Minimax design of two-channel IIR QMF banks with arbitrary group delay

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Abstract: The paper deals with the minimax design of two-channel infinite impulse response (IIR) QMF banks with arbitrary group delay, for which the IIR analysis filters and the resulting filter bank possess the frequency response optimal in the minimax (L_∞) sense. Utilising a lattice structure for the denominators of the IIR analysis filters, a design technique is presented based on an approximation scheme and a weighted least-squares (WLS) algorithm, previously developed by one of the authors for solving the resulting design problem that is basically a nonlinear optimisation problem. During the design process, this technique finds the tap coefficients for the numerator and the reflection coefficients for the denominator of the prototype IIR analysis filter simultaneously. The stability of the designed prototype IIR analysis filter is ensured by incorporating an efficient stabilisation procedure to make all of the reflection coefficient values fall between -1 and $+1$. Computer simulations show the effectiveness of the proposed design technique.

1 Introduction

Quadrature mirror filter (QMF) banks have been widely used in the areas of subband coding of speech signals [1], communication systems [2], short-time spectral analysis [3] and subband coding of image signals [4]. In these applications, a QMF bank is used to decompose a signal into subbands and the subband signals in the analysis system are decimated by an integer which is equal to the number of subbands. It is known that two-channel QMF banks can be easily employed for constructing M-channel QMF banks based on a tree structure. Hence, it is worth exploiting the design problem of two-channel QMF banks.

In general, the overall system delay of a linear-phase finite impulse response (LP-FIR) filter bank is totally determined by the lengths of the FIR filters used, although LP-FIR filter banks have been widely considered in the literature [5–9]. Therefore the inherent long system delay caused by using the two-channel LP-FIR filter banks, designed by using the techniques of [5–9], may make the overall tree-structure system impractical. However, the overall system delay of a tree-structure filter bank can be significantly reduced by imposing a low delay on the splitting stages located deep inside the tree.

Many techniques have been presented for designing two-channel low-delay FIR filter banks [10, 11]. An FIR filter bank with low delay and filter length N has a system delay of k_d, which is less than $N-1$ [9]. The time-domain technique of [10] can design an FIR filter bank with an adjustable delay. Two techniques have been presented in [11] for designing two-channel low-delay FIR filter banks optimally in the L_2 sense. Nevertheless, an IIR filter requires a lower order than an FIR filter under the same stopband energy. It is thus expected that QMF banks with IIR filters will require less computational complexity than QMF banks with FIR filters under the same stopband energy for the analysis and synthesis filters, for example, the authors of [12, 13] have shown that IIR filters can have computational advantages over FIR filters when used in a QMF configuration for image compression.

Several design results for IIR filter banks optimal in the least-squares (L_2) sense have been reported in [14–19]. However, these IIR filter banks are designed based on the linear-phase property imposed on the analysis and synthesis filters. Although designing a QMF bank with a frequency-response error minimised in the L_2 sense is a popular task, the peak reconstruction error of the resulting QMF bank may not be as small as possible. Moreover, it is known that designing filters with a minimax response results in the advantage of a smaller error ripple over the least-squares response at the same filter order. Nevertheless, there are practically no papers in the literature concerning the minimax design of IIR QMF banks with arbitrary group delay.

In this paper, we consider the optimal design of two-channel IIR QMF banks with arbitrary group delay under the minimax (L_∞) error criteria. Utilising a lattice structure for the denominators of the IIR analysis filters, a design technique based on an approximation scheme and the Lim–Lee–Chen–Yang (LLCY) WLS algorithm developed in [20] is presented for efficiently solving the resulting design problem that is basically a nonlinear optimisation problem. During the design process, this technique finds the tap coefficients for the numerator and the reflection coefficients for the denominator of the prototype IIR analysis filter $H_0(z)$ simultaneously. It ensures the stability of the designed prototype IIR analysis filters by incorporating an efficient stabilisation procedure to make the magnitude of each reflection coefficient within -1 and

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+1. Although the IIR QMF bank designed by using the proposed technique is not a perfect reconstruction filter bank, simulation results show that it outperforms the existing FIR QMF banks in several aspects, namely, peak stopband ripple and the peak variation of group delay in $H_0(z)$, peak reconstruction error and the peak variation of group delay in the resulting IIR QMF bank and the peak variation of the filter-bank response.

2 Two-channel QMF banks with arbitrary group delay

Consider the two-channel filter bank with a basic structure shown in Fig. 1. $H_0(z)$ and $H_1(z)$ designate the low-pass and high-pass analysis filters, respectively, and $F_0(z)$ and $F_1(z)$ designate the low-pass and high-pass synthesis filters, respectively. It is easy to show that the input-output relationship in the Z transform is given by

$$\hat{X}(z) = \frac{1}{2}[H_0(z)F_0(z) + H_1(z)F_1(z)]X(z) + \frac{1}{2}[H_0(-z)F_0(z) + H_1(-z)F_1(z)]X(-z) \quad (1)$$

