{rd} order distribution, we let $m = 3$ and $n \leq 1$. $H(s)$ should be modified as follows:

$$Y(s) = \frac{1 + a_1s}{1 + b_1s + b_2s^2 + b_3s^3}$$

The final moment matching matrix can be written as follows:

$$\begin{bmatrix} 0 & m_0' & -m_1' \\ m_0' & -m_1' & \frac{1}{2!}m_2' \\ -m_1' & \frac{1}{2!}m_2' & -\frac{1}{3!}m_3' \end{bmatrix} \begin{bmatrix} b_3 \\ b_2 \\ b_1 \end{bmatrix} = - \begin{bmatrix} \frac{1}{2!}m_2' \\ -\frac{1}{3!}m_3' \\ \frac{1}{4!}m_4' \end{bmatrix}$$

The Modified SMM method can match the original point and $k_1 + k_2 + k_3$ is guaranteed to be zero. Following is an example to match a Gaussian distribution with 3^{rd} order approximation. Because our method models only the situation when $t > 0$, we shift right the Gaussian distribution with 3σ. We get $p_1 = -0.6241 + 0.7261i$, $p_2 = -0.6241 - 0.7291i$, $p_3 = -0.8142$, $k_1 = -0.8847 - 0.0163i$, $k_2 = -0.8847 + 0.0163i$ and $k_3 = 1.7693$. It can be easily observed that in Figure 8, the Modified SMM has much better match than SMM for a 3-σ shifted Gaussian.

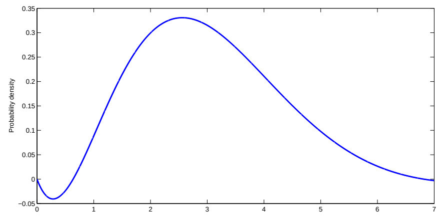

Fig. 8. Model the Gaussian distribution by the Modified SMM

Although we just show how to match zero initial condition case, non-zero initial condition formulae can also be obtained in similar fashion (just set a_2/b_3 equal to the initial value).

One might doubt that the defining range of PDF is from negative infinity to postive infinity and our SMM method seems to be defined from zero to positive infinity. We can just modify the PDF as follows:

$$\begin{cases} 0 & t < 0 \\ y(t) & t \geq 0 \end{cases} \tag{18}$$

Since most of the parameters variation such as threshold voltage and effective gate length are greater than zero, our SMM method can be operated correctly.

IV. SSTA WITH CQGP TIMING MODEL

In block based timing analysis, the arrival time random variable propagation involves two elemental operations: *ADD* and *MAX*. We can use the results from [12] and [13] directly.

A. ADD Operation

If both X and Y are expressed in the quadratic form of (5) $X \sim Q(m_X, \alpha_X, \beta_X, \gamma_X)$ and $Y \sim Q(m_Y, \alpha_Y, \beta_Y.\gamma_Y)$, then the output of the ADD operator is very straightforward written as:

$$Z = X + Y \sim Q(m_Z, \alpha_Z, \beta_Z, \gamma_Z)$$

where the quadratic parameters are computed as:

$$m_Z = m_X + m_Y; \qquad \alpha_Z = \alpha_X + \alpha_Y \tag{19}$$

$$\beta_Z = \beta_X + \beta_Y; \qquad \gamma_Z = \gamma_X + \gamma_Y \tag{20}$$

B. MAX Operation

MAX operator, however, is more complicated since it is generally a non-linear operator and error will hapen if we approximate it with a linear one. Fortunately, we can borrow the idea from [13]. Since we can use the methods proposed in [13] to estimate the first three moments - mean, variance and skewness, we can pass these moments to equation (6) and then complete the MAX operation.

V. Experimental Result

We will use our CQGP and CCGP methods to model various distributions such as uniform distribution. Although some of them look like a bell shape or even Gaussian distribution, they are actually faraway from being Gaussian. One way to check whether a distribution is close to Gaussian distribution is the Q-Q plot. Q-Q plot is a scheme that finds the correlation between ordered observations and standard quantiles then it tests the normality of the observation. If the correlation coefficients are approximate to 1, the observation is near to Gaussian distribution.

Figure 9 is a Q-Q plot to test 100000 observations. We observe that they can not form a straight line and thus they are not close to a Gaussian distribution. Therefore, using a linear Gaussian model will not be accurate. In this case, CQGP or CCGP should be applied. Figure 10 shows the result of using CQGP which achieves a very accurate result. Furthermore, CCGP could get a better fit. Figure 11 shows the two comparisons in detail. Since CCGP can match the Kurtosis, so it can model the distribution peak more accurately than CQGP.

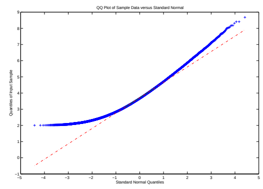

Fig. 9. Q-Q plot for a non-Gaussian distribution

VI. Conclusion

In this paper, we propose several new process data modeling method using complex high order Gaussian polynomials such as CQGP, CCGP, and CHGP. Complex coefficient allows much more freedom during moment match while is compatible with main stream SSTA method. Furthermore, we also develop a Modified Statistical Moment Match method to match the initial condition. Experimental results demonstrate the accuracy and correctness of our method.

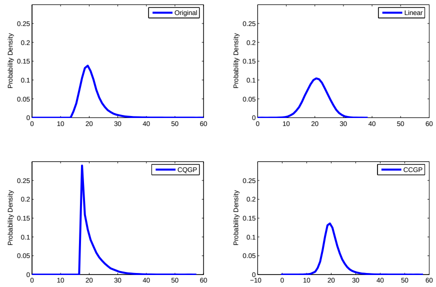

Fig. 10. Model the sample from a non-Gaussian distribution

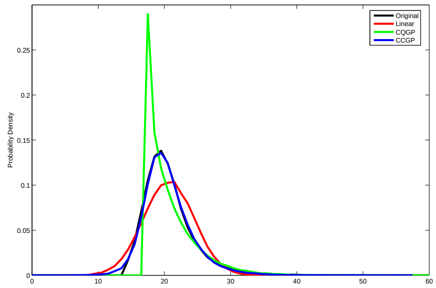

Fig. 11. To compare figure 10 in detail.

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國外研究機構	aspdac

工作記要： 2006 年的 13th Asia and South Pacific Design Automation Conference ASP-DAC 2008,在日本橫濱市舉行,我們發表了一篇論文,我個人並擔任會議議程委員,此次 aspdac 會有一些 ssta 的文章不錯,學習效果相當好.本人臨時被指派爲 session chair 並爲 panelist,收穫良多.