The first term of eqn. 1 represents a linear shift-invariant system response, which is the desired signal translation from $x(n)$ to $\hat{x}(n)$, and the second term represents the aliasing error due to the change of sampling rate in the QMF bank. Setting the synthesis filters $F_0(z) = 2H_1(-z)$ and $F_1(z) = -2H_0(-z)$ eliminates the aliasing term. As the mirror-image symmetry about the frequency $\omega = \pi/2$ exists between $H_0(z)$ and $H_1(z)$, we have $H_0(z) = H_1(-z)$. Hence, eqn. 1 becomes

$$\hat{X}(z) = [H_0(z)H_0(z) - H_0(-z)H_0(-z)]X(z) \quad (2)$$

Letting $z = e^{j\omega}$ into eqn. 2, we obtain

$$\hat{X}(e^{j\omega}) = [H_0^2(e^{j\omega}) - H_0^2(e^{j(\omega+\pi)})]X(e^{j\omega}) \quad (3)$$

Let $T(e^{j\omega})$ denote the frequency response of the QMF bank. Eqn. 3 reveals that producing a reconstructed signal $\hat{x}(n)$ that is a delayed replica of $x(n)$ requires

$$T(e^{j\omega}) = H_0^2(e^{j\omega}) - H_0^2(e^{j(\omega+\pi)}) = e^{-jg_d\omega} \text{ for all } \omega \quad (4)$$

where g_d is the system delay of the QMF bank. This imposes constraints not only that $H_0(z)$ should be an ideal low-pass filter, but also that its behaviour for all ω should satisfy the condition given in eqn. 4. Therefore $H_0(z)$ is generally called the low-pass prototype filter for the QMF bank.

3 Formulation of the design problem

Here, we consider the minimax design of the two-channel QMF banks, as shown in Fig. 1. Let the low-pass prototype filter be an IIR filter with order M/N (i.e. M zeros and N poles) and transfer function $H_0(z) = A(z)/B_N(z)$, where the numerator $A(z)$ is an Mth-order polynomial with tap coefficient vector $\boldsymbol{A} = [a_0, a_1, \ldots, a_M]^T$, $B_N(z)$ is an Nth-order FIR lattice filter with reflection coefficient vector $\boldsymbol{K} = [k_1, k_2, \ldots, k_N]^T$. The superscript T denotes the transpose operation. Fig. 2 shows the system structure for $B_N(z)$ which can be obtained from the following recursive formula [21]:

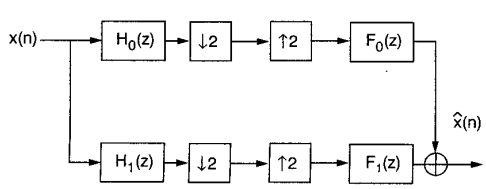

Fig. 1 *Two-channel QMF bank*

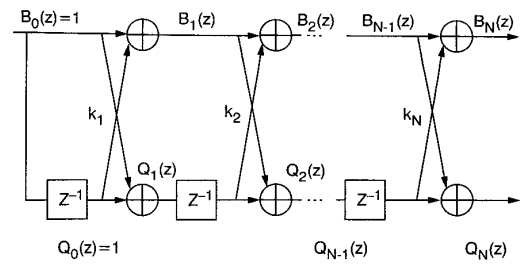

Fig. 2 *Lattice structure for an Nth-order FIR lattice filter*

$$B_0(z) = Q_0(z) = 1$$
$$B_n(z) = B_{n-1}(z) + k_n z^{-1} Q_{n-1}(z)$$
$$Q_n(z) = k_n B_{n-1}(z) + z^{-1} Q_{n-1}(z) \quad (5)$$

From eqn. 4, the overall design task is to find the optimal tap and reflection coefficients $\{a_m, k_n\}$ for the stable low-pass prototype filter $H_0(z)$, such that the condition shown in eqn. 4 must be satisfied. Let the reconstruction error for the QMF bank be defined as

$$e_r(e^{j\omega}) = T(e^{j\omega}) - e^{-jg_d\omega} \quad (6)$$

For achieving the minimax design, we define the overall error function E to be minimised as following the weighted sum of two terms:

$$E = \int_{\omega=0}^{\pi/2} W_1(\omega)|e^{-jg_d\omega} - T(e^{j\omega})|^2 \, d\omega + \alpha \int_{\omega=\omega_s}^{\pi} W_2(\omega)|H_0(e^{j\omega})|^2 \, d\omega \quad (7)$$

where $W_1(\omega)$ and $W_2(\omega)$ are two frequency response weighting functions used for the minimisation of the corresponding reconstruction error and the stopband error of $H_0(z)$, respectively, ω_s is the frequency of the stopband edge and α is the relative weight between these two error terms. We note from eqn. 7 that the overall error function E is a function of the fourth degree in the tap and reflection coefficients $\{a_m, k_n\}$. Therefore minimising E directly leads to a very highly nonlinear programming problem in addition to the stability problem for $H_0(z)$. Moreover, to obtain the minimax design, $W_1(\omega)$ and $W_2(\omega)$ must be appropriately chosen.

4 Proposed design technique

Here, we consider the design problem shown in eqn. 7. The proposed design technique is based on an iterative process to find the optimal tap and reflection coefficients for eqn. 7. Let $S_1 = \{\omega_1 = 0, \omega_2, \ldots, \ldots, \omega_K = \pi/2\}$ and $S_2 = \{\omega_{K+1} = \omega_s, \omega_{K+2}, \ldots, \omega_{K+P} = \pi\}$ be two dense grids of frequency points linearly distributed over $[0, \pi/2]$ and $[\omega_s, \pi]$, respectively, for evaluating the related error functions. Then eqn. 7 can be rewritten as

$$\hat{E} = \sum_{\omega_i \in S_1} W_1(\omega_i)|e^{-jg_d\omega_i} - T(e^{j\omega_i})|^2 + \alpha \sum_{\omega_i \in S_2} W_2(\omega_i)|H_0(e^{j\omega_i})|^2 \quad (8)$$

4.1 Determination of initial guess for $H_0(z)$

To initiate the design process, we propose the following procedure for determining an appropriate initial guess $H_0^0(z)$ for $H_0(z)$. According to the principle of a two-channel QMF bank, we define a desired frequency response $D(e^{j\omega})$ as

$$D(e^{j\omega}) = \begin{cases} e^{-j(g_d/2)\omega}, & \text{for } \omega \in [0, \pi - \omega_s] \\ \dfrac{1}{\sqrt{2}} e^{-j(g_d/2)\omega}, & \text{for } \omega = \dfrac{\pi}{2} \\ 0, & \text{for } \omega \in [\omega_s, \pi] \end{cases} \tag{9}$$

An FIR filter $G(z)$ with order $2N$ is first designed to optimally approximate the frequency response $D(e^{j\omega})$, shown by eqn. 9, using conventional least-squares (L_2) error criteria. Let the resulting filter coefficients be given by $\{h_0, h_1, \ldots, h_{2N}\}$. Through the use of the balanced model reduction algorithm presented in [22], we find an IIR filter with order N/N from $G(z)$. Assume that the IIR filter has denominator $C(z)$ with coefficients $\{c_0, c_1, \ldots, c_N\}$, then, the initial lattice system $B_N^0(z)$ with reflection coefficients $\{k_1^0, k_2^0, \ldots, k_N^0\}$ corresponding to $C(z)$ can be found, since there exists a one-to-one correspondence between $\{c_0, c_1, \ldots, c_N\}$ and $\{k_1^0, k_2^0, \ldots, k_N^0\}$ [23]. Finally, the best L_2 solution for the corresponding initial numerator $A^0(z)$ can be obtained by solving the following optimisation problem:

$$\text{Minimise} \left| D(e^{j\omega}) - \frac{A^0(e^{j\omega})}{B_N^0(e^{j\omega})} \right|^2 \quad \text{for all } \omega \in [0, \pi - \omega_s] \cup \frac{\pi}{2} \cup [\omega_s, \pi] \tag{10}$$

For evaluating the related error function given by expr. 10, we again take a set of discrete frequency points linearly distributed over $S = [0, \pi - \omega_s] \cup \pi/2 \cup [\omega_s, \pi]$. Let $S_d = \{\omega_1, \omega_2, \ldots, \omega_P = \pi - \omega_s, \omega_{P+1} = \pi/2, \omega_{P+2} = \omega_s, \ldots, \omega_{2P+1} = \pi\}$ be the dense grid of frequency points, and $\boldsymbol{U}$ be a complex $(2P+1) \times (M+1)$ matrix with its (m, n)th element given by

$$U(m, n) = \frac{e^{-j\omega_m}(n-1)}{B_N^0(e^{j\omega_m})}, \; 1 \le m \le 2P+1, \; 1 \le n \le M+1 \tag{11}$$

and $\boldsymbol{d}$ be a complex $(2P+1) \times 1$ vector with its mth element given by

$$d(m) = D(e^{j\omega_m}), \; 1 \le m \le 2P+1 \tag{12}$$

Then, the initial coefficient vector $\boldsymbol{A}^0 = [a_0^0, a_1^0, \ldots, a_M^0]^T$ of $A^0(z)$ optimal in the L_2 sense for expr. 10 can be found by minimising $|\boldsymbol{U}\boldsymbol{A}^0 - \boldsymbol{d}|^2 = (\boldsymbol{U}\boldsymbol{A}^0 - \boldsymbol{d})^H(\boldsymbol{U}\boldsymbol{A}^0 - \boldsymbol{d})$, where the superscript H denotes the complex conjugate transpose. Clearly, this leads to the optimal solution given by

$$\boldsymbol{A}^0 = [(\mathrm{Re}(\boldsymbol{U}))^T \mathrm{Re}(\boldsymbol{U}) + (\mathrm{Im}(\boldsymbol{U}))^T \mathrm{Im}(\boldsymbol{U})]^{-1} \times [(\mathrm{Re}(\boldsymbol{U}))^T \mathrm{Re}(\boldsymbol{d}) + (\mathrm{Im}(\boldsymbol{U}))^T \mathrm{Im}(\boldsymbol{d})]$$

where $\mathrm{Re}(\boldsymbol{X})$ and $\mathrm{Im}(\boldsymbol{X})$ represent the real and imaginary parts of the matrix $\boldsymbol{X}$, respectively. After finding the appropriate initial guess $H_0^0(z) = A^0(z)/B_N^0(z)$, we present an iterative procedure step by step for computing $A(z)$ and $B_N(z)$ during the design process.

4.2 Iterative procedure

The required initial $W_1(\omega)$ and $W_2(\omega)$ are both set to the identity matrices with appropriate sizes, respectively.

Step 1: At the lth iteration, the gradient matrices $\boldsymbol{H}_t = [\nabla T^l(e^{j\omega_i})]$ for $\omega_i \in S_1$ and $\boldsymbol{H}_h = [\nabla H_0^l(e^{j\omega_i})]$ for $\omega_i \in S_2$ are computed, where ∇ represents the gradient operator $[\partial/\partial k_1, \partial/\partial k_2, \ldots, \partial/\partial k_N, \partial/\partial a_0, \partial/\partial a_1, \ldots, \partial/\partial a_M]$. Accordingly, $\boldsymbol{H}_t$ and $\boldsymbol{H}_h$ are given by

$$\boldsymbol{H}_t = \begin{bmatrix} \dfrac{\partial T^l(e^{j\omega_1})}{\partial k_1} & \cdots & \dfrac{\partial T^l(e^{j\omega_1})}{\partial k_N} & \dfrac{\partial T^l(e^{j\omega_1})}{\partial a_0} & \cdots & \dfrac{\partial T^l(e^{j\omega_1})}{\partial a_M} \\ \dfrac{\partial T^l(e^{j\omega_2})}{\partial k_1} & \cdots & \dfrac{\partial T^l(e^{j\omega_2})}{\partial k_N} & \dfrac{\partial T^l(e^{j\omega_2})}{\partial a_0} & \cdots & \dfrac{\partial T^l(e^{j\omega_2})}{\partial a_M} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \dfrac{\partial T^l(e^{j\omega_K})}{\partial k_1} & \cdots & \dfrac{\partial T^l(e^{j\omega_K})}{\partial k_N} & \dfrac{\partial T^l(e^{j\omega_K})}{\partial a_1} & \cdots & \dfrac{\partial T^l(e^{j\omega_K})}{\partial a_N} \end{bmatrix}$$

$$\boldsymbol{H}_h = \begin{bmatrix} \dfrac{\partial H_0^l(e^{j\omega_{K+1}})}{\partial k_1} & \cdots & \dfrac{\partial H_0^l(e^{j\omega_{K+1}})}{\partial k_N} & \dfrac{\partial H_0^l(e^{j\omega_{K+1}})}{\partial a_0} & \cdots & \dfrac{\partial H_0^l(e^{j\omega_{K+1}})}{\partial a_M} \\ \dfrac{\partial H_0^l(e^{j\omega_{K+2}})}{\partial k_1} & \cdots & \dfrac{\partial H_0^l(e^{j\omega_{K+2}})}{\partial k_N} & \dfrac{\partial H_0^l(e^{j\omega_{K+2}})}{\partial a_0} & \cdots & \dfrac{\partial H_0^l(e^{j\omega_{K+2}})}{\partial a_M} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \dfrac{\partial H_0^l(e^{j\omega_{K+P}})}{\partial k_1} & \cdots & \dfrac{\partial H_0^l(e^{j\omega_{K+P}})}{\partial k_N} & \dfrac{\partial H_0^l(e^{j\omega_{K+P}})}{\partial a_0} & \cdots & \dfrac{\partial H_0^l(e^{j\omega_{K+P}})}{\partial a_M} \end{bmatrix}$$

We have derived the following equations for computing these two gradient matrices:

$$\begin{aligned} \frac{\partial T^l(e^{j\omega_i})}{\partial a_m} &= \frac{\partial[(H_0^l(e^{j\omega_i}))^2 - (H_0^l(e^{j(\omega_i+\pi)}))^2]}{\partial a_m} \\ &= 2H_0^l(e^{j\omega_i}) \frac{1}{B_N^l(e^{j\omega_i})} \frac{\partial A^l(e^{j\omega_i})}{\partial a_m} \\ &\quad - 2H_0^l(e^{j(\omega_i+\pi)}) \frac{1}{B_N^l(e^{j(\omega_i+\pi)})} \\ &\quad \times \frac{\partial A^l(e^{j(\omega_i+\pi)})}{\partial a_m} \end{aligned}$$

$$\text{for } \omega_i \in S_1;\ m = 0, 1, \ldots, M$$

$$\frac{\partial T^l(e^{j\omega_i})}{\partial k_n} = \frac{\partial[(H_0^l(e^{j\omega_i}))^2 - (H_0^l(e^{j(\omega_i+\pi)}))^2]}{\partial k_n}$$

$$= -2H_0^l(e^{j\omega_i}) \frac{A^l(e^{j\omega_i})}{(B_N^l(e^{j\omega_i}))^2} \frac{\partial B_N^l(e^{j\omega_i})}{\partial k_n} + 2H_0^l(e^{j(\omega_i+\pi)}) \frac{A^l(e^{j(\omega_i+\pi)})}{(B_N^l(e^{j(\omega_i+\pi)}))^2} \times \frac{\partial B_N^l(e^{j(\omega_i+\pi)})}{\partial k_n}$$

$$\text{for } \omega_i \in S_1;\ n = 1, 2, \ldots, N$$

$$\frac{\partial H_0^l(e^{j\omega_i})}{\partial a_m} = \frac{1}{B_N^l(e^{j\omega_i})} \frac{\partial A^l(e^{j\omega_i})}{\partial a_m}$$

$$\text{for } \omega_i \in S_2;\ m = 0, 1, \ldots, M$$

and

$$\frac{\partial H_0^l(e^{j\omega_i})}{\partial k_n} = \frac{-A^l(e^{j\omega_i})}{(B_N^l(e^{j\omega_i}))^2} \frac{\partial B_N^l(e^{j\omega_i})}{\partial k_n}$$

$$\text{for } \omega_i \in S_2;\ n = 1, 2, \ldots, N. \quad (13)$$

Step 2: Use a linearisation scheme to approximate the frequency response error

$$\hat{E}^l = \sum_{\omega_j \in S_1} W_1(\omega_i) |e^{-jg_d\omega_i} - T^l(e^{j\omega_i})|^2 + \alpha \sum_{\omega_i \in S_2} W_2(\omega_i) |H_0^l(e^{j\omega_i})|^2$$

due to a perturbation in the coefficient vectors in the linear subspace spanned by the gradient matrices $\boldsymbol{H}_t$ and $\boldsymbol{H}_h$. That is, the approximation error given by

$$\hat{E}_a^l = \sum_{\omega_i \in S_1} W_1(\omega_i) |e^{-jg_d\omega_i} - T^l(e^{j\omega_i}) - \nabla T^l(e^{j\omega_i})\boldsymbol{v}|^2 + \alpha \sum_{\omega_i \in S_2} W_2(\omega_i) |H_0^l(e^{j\omega_i}) + \nabla H_0^l(e^{j\omega_i})\boldsymbol{v}|^2 \quad (14)$$

is computed, where the vector $\boldsymbol{v} = [\Delta\boldsymbol{K}, \Delta\boldsymbol{A}] = [\Delta k_1, \Delta k_2, \ldots, \Delta k_N, \Delta a_0, \Delta a_1, \ldots, \Delta a_M]^T$ contains the increments of the independent coefficients to be found. Then solve the optimisation problem of eqn. 14 to obtain the increment coefficient vector $\boldsymbol{v}$. Let $\boldsymbol{R}_1$ be a complex vector with size $K \times 1$ and the ith element given by $R_1(i) = e^{jg_d\omega_i} - T^l(e^{j\omega_i})$, for $\omega_i \in S_1$, and $\boldsymbol{R}_2$ be a complex vector with size $P \times 1$ and the ith element given by $R_2(i) = -H_0^l(e^{j\omega_{i+K}})$, for $\omega_{i+K} \in S_2$. Then, it is easy to show that the optimal solution $\boldsymbol{v}$ for eqn. 14 is given by

$$\boldsymbol{v} = \{[(\mathrm{Re}(\boldsymbol{H}_t))^T \boldsymbol{W}_1 \mathrm{Re}(\boldsymbol{H}_t) + (\mathrm{Im}(\boldsymbol{H}_t))^T \boldsymbol{W}_1 \mathrm{Im}(\boldsymbol{H}_t)] + \alpha[(\mathrm{Re}(\boldsymbol{H}_h))^T \boldsymbol{W}_2 \mathrm{Re}(\boldsymbol{H}_h) + (\mathrm{Im}(\boldsymbol{H}_h))^T \boldsymbol{W}_2 \mathrm{Im}(\boldsymbol{H}_h)]\}^{-1} \times \{[(\mathrm{Re}(\boldsymbol{H}_t))^T \boldsymbol{W}_1 \mathrm{Re}(\boldsymbol{R}_1) + (\mathrm{Im}(\boldsymbol{H}_t))^T \boldsymbol{W}_1 \mathrm{Im}(\boldsymbol{R}_1)] + \alpha[(\mathrm{Re}(\boldsymbol{H}_h))^T \boldsymbol{W}_2 \mathrm{Re}(\boldsymbol{R}_2) + (\mathrm{Im}(\boldsymbol{H}_h))^T \boldsymbol{W}_2 \mathrm{Im}(\boldsymbol{R}_2)]\} \quad (15)$$

where $\boldsymbol{W}_1 = \mathrm{diag}(W_1(\omega_1), W_1(\omega_2), \ldots, W_1(\omega_K))$ and $\boldsymbol{W}_2 = \mathrm{diag}(W_2(\omega_{K+1}), W_2(\omega_{K+2}), \ldots, W_2(\omega_{K+P}))$. $\mathrm{diag}(\cdot)$ denotes a diagonal matrix.

Step 3: Perform a line search by using the Nelder–Meade simplex algorithm of [24] to find the best step size t with $0 < t < 1$ to update the numerator and denominator of $H_0^l(z)$ such that the following cost function reaches its minimum:

$$\sum_{\omega_i \in S_1} W_1(\omega_i) |e^{-jg_d\omega_i} - T^{l+1}(e^{j\omega_i})|^2 + \alpha \sum_{\omega_i \in S_2} W_2(\omega_i) |H_0^{l+1}(e^{j\omega_i})|^2 \quad (16)$$

subject to the constraints of $\max|k_j^l + t\Delta k_j| < k_{max}$, $j = 1, 2, \ldots, N$, where $H_0^{l+1}(e^{j\omega})$ has tap and reflection coefficient vectors given by $\boldsymbol{A}^{l+1} = [a_0^l + t\Delta a_0, a_1^l + t\Delta a_1, \ldots, a_M^l + t\Delta a_M]^T$ and $\boldsymbol{K}^{l+1} = [k_1^l + t\Delta k_1, k_2^l + t\Delta k_2, \ldots, k_N^l + t\Delta k_N]^T$, respectively, and $T^{l+1}(e^{j\omega})$ is the corresponding filter bank response. Moreover, k_{max} is a preset maximal absolute value and must be less than 1 for the reflection coefficients in order to ensure the stability of the designed IIR QMF bank.

Step 4: Compute the overall error function of eqn. 8 corresponding to $H_0^{l+1}(e^{j\omega_i})$ and $T^{l+1}(e^{j\omega_i})$, which is in fact given by expr. 16 and denoted by $\hat{E}^{l+1}$.

Step 5: Compute the ratio $|\hat{E}^l - \hat{E}^{l+1}|/\hat{E}^l$. If this ratio is less than a preset positive number ϵ, then go to *step 6*. Otherwise, we continue this procedure and go to *step 1*.

Step 6: Let $\mathrm{Max}(V_r)$ and $\mathrm{Min}(V_r)$ be the maximum and minimum of $|e^{-jg_d\omega_i} - T^{l+1}(e^{j\omega_i})|$ over all the extreme frequencies for $\omega_i \in S_1$, respectively, and $\mathrm{Max}(V_0)$ and $\mathrm{Min}(V_0)$ be the maximum and minimum of $|H_0^{l+1}(e^{j\omega_i})|$ over all the extreme frequencies for $\omega_i \in S_2$, respectively. If all of the following stopping criteria:

$$\frac{\mathrm{Max}(V_r) - \mathrm{Min}(V_r)}{\mathrm{Max}(V_r)} \le \kappa_1$$

$$\frac{\mathrm{Max}(V_0) - \mathrm{Min}(V_0)}{\mathrm{Max}(V_0)} \le \kappa_2$$

are satisfied, then terminate the design process. Otherwise, go to *step 7*.

Step 7: Adjust $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ by using the LLCY algorithm of [20].

Step 7.1: If the stopping criterion

$$\frac{\mathrm{Max}(V_r) - \mathrm{Min}(V_r)}{\mathrm{Max}(V_r)} \le \kappa_1$$

is not satisfied, then construct the envelope function $F_r(\omega)$ of $|e^{-jg_d\omega} - T^{l+1}(e^{j\omega})|$ for $\omega \in [0, \pi/2]$. Compute the updating function $v_1(\omega)$ as follows:

$$v_1(\omega) = \frac{K\{F_r(\omega)\}^{1.5}}{\sum_{\omega_i \in S_1} W_1(\omega_i)\{F_r(\omega_i)\}^{1.5}}$$

and update $W_1(\omega)$ by computing $W_1(\omega)v_1(\omega)$ and the corresponding weighting matrix $\boldsymbol{W}_1$.

Step 7.2: If the stopping criterion

$$\frac{\mathrm{Max}(V_0) - \mathrm{Min}(V_0)}{\mathrm{Max}(V_0)} \le \kappa_2$$

is not satisfied, then construct the envelope function $F_0(\omega)$ of $|H_0^{l+1}(\omega)|$ for $\omega \in [\omega_s, \pi]$. Compute the updating function $v_2(\omega)$ as follows:

$$v_2(\omega) = \frac{P\{F_0(\omega)\}^{1.5}}{\sum_{\omega_i \in S_2} W_2(\omega_i)\{F_0(\omega_i)\}^{1.5}}$$

and update $W_2(\omega)$ by computing $W_2(\omega)v_2(\omega)$ and the corresponding weighting matrix $\boldsymbol{W}_2$. Then, set $l = l + 1$ and go to *step 1*.

4.2.1 Remarks:

There are two situations where the gradient matrices $\boldsymbol{H}_t$ and $\boldsymbol{H}_h$ may degenerate.

Case 1: The columns of $\boldsymbol{H}_t$ and $\boldsymbol{H}_h$ are not linearly independent. Then, the optimal solution for eqn. 14 will

not be unique. To find an appropriate optimal solution, we construct matrices $\boldsymbol{G}_t$ and $\boldsymbol{G}_h$ by choosing the independent columns from $\boldsymbol{H}_t$ and $\boldsymbol{H}_h$, and a vector $\boldsymbol{u}$ by choosing the components of $\boldsymbol{v}$ corresponding to the independent columns. Then use $\boldsymbol{G}_t$, $\boldsymbol{G}_h$ and $\boldsymbol{u}$ to replace $\boldsymbol{H}_t$, $\boldsymbol{H}_h$ and $\boldsymbol{v}$ in eqn. 14. On the other hand, if only $\boldsymbol{H}_t$ (or $\boldsymbol{H}_h$) has columns not linearly independent, then, we construct a matrix $\boldsymbol{G}_t$ (or $\boldsymbol{G}_h$) by replacing the elements of those columns that are not linearly independent with zero elements from $\boldsymbol{H}_t$ (or $\boldsymbol{H}_h$). Then use $\boldsymbol{G}_t$ (or $\boldsymbol{G}_h$) to replace $\boldsymbol{H}_t$ (or $\boldsymbol{H}_h$) in eqn. 14 to obtain an appropriate optimal solution.

Case 2: At the lth iteration, the ith reflection coefficient k_i may have the absolute value equal to k_{max}. To tackle this difficulty, we construct a vector $\boldsymbol{u}$ by eliminating Δk_i of the vector $\boldsymbol{v}$ and two matrices $\boldsymbol{G}_t$ and $\boldsymbol{G}_h$ by eliminating the columns of $\boldsymbol{H}_t$ and $\boldsymbol{H}_h$ corresponding to Δk_i, respectively. Then, we use the $\boldsymbol{G}_t$, $\boldsymbol{G}_h$, and $\boldsymbol{u}$ to replace $\boldsymbol{H}_t$, $\boldsymbol{H}_h$ and $\boldsymbol{v}$ in eqn. 14.

5 Design example

In this Section, simulation results are presented for illustration and comparison. These simulations were performed on a personal computer with a Pentium CPU using MATLAB programming language. For the design example, the spacing for two adjacent frequency grid points in $[0, \pi]$ is set to $\pi/299$, i.e. the number of frequency grid points taken in $[0, \pi]$ is 300. The value of ϵ used for terminating the design process is set to 10^{-3}. The performance for each of the designed filter banks is evaluated in terms of the peak stopband ripple (PSR) and the maximal variation of the passband group delay in $H_0(e^{j\omega})$, the peak reconstruction error (PRE) and the maximal variation of the group delay in $T(e^{j\omega})$ and the maximal variation of the filter-bank response. They are defined as

$$\mathrm{PSR} = -20\log_{10}\left(\max_{\omega_i \in [\omega_s, \pi]} |H_0(e^{j\omega_s})|\right) \text{ (dB)}$$

$$\text{max. variation of passband } GD(H_0(e^{j\omega})) = \max_{\omega_i \in [0, \omega_p]} \left| GD(H_0(e^{j\omega_i})) - \frac{g_d}{2} \right| \text{ samples}$$

$$\mathrm{PRE} \text{ in } |T(e^{j\omega})| = \max_{\omega_i \in [0, (\pi/2)]} |20\log_{10}|T(e^{j\omega_i})|| \text{ (dB)}$$

$$\text{max. variation of } GD(T(e^{j\omega})) = \max_{\omega_i \in [0, (\pi/2)]} |GD(T(e^{j\omega_i})) - g_d| \text{ samples}$$

$$\text{max. variation of filter bank response} = \max_{\omega_i \in [0, (\pi/2)]} |e^{-jg_d\omega_i} - T(e^{j\omega_i})| \tag{17}$$

where $GD(x)$ denotes the group delay of x and ω_p the passband edge frequency of $H_0(e^{j\omega})$. For comparison, we also utilise the technique of [9] for the design example with the FIR QMF bank.

Design example: We use the design specifications: the desired group delay g_d for $T(z)$ is 11, ω_p and ω_s are 0.4π and 0.6π, respectively, $M = N = 8$, the order of $H_0(z)$ for the FIR QMF bank designed by using the technique of [9] is 25. Hence, the numbers of independent coefficients for the IIR QMF bank designed by using the proposed technique and the FIR QMF bank designed by using the technique of [9] are 17 and 26, respectively. Table 1

Table 1: Significant design results for the design example

	IIR QMF bank	FIR QMF bank [9]
Filter order	8/8	25
No. of coefficients	17	26
PSR, dB	27.91	22.05
Max. variation of passband group delay of $H_0(e^{j\omega})$	0.1035	0.1679
PRE in $\|T(e^{j\omega})\|$, dB	0.0357	0.1359
Max. variation of group delay of $T(e^{j\omega})$	0.1578	0.3513
No. of iterations	29	7
Max. variation of filter bank response	4.10×10^{-3}	1.59×10^{-2}

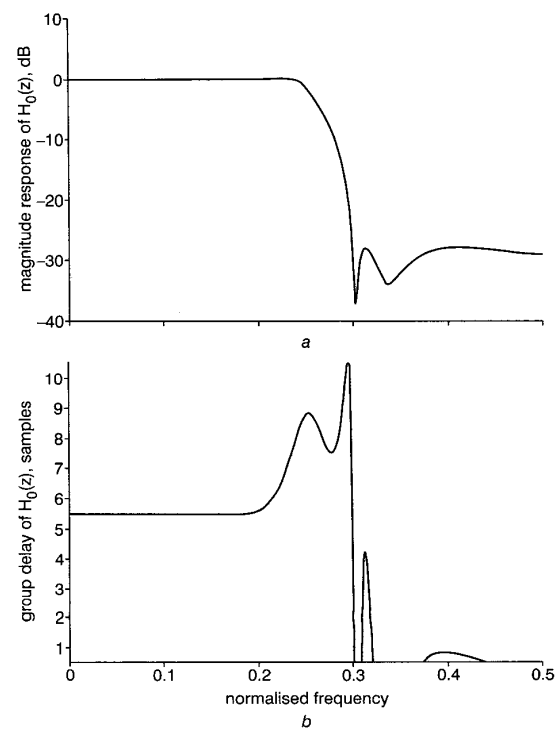

Fig. 3 *Magnitude response and group delay variation of $H_0(z)$ for the designed IIR QMF bank*

a Magnitude response
b Group delay response

Table 2: Tap and reflection coefficients for the design example

i	a_i	k_i
0	$-1.2598426 \times 10^{-2}$	
1	4.8936644×10^{-2}	9.1739807×10^{-2}
2	$-7.5870625 \times 10^{-2}$	8.3686351×10^{-1}
3	3.2289560×10^{-2}	$-4.0153005 \times 10^{-1}$
4	4.0767693×10^{-2}	4.7969762×10^{-1}
5	3.3775573×10^{-1}	$-4.5653815 \times 10^{-1}$
6	3.5931872×10^{-1}	3.4155385×10^{-1}
7	2.8686857×10^{-1}	$-1.6809614 \times 10^{-1}$
8	1.0147014×10^{-1}	4.0305107×10^{-2}

shows the significant design results with $\alpha = 0.01$ and 2 for the designed IIR QMF bank and the FIR QMF bank designed by using the technique of [9], respectively. Moreover, both κ_1 and κ_2 are set to 0.01. The resulting tap and reflection coefficients for the designed IIR QMF bank are listed in Table 2. Although the number of independent coefficients used for the IIR QMF bank is only about two thirds of that used for the FIR QMF bank, we observe from Table 1 that the IIR QMF bank designed by using the proposed technique significantly outperforms the FIR QMF bank presented in [9]. Fig. 3 depicts the magnitude response and group delay, respectively, of $H_0(z)$, for the designed IIR QMF bank. Fig. 4 shows the magnitude response and the group delay deviation of $T(z)$. Finally, the corresponding variation of the designed filter bank response is also depicted in Fig. 5.

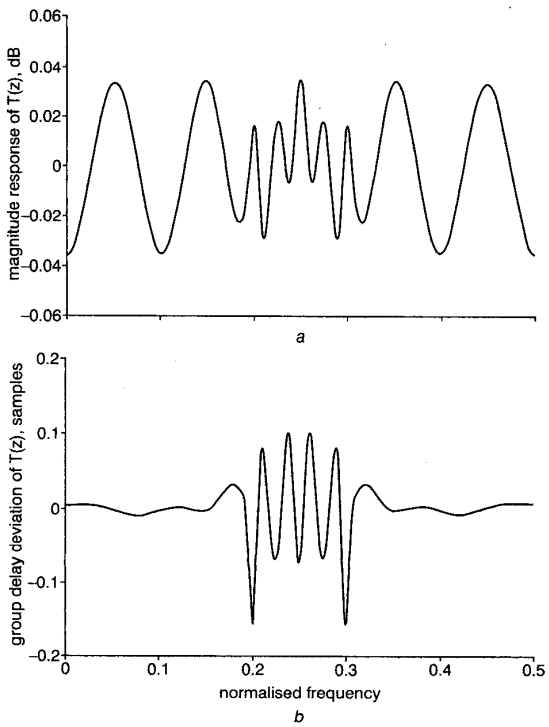

Fig. 4 *Magnitude response and group delay variation of T(z)*
a Magnitude response
b Group delay variation

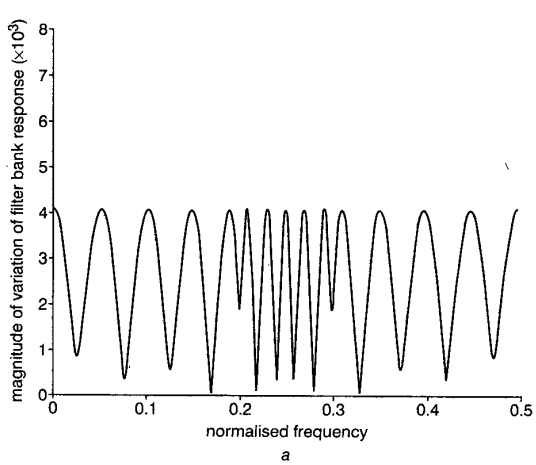

Fig. 5 *Magnitude of variation of the filter-bank response*

In fact, the above design is the example related to the optimal design of two-channel IIR QMF banks with low group delay.

6 Conclusion

This paper has presented a technique for the minimax design of two-channel IIR QMF banks with arbitrary group delay. We employ a recursive filter with a lattice denominator to be the low-pass analysis filter of the IIR QMF banks. At each iteration of the proposed technique, the core design work includes appropriately adjusting the tap and reflection coefficients for the low-pass analysis filter to reduce the resulting peak error and keep the designed low-pass analysis filter stable. For the first task, a linearisation scheme and a WLS algorithm have been utilised to solve the resulting highly nonlinear programming problem. For the second task, the stability of the designed recursive low-pass analysis filter is ensured by incorporating an efficient stabilisation procedure to make all of the reflection coefficient values fall between -1 and $+1$. Simulation results have shown that the proposed technique provides very satisfactory design results in the case of low group delay.

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